A differentiable dynamic network loading model that yields queue length distributions and accounts for spillback

Carolina Osorio, Gunnar Flötteröd, and Michel Bierlaire

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Motivation

- stochastic queue lengths
  - risk-averse route choice models
  - analysis of network breakdowns
  - ...
- specified through differentiable equations
  - linearization of network loading map
  - optimization, calibration, OD estimation
  - ...

Outline

Node model (in symmetry to the LWR model)

Link model

Experiments

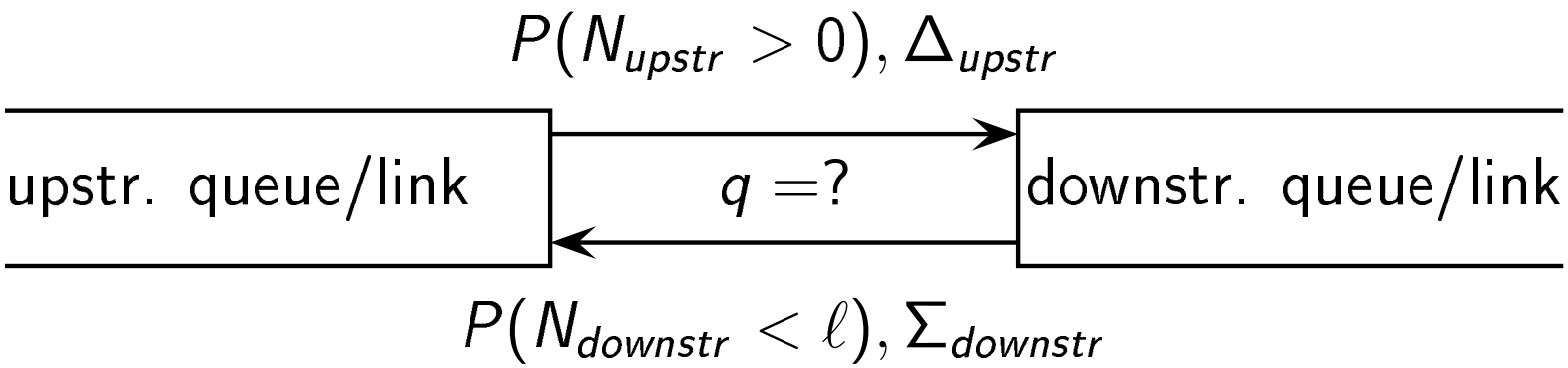
Summary, conclusion, outlook

Outline

Model entities

queueing theory	LWR model (Lebacque, '96, '05)
arrival rate λ	upstream demand Δ
service rate μ	downstream supply Σ
number of jobs N	traffic density ϱ
max. number of jobs ℓ	maximum density $\hat{\varrho}$

Boundary conditions



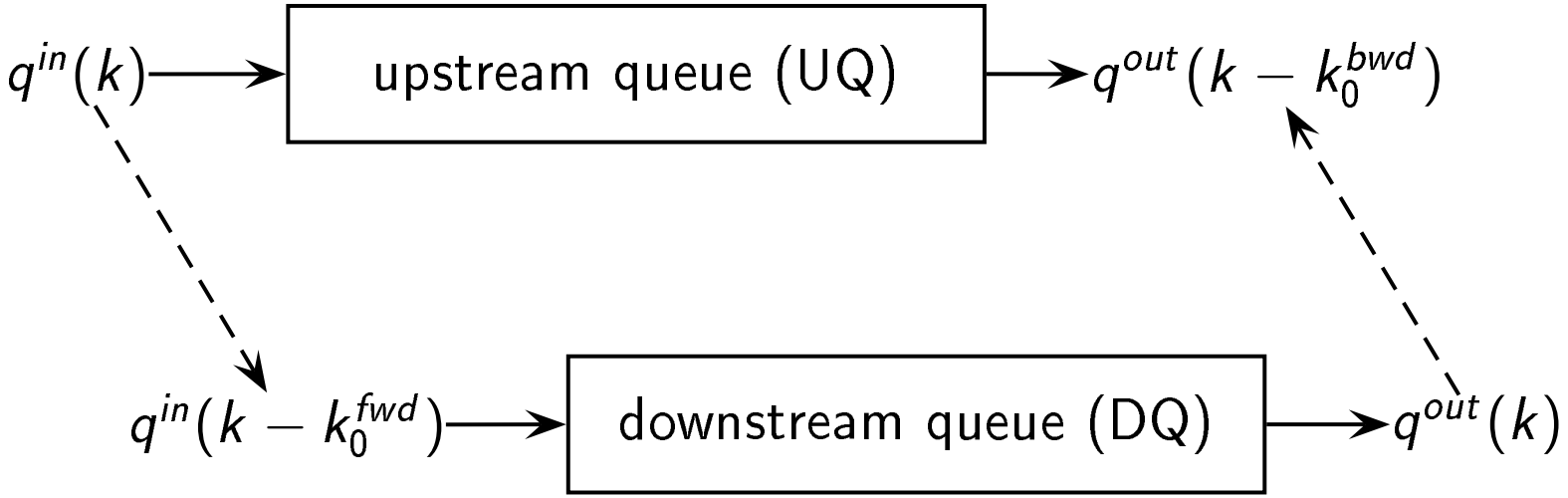
queueing theory	LWR model
waiting prob. $P(N_{upstr} > 0)$	upstr. demand Δ_{upstr}
non-blocking prob. $P(N_{downstr} < \ell)$	downstr. supply $\Sigma_{downstr}$
$q \propto P(N_{upstr} > 0)P(N_{downstr} < \ell)$	$q = \min\{\Delta_{upstr}, \Sigma_{downstr}\}$

