Optimization of Uncertainty Features for Transportation Problems

Niklaus Eggenberg
Matteo Salani and Prof. M. Bierlaire

Transport and Mobility Laboratory, EPFL, Switzerland

STRC, Monte-Verità 2008
Outline

- Optimization under Uncertainty: Existing Methods
- Uncertainty Feature Optimization (UFO)
- UFO: generalized framework
- Example: Multi-Dimensional Knapsack Problem
- Simulation Results for MDPK
- Future Work and Conclusions
Optimization with Noisy Data
Typical Examples

- Portfolio Optimization
- Vehicle Routing (GPS, transport problems, ...)
- Project Management
- Many others!
Four Approaches

1. Neglect and solve deterministic problem

- Not realistic (Herroelen 2005, Sahinidis 2004)
Four Approaches

1. Neglect and solve deterministic problem

2. On-line Optimization
   - Data-driven
   - Not feasible for some problems (e.g. airline schedules)
Existing Methods III

Four Approaches

1. Neglect and solve deterministic problem

2. On-line Optimization

3. Characterize the Uncertainty and solve robust or stochastic problems
   - Need explicit Uncertainty characterization
   - Hard to characterize/model in general
   - Leads to difficult problems
   - Sensitive to uncertainty characterization
   - Solutions tend to “simple” properties
Examples from Airline Scheduling

- Increase plane’s idle time (Al-Fawzana & Haouari 2005)
- Decrease plane rotation length (Rosenberger et al. 2004)
- Departure de-peaking (Jiang 2006, Frank et al. 2005)
- More plane crossings (Bian et al. 2004, Klabjan et al. 2002)
- ...
Four Approaches

1. Neglect and solve deterministic problem
2. On-line Scheduling
3. Characterize the Uncertainty
4. Model Uncertainty Implicitly => **Uncertainty Features**
Uncertainty Feature Optimization

I. Increase robustness/stability (e.g. idle time)

II. Increase recoverability (e.g. plane crossings)
UF: Definition

Given a problem with Decision Variables $x$

**UF**: a function $\mu(x)$ measuring the “quality” of a solution $x$

**OBJECTIVE**: $\text{MAX } \mu(x)$

s.t. $x$ feasible solution to initial problem
General Optimization Problem

\[ \text{MIN } f(x) \]
\[ s.t. \quad a(x) \leq b \]
\[ x \in X \]
UF and Optimality Budget

\[ \mu(x) \] Uncertainty Feature

\[ f^* \] Original Optimum

\[ \rho \] Maximal Optimality Gap
UFO: Multi-Objective Problem

\[
\text{OPT} \ [f(x), \mu(x)] \\
\text{s.t.} \quad a(x) \leq b \\
x \in X
\]
UFO with Budget Relaxation

\[ \max \mu(x) \]

\[ \text{s.t.} \quad a(x) \leq b \]

\[ f(x) \leq (1 + \rho)f^* \]

\[ x \in X \]
UFO Properties

I. Complexity not changed if $\mu(x)$ similar to $f(x)$
II. Implicit modeling of uncertainty
III. Differentiate solutions on optimal facet
IV. “Plug” tool for any existing method
V. Can use UF based on explicit uncertainty set
VI. Generalizes existing methods
Stochastic Problem as an **UFO**

Given an Uncertainty Set $U$ with a probability measure on it

$$\min \ E_U\{f(x)\}$$

s.t. $a(x) \leq b$

$x \in X$
Stochastic Problem as an **UFO**

\[ \text{MAX } \mu(x) = -E_U\{f(x)\} \]

\[ \text{s.t. } a(x) \leq b \]

\[ f(x) \leq (1 + \infty)f^* \]

\[ x \in X \]
Robust Optimization
(Bertsimas & Sim 2004)

• Solving Linear Problems with noisy data

• Solution is feasible in the worst case

• Worst case parametrized and solution-dependent
BONUS

• Methodology to compute maximal values for the parameters to ensure a robust solution exists

