Choice set generation for iterated DTA simulations

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Introduction

- A path is a connected sequence of nodes in a network
- Concept of “path” carries over to “travel plan”
- DTA simulations: huge (path) choice sets
- Objective: efficient path sampling from arbitrary distributions
Outline

Relevance of choice set modeling

The Metropolis-Hastings algorithm

Metropolis-Hastings sampling of paths

Simple example

Tel-Aviv example

Outlook & summary
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Outlook & summary
Choice process

- choice process: decision maker $n$...
  1. considers a set $C_n$ of alternatives
  2. selects one alternative $i$ from that set
- two modeling questions:
  1. what choice set $C_n$ is considered?
  2. given $C_n$, what choice $i$ is made?
Choice set and choice

- choice in the presence of an uncertain choice set

\[ P_n(i) = \sum_{C_n \subseteq C} P_n(i|C_n)P_n(C_n) \]

- simulation: draw from \( P_n(i) \) by
  1. drawing \( C_n \) from \( P_n(C_n) \)
  2. drawing \( i \) from \( P_n(i|C_n) \)

- the choice set is decisive for the simulated choice!
Modeling of path choice sets

- difficult because real choice set is typically not observable
- two broad classes of methods
  - modeling of consideration sets
    - deterministic (e.g., K-SP) or stochastic (randomized SP)
    - unrealistic: fail to capture the chosen alternative
  - assume that decision maker considers all alternatives
    - also unrealistic
    - sampling protocol generates operational subset
    - correct for sampling in the estimation
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How to sample from large (path) choice sets?

• approach
  – give every path \( i \in C \) a weight \( b(i) > 0 \)
  – sampling probability \( q(i) \) shall be \( \propto b(i) \)

• direct sampling from \( q(i) \) requires path enumeration

\[
q(i) = \frac{b(i)}{\sum_{j \in C} b(j)}
\]

• but pair-wise comparison of paths is easily done

\[
\frac{q(i)}{q(j)} = \frac{b(i)}{b(j)}
\]
Metropolis-Hastings (MH) algorithm

1. set iteration counter \( k = 0 \)
2. select arbitrary initial state \( i^k \)
3. repeat beyond stationarity
   3.1 draw candidate state \( j \) from proposal distribution \( q(i^k, j) \)
   3.2 compute acceptance probability
   \[
   \alpha(i^k, j) = \min \left( \frac{b(j)q(j, i^k)}{b(i^k)q(i^k, j)}, 1 \right)
   \]
   3.3 with probability \( \alpha(i^k, j) \), let \( i^{k+1} = j \); else, let \( i^{k+1} = i^k \)
   3.4 increase \( k \) by one
Convergence of MH algorithm

• given
  – a finite state space
  – positive weights $b(i)$
  – an irreducible\textsuperscript{1} proposal distribution $q(i, j)$

MH converges to stationary distribution\textsuperscript{2} $b(i)/\sum_j b(j)$

• proposal distribution $q(i, j)$ crucial for convergence speed
  – too little variability: slow convergence
  – too much variability: random search

\textsuperscript{1} every state can (eventually) reach every other state
\textsuperscript{2} long-term state coverage of the process
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State space

• a state $i = (\Gamma, a, b, c)$ consists of
  – a path $\Gamma$
  – three node indices $a < b < c$ within that path

• node indices simplify computation of transition probabilities
Weights

- intuitive: weight $\exp[-\mu \delta(\Gamma)]$ with path cost $\delta(\Gamma)$ and $\mu \geq 0$
- there are $|\Gamma|(|\Gamma| - 1)(|\Gamma| - 2)/6$ states with the same $\Gamma$
- corrected weights:

$$b(i) = \frac{\exp[-\mu \delta(\Gamma)]}{|\Gamma|(|\Gamma| - 1)(|\Gamma| - 2)/6}$$
Proposal distribution

- SHUFFLE operation
  - re-sample (uniformly) $a < b < c$ within path $\Gamma$

- SPLICE operation
  - sample a node $v$ “near” the path segment $\Gamma(a) \ldots \Gamma(c)$
  - connect $\Gamma(a)$ to $v$
  - connect $v$ to $\Gamma(c)$
  - let new $b$ point at $v$, update $c$

- combined proposal: randomly select one procedure

- [[complicated computation of proposal probabilities]]
SPLICE example

A \rightarrow B \rightarrow C \rightarrow D \rightarrow E

\begin{align*}
\text{origin} & \quad a = 2 \quad b = 3 \\
\text{destination} & \quad c = 4 
\end{align*}
SPLICE example

A \rightarrow B \rightarrow C \rightarrow D \rightarrow E

a = 2 \quad b = 3 \quad c = 4

origin \quad destination
SPLICE example

Graph:

- Nodes: A, B, C, D, E, F, G
- Edges:
  - A → B (a = 2)
  - B → C (b = 3)
  - D → E
  - G → D
  - F → G

- Parameters:
  - a = 2
  - b = 3
  - c = 4

Origin: A
Destination: E
SPLICE example

A → B → C → D → E

\(a' = 2\)
\(a = 2\)
\(b = 3\)

\(b' = 4\)

\(c' = 5\)
\(c = 4\)
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Simple example
Simple example: correlation within the chain

(a) $\mu = 0.0$  (b) $\mu = 2.0$  (c) $\mu = 4.0$

- for independent draws, extract every 2500th path
Simple example: scatterplots

\( \chi^2 \) test does not reject hypothesis: sample from target distr.
- test statistics: 198.93, 177.29, 157.69 for \( \mu = 0, 2, 4 \)
- 0.5, 0.9, 0.95 quantiles: 168.33, 192.95, and 200.33
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Tel-Aviv example: network
Tel-Aviv example: within-chain correlation

(a) $\mu = 0.01$  \hspace{1cm} (b) $\mu = 0.02$  \hspace{1cm} (c) $\mu = 0.04$

- for independent draws, extract every 10 000th route
Tel-Aviv example: length distribution

Squares: $\mu = 0.01$, circles: $\mu = 0.02$, triangles: $\mu = 0.04$. 
Computational performance

• performance is quite problem specific
  – narrow target distribution → faster
  – origin/destination nearby → faster
  – small overall network (or preprocessing) → faster
  – missing some routes uncritical → faster
  – simple proposal distribution → probably faster

• Tel-Aviv example: order of $10^3 \ldots 10^4$ iterations per minute
• main bottleneck: many shortest path tree computations
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Outlook & summary
Outlook: choice set formation in the simulation loop

travel behavior

travel demand
- route choice
- dpt. time choice
- mode choice
- ...

network supply
- traffic flow
- congestion, delay
- reliability
- ...

network conditions
Outlook: choice set formation in the simulation loop

travel demand

- choice set
- choice

travel behavior

network supply
- traffic flow
- congestion, delay
- reliability
- ...

network conditions
Summary

• Metropolis-Hastings sampling of paths
  – generalizes to all-day travel plans
  – well-specified choice set distributions

• operational implementation
  – upcoming re-implementation of BIOROUTE
  – quite efficient (but can be tuned further)

• consistency with iterated DTA simulations
  – iterated simulation constitutes Markov chain
  – so does Metropolis-Hastings algorithm
  – specification of one joint chain