Outline

Discrete-time network simulation

- Morse ('58) gives closed-form expression of queue evolution in continuous time for
  - constant arrival and service rate λ and μ
  - exponential arrivals and services
- network simulation, one discrete time step:
  1. update all λ and μ from all $P(N > 0)$ and $P(N < \ell)$
  2. keep λ and μ constant during time step
  3. update queue distributions, get new $P(N > 0)$, $P(N < \ell)$

One link consists of two queues

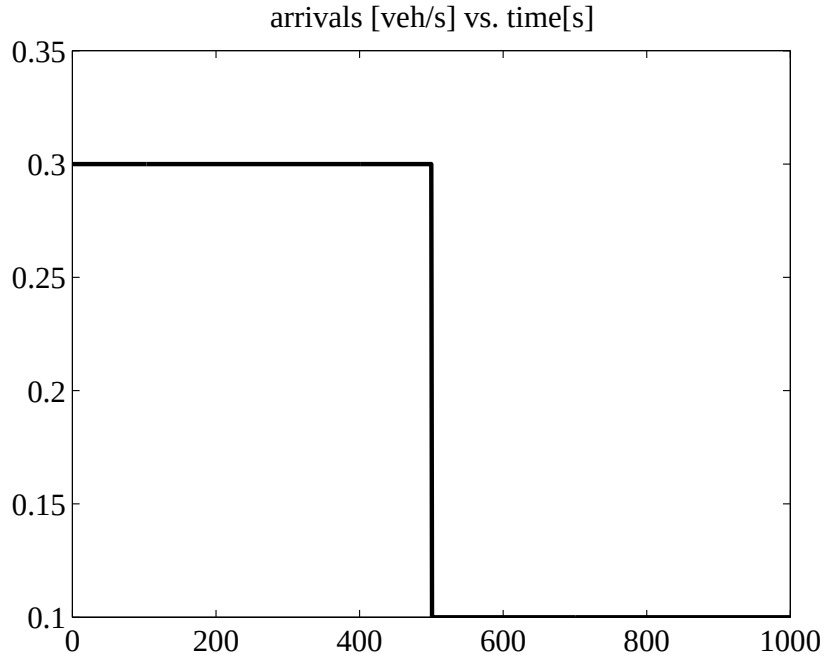


(Charypar et al., '07)

- k is discrete time index
- forward lag k_0^{fwd} generates minimum link travel time
- backward lag k_0^{bwd} generates slow queue dissipation