• Similar to Fischetti & Monaci, 2008 in this context
Multi-Dimensional Knapsack Problem

\[
\begin{align*}
\text{max} & \quad \sum_{i=1}^{N} p_i x_i \\
\text{s.t.} & \quad \sum_{j=1}^{N} a_{ij} x_i \leq b_i \quad \forall i = 1, \ldots, M \\
& \quad x \in \mathbb{Z}_+^N
\end{align*}
\]
MDKP with Max Taken Object UFO

\[
\begin{align*}
\min & \quad \left\{ \mu(x) = \max_{i=1,\ldots,N} \{x_i\} \right\} \\
\text{s.t.} & \quad Ax \leq b \\
& \quad p^T x \geq (1 - \rho)p^* \\
& \quad x \in \mathbb{Z}^N_+
\end{align*}
\]
Other derived UF

• Max Taken (MTk): \( \mu(x) = \max_{i=1,...,N} \{x_i\} \)

• Diversification (Div): \( \mu(x) = \sum (\min \{1, x_i\}) \)

• Impact Ratio (IR): \( \mu(x) = -\max_i \frac{a_{ij}x_j}{b_i} \)

• 2Sum: \( \mu(x) = -\max_{i,j \neq k} \frac{a_{ij}x_j + a_{ik}x_k}{b_i} \)
Instances with 50 objects

- 1, 5 or 10 constraints
- Profit-weight correlation or not
- Marginal Profit Distribution: clustered, normal, wide
- Deviation Matrix $\hat{A}$ proportional to $A$ (0.2, 0.5, 0.8)
- Maximal varying coefficients: 2 or 50

IN TOTAL: 3240 Instances
Described by $p$, $b$, $A$ and $\hat{A}$
Simulation

A scenario is characterized by its realized constraint matrix $\tilde{A}$:

- $\tilde{A} : \tilde{A} \sim \rho \tilde{A}$ matrix ($\rho = 0.75, 1.0$)
- $A : \tilde{A} \sim \rho A$ matrix ($\rho = 0.1, 0.2, 0.5$)
- $R : \tilde{A}$ randomly with average coefficient $\tilde{a}_{ij} = 10, 20, 30$

5 scenarios per instance $\Rightarrow 129'600$ scenarios
Comparison Criteria

- normalized UF value (max is always 1.0)
- # unfeasible scenarios (and percentage)
- Optimality gap to scenario’s optimal solution
- Maximal number of violated constraints
MDKP Package

• Generation of problems
• Solve Models inc. Robust (combining possible)
• Simulation with user-defined parameters

Planned to be online soon.

TESTERS ARE WELKOME!!!
Different Simulations for clustered profit-correlated instances with 10 constraints

