Advances in route choice modelling

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Route choice model

Given

- a mono- or multi-modal transportation network (nodes, links, origin, destination)
- an origin-destination pair
- link and path attributes

identify the route that a traveler would select.

Choice model

Assumptions about

1. the decision-maker: n
2. the alternatives
   - Choice set $\mathcal{C}_n$
   - $p \in \mathcal{C}_n$ is composed of a list of links (i,j)
3. the attributes
   - link-additive: length, travel time, etc.

$$x_{kp} = \sum_{(i,j)\in P} x_{k(i,j)}$$

   - non link-additive: scenic path, usual path, etc.
4. the decision-rules: $\Pr(p|\mathcal{C}_n)$

Shortest path

Decision-makers all identical

Alternatives

- all paths between O and D
- $\mathcal{C}_n = \mathcal{U} \quad \forall n$
- $\mathcal{U}$ can be unbounded when loops are present

Attributes one link additive generalized cost

$$c_p = \sum_{(i,j) \in P} c_{(i,j)}$$

- traveler independent
- link cost may be negative
- no loop with negative cost must be present so that $c_p > -\infty$ for all p

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Shortest path

Decision-rules path with the minimum cost is selected

$$\Pr(p) = \begin{cases} K & \text{if } c_p \leq c_q \quad \forall c_q \in \mathcal{U} \\ 0 & \text{otherwise} \end{cases}$$

- K is a normalizing constant so that $\sum_{p \in \mathcal{U}} \Pr(p) = 1$.
- $K = 1/S$, where S is the number of shortest paths between O and D.
- Some methods select one shortest path p^*

$$\Pr(p) = \begin{cases} 1 & \text{if } p = p^* \\ 0 & \text{otherwise} \end{cases}$$

Shortest path

Advantages:

- well defined
- no need for behavioral data
- efficient algorithms (Dijkstra)

Disadvantages

- behaviorally unrealistic
- instability with respect to variations in cost
- calibration on real data is very difficult
  - inverse shortest path problem is NP complete
  - Burton, Pulleyblank and Toint (1997) The Inverse Shortest Paths Problem With Upper Bounds on Shortest Paths Costs *Network Optimization* , Series: Lecture Notes in Economics and Mathematical Systems , Vol. 450, P. M. Pardalos, D. W. Hearn and W. W. Hager (Eds.), pp. 156-171, Springer

Dial's approach

Dial R. B. (1971) A probabilistic multipath traffic assignment model which obviates path enumeration *Transportation Research* Vol. 5, pp. 83-111.

Decision-makers all identical

Alternatives efficient paths between O and D

Attributes link-additive generalized cost

Decision-rules multinomial logit model

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Dial's approach

- Def 1: A path is efficient if every link in it has
  - its initial node closer to the origin than its final node, and
  - its final node closer to the destination than its initial node.
- Def 2: A path is efficient if every link in it has its initial node closer to the origin than its final node.

Efficient path: a path that does not backtrack.

Dial's approach

- Choice set $\mathcal{C}_n$ = set of efficient paths (finite, no loop)
- No explicit enumeration
- Every efficient path has a non zero probability to be selected
- Probability to select a path

$$\Pr(p) = \frac{e^{\theta(\sum_{(i,j)\in p^*} c(i,j) - \sum_{(i,j)\in p} c(i,j))}}{\sum_{q\in\mathcal{C}_n} e^{\theta(\sum_{(i,j)\in p^*} c(i,j) - \sum_{(i,j)\in p} q(i,j))}}$$

where p^* is the shortest path and θ is a parameter

Dial's approach

Note: the length of the shortest path is constant across $\mathcal{C}_n$

$$\Pr(p) = \frac{e^{-\theta \sum_{(i,j)\in p} c(i,j))}}{\sum_{q\in\mathcal{C}_n} e^{-\theta \sum_{(i,j)\in q} q(i,j))}} = \frac{e^{-\theta c_p}}{\sum_{q\in\mathcal{C}_n} e^{-\theta c_q}}$$