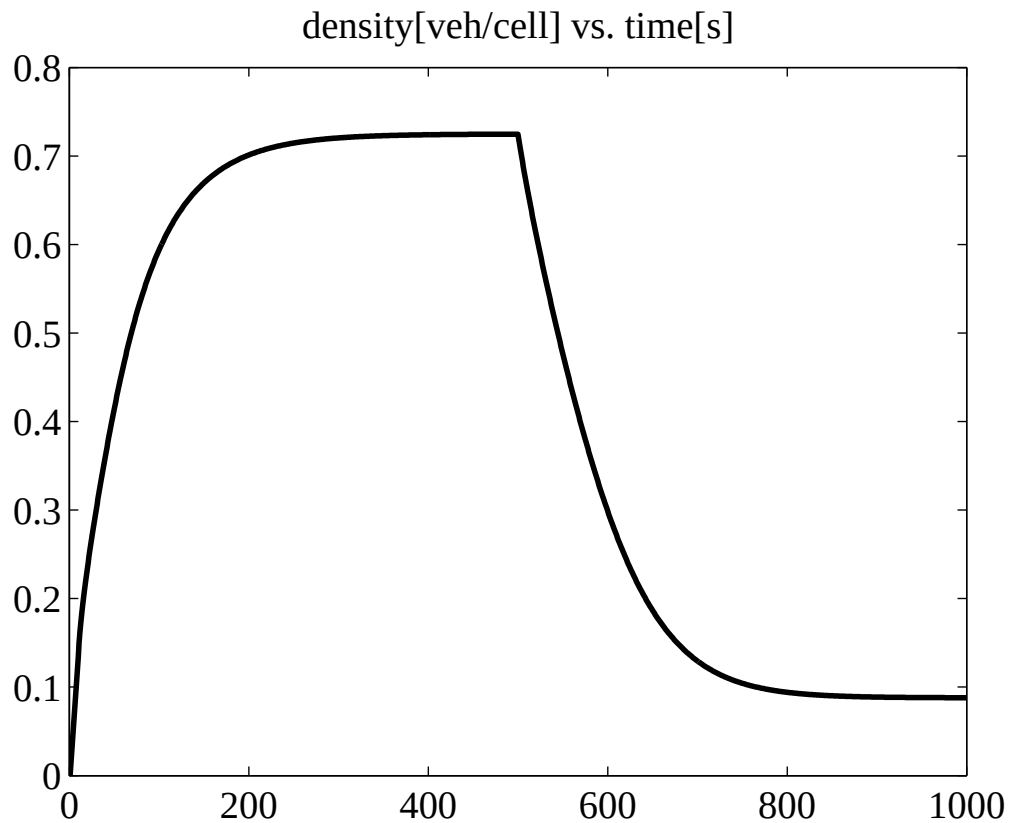
Outline

Arrival profile – $\lambda(k)$

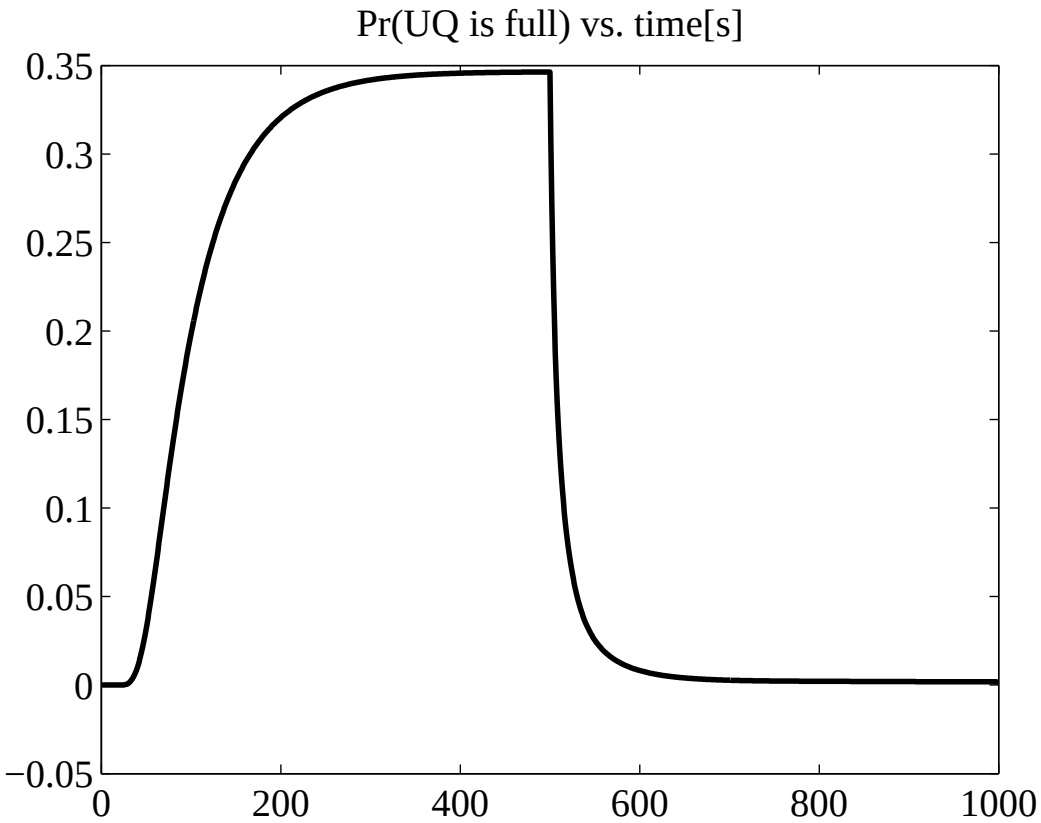


- initially empty link with 0.2 veh/s downstream bottleneck
- piecewise constant arrivals: $0.0 \longrightarrow 0.3 \longrightarrow 0.1$ veh/s