<table>
<thead>
<tr>
<th></th>
<th>Det</th>
<th>Rob_Å</th>
<th>Rob_A_0.1</th>
<th>MTk_0.2</th>
<th>Div_0.1</th>
<th>IR_0.3</th>
<th>2Sum_0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>A 75, 100</td>
<td></td>
<td></td>
<td></td>
<td>0.974</td>
<td>0.610</td>
<td>0.962</td>
<td>0.932</td>
</tr>
<tr>
<td>(1800 Scen.)</td>
<td>1642</td>
<td>166</td>
<td>1014</td>
<td>914</td>
<td>1199</td>
<td>85</td>
<td>1174</td>
</tr>
<tr>
<td></td>
<td>91.22</td>
<td>9.22</td>
<td>56.33</td>
<td>50.78</td>
<td>66.61</td>
<td>4.72</td>
<td>65.22</td>
</tr>
<tr>
<td></td>
<td>0.56</td>
<td>20.93</td>
<td>4.72</td>
<td>10.53</td>
<td>5.29</td>
<td>31.04</td>
<td>5.56</td>
</tr>
<tr>
<td></td>
<td>25.21</td>
<td>68.36</td>
<td>38.91</td>
<td>49.81</td>
<td>50.47</td>
<td>59.53</td>
<td>41.99</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>3</td>
<td>7</td>
<td>4</td>
<td>5</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A 10, 25, 50</td>
<td></td>
<td></td>
<td></td>
<td>0.974</td>
<td>0.610</td>
<td>0.962</td>
<td>0.932</td>
</tr>
<tr>
<td>(2700 Scen.)</td>
<td>2404</td>
<td>489</td>
<td>1232</td>
<td>1141</td>
<td>1544</td>
<td>76</td>
<td>1566</td>
</tr>
<tr>
<td></td>
<td>89.04</td>
<td>18.11</td>
<td>45.63</td>
<td>42.26</td>
<td>57.19</td>
<td>2.81</td>
<td>58.00</td>
</tr>
<tr>
<td></td>
<td>0.56</td>
<td>18.13</td>
<td>5.09</td>
<td>11.38</td>
<td>6.03</td>
<td>30.04</td>
<td>6.03</td>
</tr>
<tr>
<td></td>
<td>34.03</td>
<td>57.07</td>
<td>40.47</td>
<td>47.12</td>
<td>40.39</td>
<td>52.16</td>
<td>40.19</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>6</td>
<td>7</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R 10, 20, 30</td>
<td></td>
<td></td>
<td></td>
<td>0.974</td>
<td>0.610</td>
<td>0.962</td>
<td>0.932</td>
</tr>
<tr>
<td>(2700 Scen.)</td>
<td>2616</td>
<td>1079</td>
<td>2100</td>
<td>1506</td>
<td>1974</td>
<td>171</td>
<td>1962</td>
</tr>
<tr>
<td></td>
<td>96.89</td>
<td>39.96</td>
<td>77.78</td>
<td>55.78</td>
<td>73.11</td>
<td>6.33</td>
<td>72.67</td>
</tr>
<tr>
<td></td>
<td>0.45</td>
<td>17.32</td>
<td>3.36</td>
<td>11.24</td>
<td>5.36</td>
<td>33.33</td>
<td>5.53</td>
</tr>
<tr>
<td></td>
<td>46.67</td>
<td>62.71</td>
<td>51.87</td>
<td>57.25</td>
<td>51.81</td>
<td>61.33</td>
<td>51.65</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>7</td>
<td>8</td>
<td>4</td>
<td>6</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>
### MDKP – Results IV