Multinomial logit model with

$$V_p = -\theta c_p$$

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Dial's approach

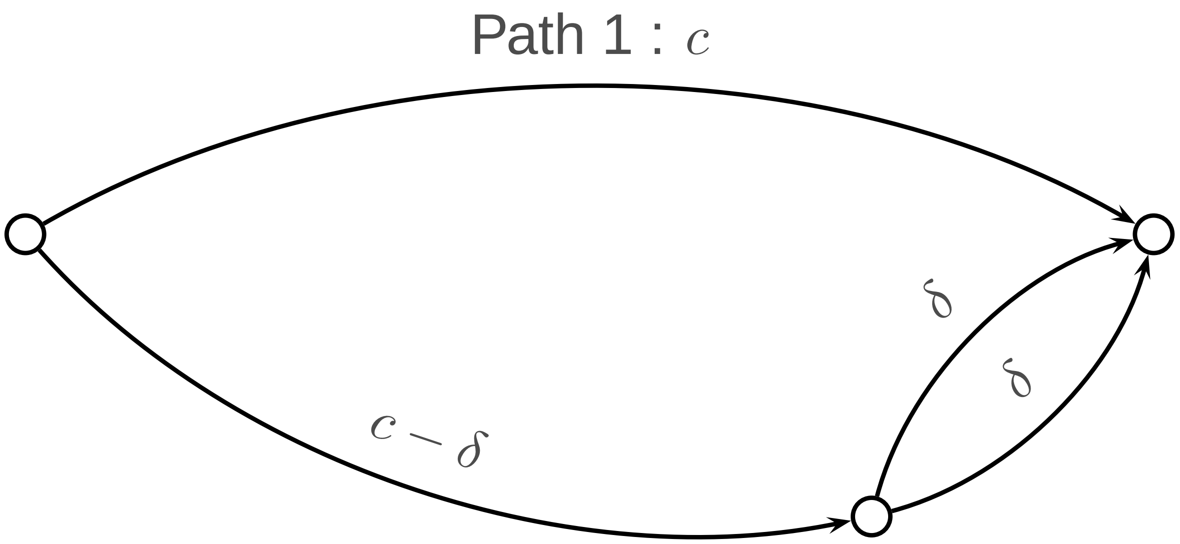
Advantages:

- probabilistic model, more stable
- calibration parameter θ
- avoid path enumeration
- designed for traffic assignment

Disadvantages:

- MNL assumes independence among alternatives
- efficient paths are mathematically convenient but not behaviorally motivated

Dial's approach



$$\Pr(1) = \frac{e^{-\theta c_1}}{\sum_{q \in \mathcal{C}} e^{-\theta c_q}} = \frac{e^{-\theta c}}{3e^{-\theta c}} = \frac{1}{3} \text{ for any } c, \delta, \theta$$

Path Size Logit

- With MNL, the utility of overlapping paths is overestimated
- When δ is large, there is some sort of “double counting”
- Idea: include a correction