Density profile – $\varrho(k)$



Blocking probability – $P(N_{UQ} = \ell)$



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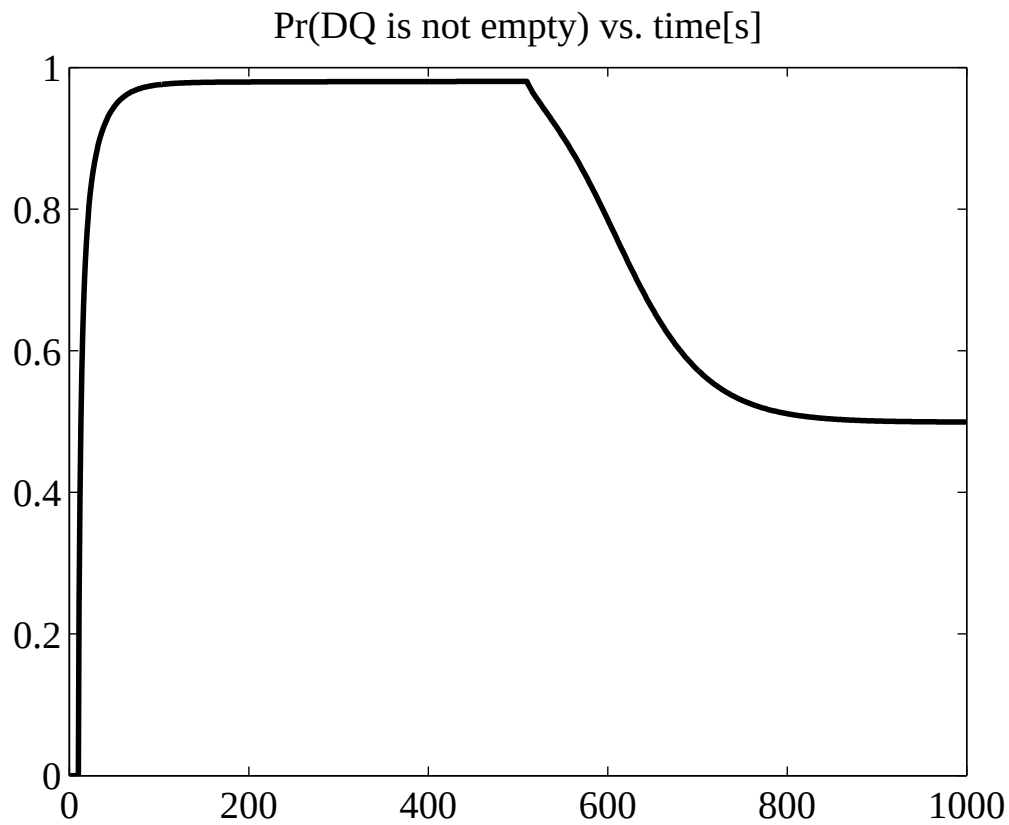