**Performance evolution for increasing budget $\rho$ (same instances)**

<table>
<thead>
<tr>
<th></th>
<th>2Sum_0.1</th>
<th>2Sum_0.2</th>
<th>2Sum_0.3</th>
<th>IR_0.1</th>
<th>IR_0.2</th>
<th>IR_0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A 75, 100</strong> (1800 Scen.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UF value</td>
<td>0.932</td>
<td>0.958</td>
<td>0.962</td>
<td>0.932</td>
<td>0.958</td>
<td>0.962</td>
</tr>
<tr>
<td># Infeas.</td>
<td>1174</td>
<td>528</td>
<td>90</td>
<td>1220</td>
<td>528</td>
<td>85</td>
</tr>
<tr>
<td>Infeas [%]</td>
<td>65.22</td>
<td>29.33</td>
<td>5.00</td>
<td>67.78</td>
<td>29.33</td>
<td>4.72</td>
</tr>
<tr>
<td>Max Opt Gap [%]</td>
<td>41.99</td>
<td>52.36</td>
<td>59.53</td>
<td>42.18</td>
<td>48.58</td>
<td>59.53</td>
</tr>
<tr>
<td>Max # Violated</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>5</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>2Sum_0.1</th>
<th>2Sum_0.2</th>
<th>2Sum_0.3</th>
<th>IR_0.1</th>
<th>IR_0.2</th>
<th>IR_0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A 10, 25, 50</strong> (2700 Scen.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UF value</td>
<td>0.932</td>
<td>0.958</td>
<td>0.962</td>
<td>0.932</td>
<td>0.958</td>
<td>0.962</td>
</tr>
<tr>
<td># Infeas.</td>
<td>1566</td>
<td>579</td>
<td>84</td>
<td>1671</td>
<td>592</td>
<td>76</td>
</tr>
<tr>
<td>Infeas [%]</td>
<td>58.00</td>
<td>21.44</td>
<td>3.11</td>
<td>61.89</td>
<td>21.93</td>
<td>2.81</td>
</tr>
<tr>
<td>Avg Opt Gap [%]</td>
<td>6.03</td>
<td>16.85</td>
<td>29.57</td>
<td>5.4</td>
<td>16.83</td>
<td>30.04</td>
</tr>
<tr>
<td>Max Opt Gap [%]</td>
<td>40.19</td>
<td>46.95</td>
<td>46.75</td>
<td>40.39</td>
<td>46.99</td>
<td>52.16</td>
</tr>
<tr>
<td>Max # Violated</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>2Sum_0.1</th>
<th>2Sum_0.2</th>
<th>2Sum_0.3</th>
<th>IR_0.1</th>
<th>IR_0.2</th>
<th>IR_0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>R 10, 20, 30</strong> (2700 Scen.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UF value</td>
<td>0.932</td>
<td>0.958</td>
<td>0.962</td>
<td>0.932</td>
<td>0.958</td>
<td>0.962</td>
</tr>
<tr>
<td># Infeas.</td>
<td>1962</td>
<td>997</td>
<td>174</td>
<td>2001</td>
<td>996</td>
<td>171</td>
</tr>
<tr>
<td>Infeas [%]</td>
<td>72.67</td>
<td>36.93</td>
<td>6.44</td>
<td>74.11</td>
<td>36.89</td>
<td>6.33</td>
</tr>
<tr>
<td>Avg Opt Gap [%]</td>
<td>5.53</td>
<td>16.73</td>
<td>32.93</td>
<td>5.33</td>
<td>16.81</td>
<td>33.33</td>
</tr>
<tr>
<td>Max Opt Gap [%]</td>
<td>51.65</td>
<td>57.11</td>
<td>60.71</td>
<td>51.81</td>
<td>57.15</td>
<td>61.33</td>
</tr>
<tr>
<td>Max # Violated</td>
<td>6</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>
Performance for combined (normalized) objectives

<table>
<thead>
<tr>
<th></th>
<th>MTK_2Sum_0.3</th>
<th>_Rob_A_0.1</th>
<th>Rob_A_0.1</th>
<th>Rob_A_0.2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Å 75, 100</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>UF value</td>
<td>0.969</td>
<td>1.475</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td># Infeas.</td>
<td>102</td>
<td>514</td>
<td>1014</td>
</tr>
<tr>
<td></td>
<td>Infeas [%]</td>
<td>5.67</td>
<td>28.56</td>
<td>56.33</td>
</tr>
<tr>
<td></td>
<td>Avg Opt Gap [%]</td>
<td>33.02</td>
<td>16.47</td>
<td>4.72</td>
</tr>
<tr>
<td></td>
<td>Max Opt Gap [%]</td>
<td>80.03</td>
<td>53.41</td>
<td>38.91</td>
</tr>
<tr>
<td></td>
<td>Max # Violated</td>
<td>2</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td><strong>Å 10, 25, 50</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>UF value</td>
<td>0.969</td>
<td>1.475</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td># Infeas.</td>
<td>111</td>
<td>542</td>
<td>1232</td>
</tr>
<tr>
<td></td>
<td>Infeas [%]</td>
<td>4.11</td>
<td>20.07</td>
<td>45.63</td>
</tr>
<tr>
<td></td>
<td>Avg Opt Gap [%]</td>
<td>31.90</td>
<td>16.93</td>
<td>5.09</td>
</tr>
<tr>
<td></td>
<td>Max Opt Gap [%]</td>
<td>76.44</td>
<td>51.97</td>
<td>40.47</td>
</tr>
<tr>
<td></td>
<td>Max # Violated</td>
<td>2</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td><strong>R 10, 20, 30</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>UF value</td>
<td>0.969</td>
<td>1.475</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td># Infeas.</td>
<td>209</td>
<td>969</td>
<td>2100</td>
</tr>
<tr>
<td></td>
<td>Infeas [%]</td>
<td>7.74</td>
<td>35.89</td>
<td>77.78</td>
</tr>
<tr>
<td></td>
<td>Avg Opt Gap [%]</td>
<td>34.94</td>
<td>17.16</td>
<td>3.36</td>
</tr>
<tr>
<td></td>
<td>Max Opt Gap [%]</td>
<td>80.20</td>
<td>61.17</td>
<td>51.87</td>
</tr>
<tr>
<td></td>
<td>Max # Violated</td>
<td>3</td>
<td>5</td>
<td>8</td>
</tr>
</tbody>
</table>
Aggregated Results