$$V_p = -\theta c_p + \beta \ln \text{PS}_p$$

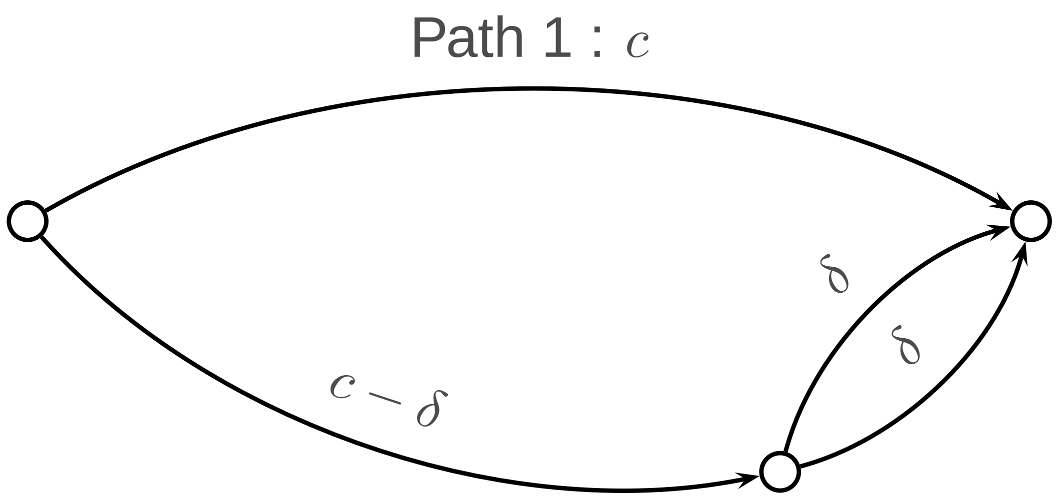
where

$$\text{PS}_p = \sum_{(i,j)\in p} \frac{c_{(i,j)}}{c_p} \frac{1}{\sum_{q\in\mathcal{C}} \delta^q_{i,j}}$$

and

$$\delta^q_{i,j} = \begin{cases} 1 & \text{if link } (i,j) \text{ belongs to path } q \\ 0 & \text{otherwise} \end{cases}$$