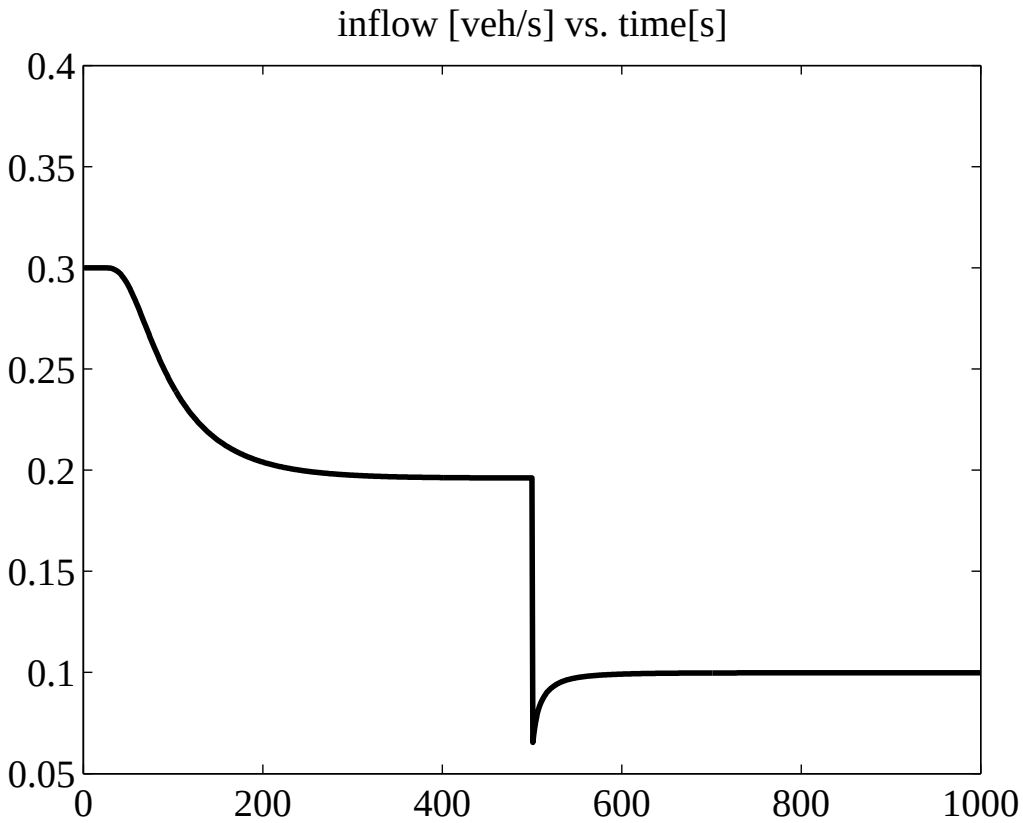
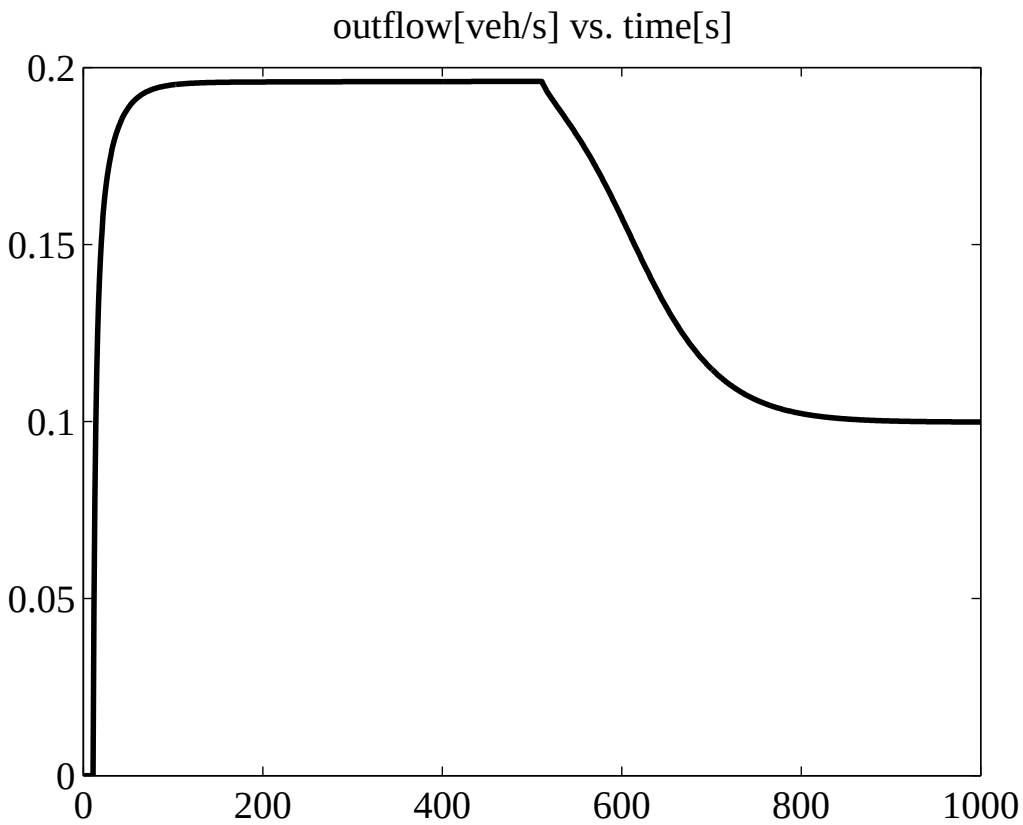
Availability probability – $P(N_{DQ} > 0)$