Number of constraints matters

Feasibility failure for the deterministic model
1 constraint  37%
5 constraints  84%
10 constraints  91%
Aggregated Results

Clustered M.P. Distribution works best for UFs

Feasibility failure for the IR_0.3 model
Clustered degeneration       29%
Normal degeneration          55%
Wide degeneration            63%

Robust less sensitive to degeneration & correlation
Aggregated Results

- UFO less sensitive to change in noise & number constraints
- Robust sensitive to noise change
- Budget is a decent optimality loss estimator
Future Work

• Application of UFO to Airline Transportation

• Find an UF generator?
Conclusions

• UFO allows to cope with uncertainty IMPLICITLY

• Using explicit uncertainty model is still possible

• UFO can be combined with any already existing method

• It is not sensitive to erroneous noise characterization
THANKS for your attention

Any Questions?
Robust problem as an **UFO**

Original LP Problem

\[
\text{MAX } c^T x \\
\text{s.t. } Ax \leq b \\
x \in X
\]
Robust problem as an **UFO**

Formulation of Bertsimas and Sim (2004)

\[
\begin{align*}
\text{MAX } & \quad c^T x \\
\text{s.t. } & \quad Ax + \beta(x, \Gamma) \leq b \\
& \quad x \in X
\end{align*}
\]
\[ \tilde{a}_{ij} \in [a_{ij} - \hat{a}_{ij}; a_{ij} + \hat{a}_{ij}] \quad \forall \ j \in J_i \]

\[ \beta_i(\chi, \Gamma_i) = \max_{\{S_i \cup \{t_i\}|S_i \in J_i, |S_i| = |\Gamma_i|, t_i \in J_i \setminus S_i\}} \]

\[ \left\{ \sum_{j \in S_i} \hat{a}_{ij} |\chi_j| + (\Gamma_i - |\Gamma_i|)\hat{a}_{it_i} |\chi_{t_i}| \right\} \]
Start with Feasibility Problem

$$f^* = \text{MIN} \ f(x)$$

$$= \text{MIN} \ [Ax + \beta(x, J)] - b$$

s.t. \ \ x \in X$$
Define ** UF and budget**

\[
\mu(x) = c^T x \quad \quad \rho = \max_i \left\{ \frac{\rho_i f_i(x^*)}{f^*} - 1 \right\}
\]

Where

\[
\rho_i = \begin{cases} 
\frac{\beta_i(x, \Gamma_i)}{f_i(x^*)} & \text{if } f_i(x^*) \neq 0 \\
0 & \text{if } f_i(x^*) = 0
\end{cases}
\]

and

\[
\beta(x, J) = \beta(x, \Gamma) + \bar{\beta}(x, \Gamma)
\]
**UFO formulation**

\[ \text{MAX } \mu(x) = c^T x \]

s.t. \[ [Ax + \beta(x, J)] - b \leq (1 + \rho)f^* \]

\[ x \in X \]
Replace Elements in Constraint

\[ [Ax + \beta(x, J)] - b \leq (1 + \rho)f^* \]

\[ = \]

\[ [Ax + \beta(x, J)] - b \leq \beta(x, \Gamma) \]

Which is equivalent to

\[ Ax + \beta(x, J) - \bar{\beta}(x, \Gamma) \leq b \]

\[ = \]

\[ Ax + \beta(x, \Gamma) \leq b \]
Retrieve Robust Formulation

\[ \text{MAX } \mu(x) = c^T x \]

s.t. \hspace{1cm} Ax + \beta(x, \Gamma) \leq b

\[ x \in X \]

Q.E.D.