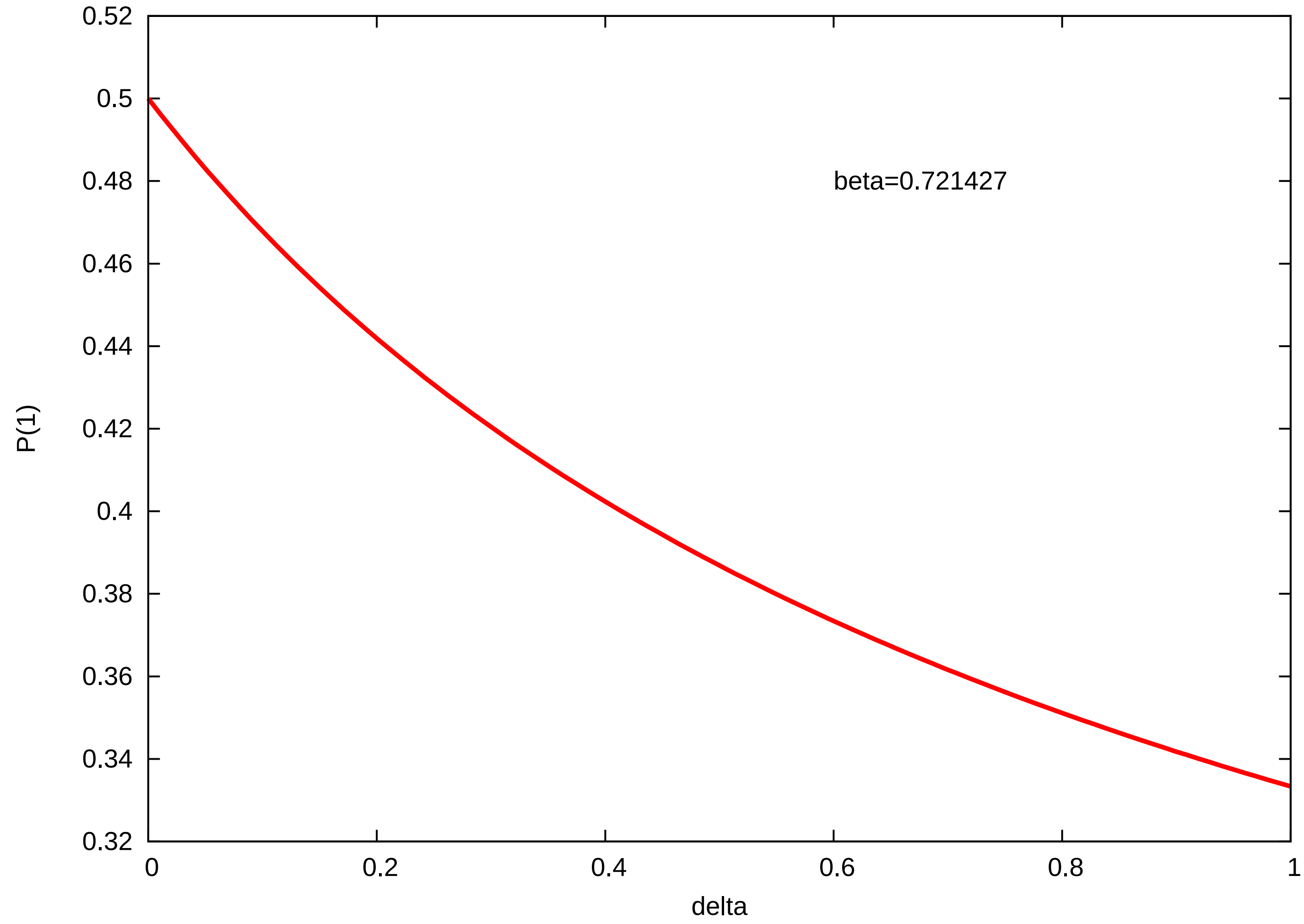
Path Size Logit



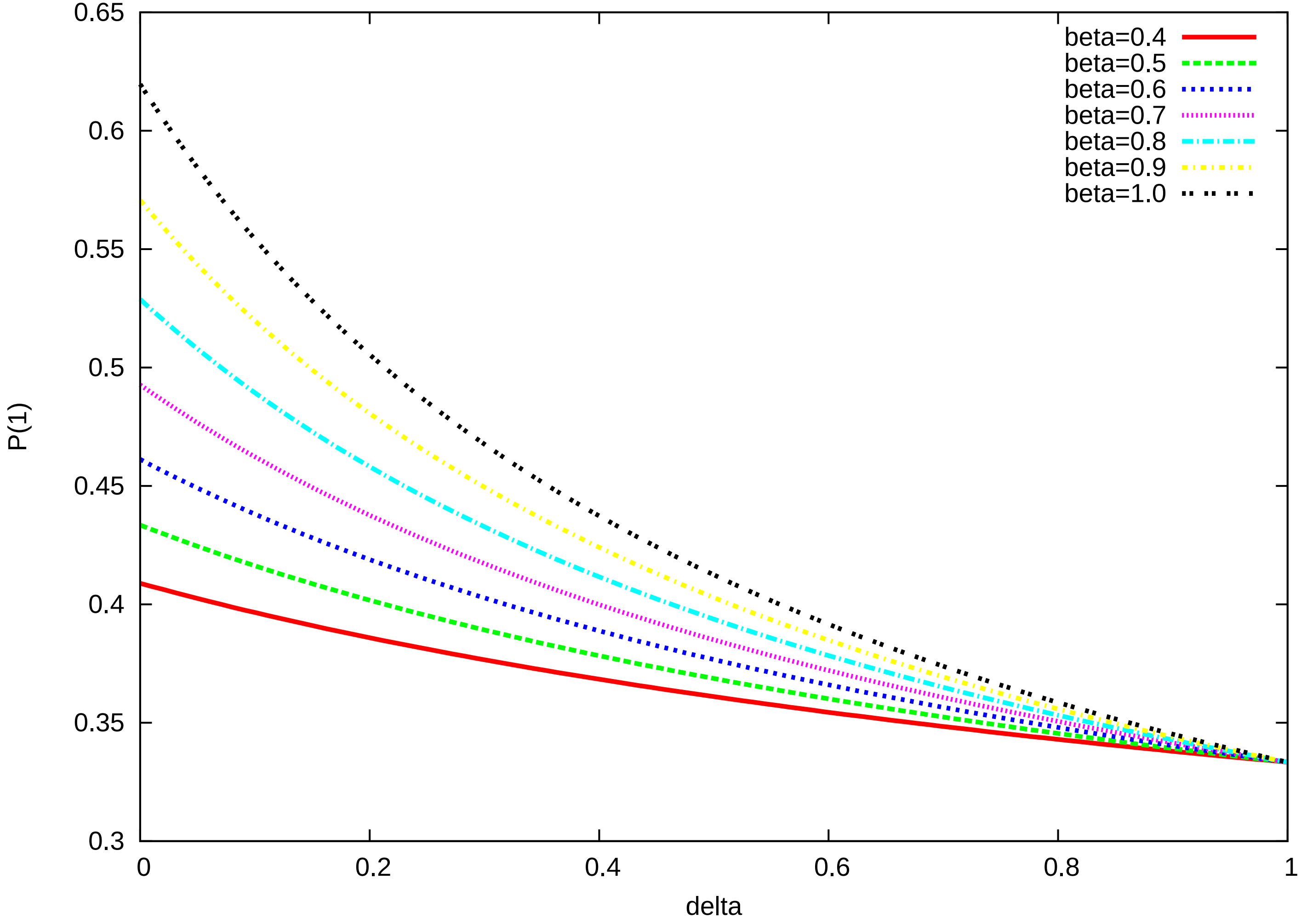
$$\mathrm{PS}_1 = \frac{c}{c}\frac{1}{1} = 1$$