Inflow profile – $q^{in}(k)$



Outflow profile – $q^{out}(k)$



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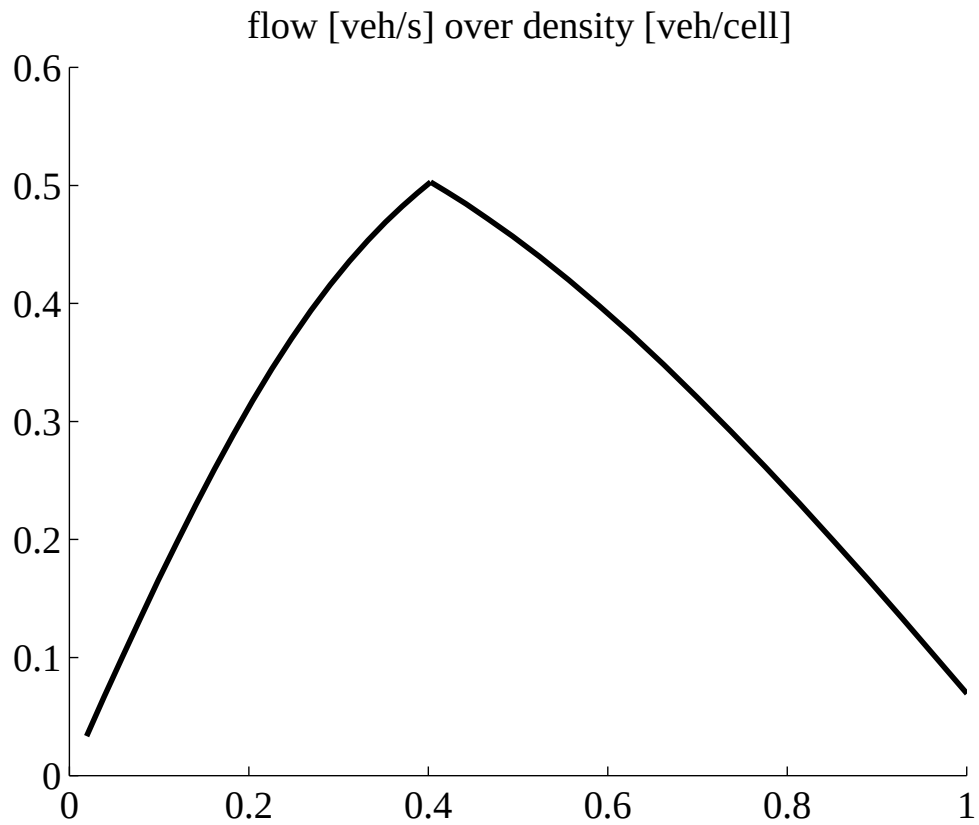
Fundamental diagram



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- dynamic network loading based on finite capacity queueing theory
  - captures queue length distributions
  - accounts for spillback
  - captures build-up and dissipation of queues
  - generates plausible fundamental diagram
- differentiability good for estimation, optimization, assignment
- ongoing work: more complex node models, network traffic