$$\mathrm{PS}_2 = \mathrm{PS}_3 = \frac{c-\delta}{c}\frac{1}{2} + \frac{\delta}{c}\frac{1}{1} = \frac{1}{2} + \frac{\delta}{2c}$$

Path Size Logit



Path Size Logit



Path Size Logit

Advantages:

- MNL formulation: simple
- Easy to compute
- Exploits the network topology
- Practical

Disadvantages:

- Derived from the theory on nested logit
- Several formulations have been proposed
- Correlated with observed and unobserved attributes
- May give biased estimates

Path Size Logit: readings

- Cascetta, E., Nuzzolo, A., Russo, F., Vitetta, A. 1996. A modified logit route choice model overcoming path overlapping problems. Specification and some calibration results for interurban networks. In Lesort, J.B. (Ed.), Proceedings of the 13th International Symposium on Transportation and Traffic Theory, Lyon, France.
- Ramming, M., 2001. Network Knowledge and Route Choice, PhD thesis, Massachusetts Institute of Technology.
- Ben-Akiva, M., and Bierlaire, M. (2003). Discrete choice models with applications to departure time and route choice. In Hall, R. (ed) *Handbook of Transportation Science, 2nd edition* pp.7-38. Kluwer.

Path Size Logit: readings

- Hoogendoorn-Lanser, S., van Nes, R. and Bovy, P. (2005) Path Size Modeling in Multimodal Route Choice Analysis. *Transportation Research Record* vol. 1921 pp. 27-34
- Frejinger, E., and Bierlaire, M. (2007). Capturing correlation with subnetworks in route choice models, *Transportation Research Part B: Methodological* 41(3):363-378. doi:10.1016/j.trb.2006.06.003

Random utility models

Decision-makers with characteristics

- value of time
- access to information
- trip purpose

Alternatives explicit set of paths

Attributes both link-additive and path specific

Decision-rules RUM designed to capture correlations

Note: MNL is a random utility model, but the independence assumption is inappropriate. We must relax it.

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In this lecture, we focus on one of the most complicated issues

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Relax the independence assumption

- MEV models:
  - Vovsha, P. and Bekhor, S., 1998 Link-nested logit model of route choice, Overcoming route overlapping problem, *Transportation Research Record* 1645, pp. 133-142.
  - Abbe, E., Bierlaire, M., and Toledo, T. (2007). Normalization and correlation of cross-nested logit models, *Transportation Research Part B: Methodological* 41(7):795-808. doi:10.1016/j.trb.2006.11.006
- Probit models: Yai, Iwakura and Morichi (1997) Multinomial probit with structured covariance for route choice behavior. *Transportation Research Part B* 31(3), pp. 195-207.

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Relax the independence assumption

- Mixtures of MNL:
  - Bekhor, S., Ben-Akiva, M. and Ramming M.S. (2002). Adaptation of Logit Kernel to Route Choice Situation. *Transportation Research Record*, 1805, 78-85.
  - Frejinger, E., and Bierlaire, M. (2007). Capturing correlation with subnetworks in route choice models, *Transportation Research Part B: Methodological* 41(3):363-378. doi:10.1016/j.trb.2006.06.003

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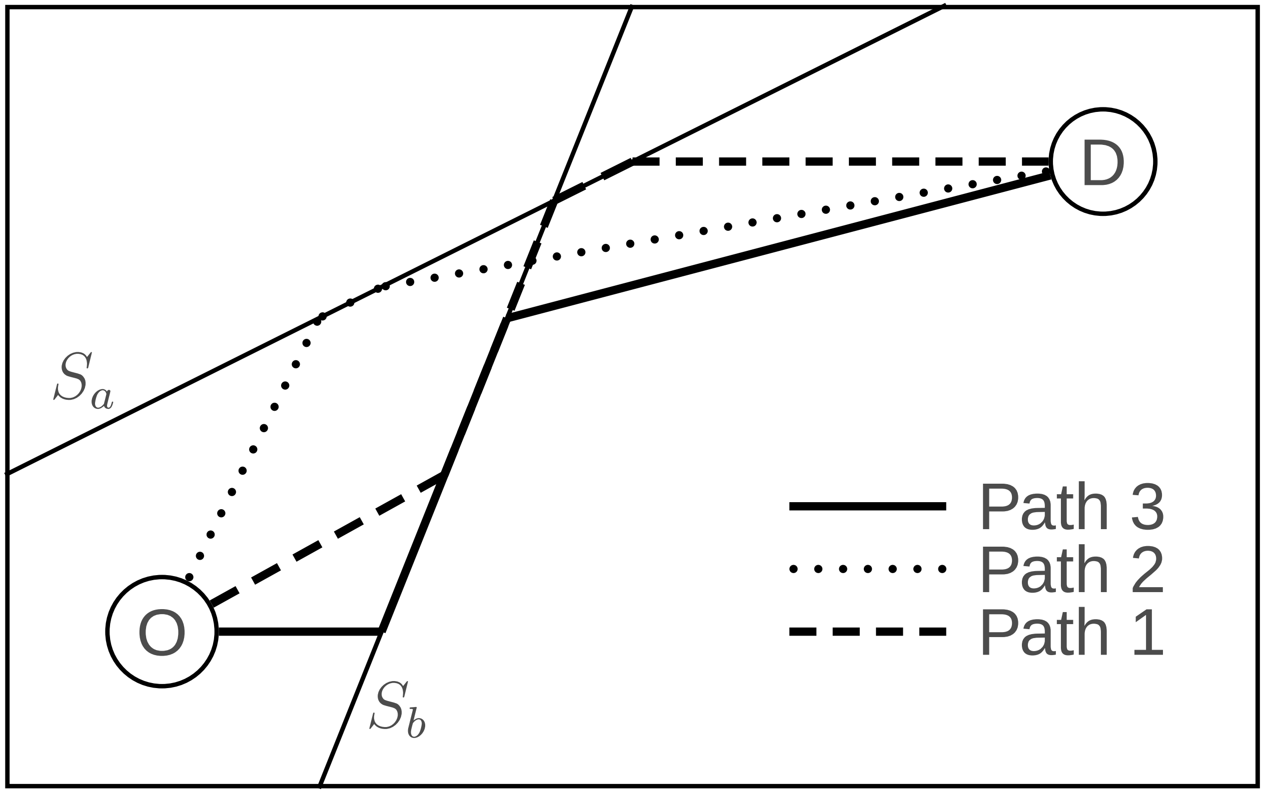
Subnetwork component

Sequence of links corresponding to a part of the network which can be easily labeled, and is behaviorally meaningful in actual route descriptions

- Champs-Elysées in Paris
- Fifth Avenue in New York
- Mass Pike in Boston
- City center in Lausanne

Paths sharing a subnetwork component are assumed to be correlated even if they are not physically overlapping

Subnetworks - Example



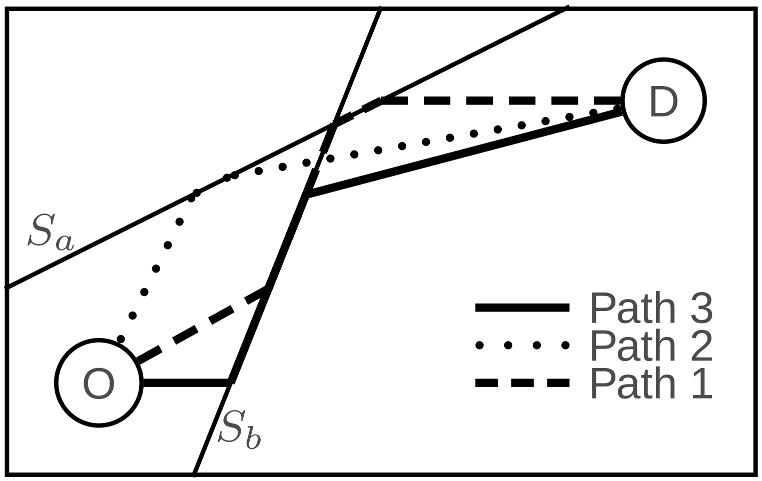
Subnetworks - Methodology

- Factor analytic specification of an error component model (based on model presented in Bekhor et al., 2002)

$$U_p = V_p + \sum_s \sqrt{c_{ps}} \sigma_s \zeta_s + \nu_p$$

  - c_{ps} is the length by which path p overlaps with subnetwork component s
  - σ_s is an unknown parameter
  - $\zeta_s \sim N(0,1)$
  - ν_p i.i.d. Extreme Value distributed

Subnetworks - Example



$$U_1 = \beta^T X_1 + \sqrt{l_{1a}}\sigma_a\zeta_a + \sqrt{l_{1b}}\sigma_b\zeta_b + \nu_1$$
$$U_2 = \beta^T X_2 + \sqrt{l_{2a}}\sigma_a\zeta_a + \nu_2$$
$$U_3 = \beta^T X_3 + \sqrt{l_{3b}}\sigma_b\zeta_b + \nu_3$$

$$\Sigma = $$
$$\begin{bmatrix} l_{1a}\sigma_a^2 + l_{1b}\sigma_b^2 & \sqrt{l_{1a}}\sqrt{l_{2a}}\sigma_a^2 & \sqrt{l_{1b}}\sqrt{l_{3b}}\sigma_b^2 \\ \sqrt{l_{1a}}\sqrt{l_{2a}}\sigma_a^2 & l_{2a}\sigma_a^2 & 0 \\ \sqrt{l_{3b}}\sqrt{l_{1b}}\sigma_b^2 & 0 & l_{3b}\sigma_b^2 \end{bmatrix}$$

Mixture of MNL

In statistics, a mixture density is a pdf which is a convex linear combination of other pdf's.
If $f(\varepsilon, \theta)$ is a pdf, and if $w(\theta)$ is a nonnegative function such that

$$\int_\theta w(\theta) d\theta = 1$$

then

$$g(\varepsilon) = \int_\theta w(\theta) f(\varepsilon, \theta) d\theta$$

is also a pdf. We say that g is a mixture of f.
If f is the pdf of a MNL model, it is a mixture of MNL

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Mixture of MNL

$$U_p = V_p + \sum_s \sqrt{c_{ps}}\sigma_s\zeta_s + \nu_p$$

If ζ is given,

$$\Pr(p|\zeta) = \frac{e^{V_p + \sum_s \sqrt{c_{ps}}\sigma_s\zeta_s}}{\sum_q e^{V_q + \sum_s \sqrt{c_{qs}}\sigma_s\zeta_s}}$$

ζ is distributed $N(0, I)$

$$\Pr(p) = \int_\zeta \frac{e^{V_p + \sum_s \sqrt{c_{ps}}\sigma_s\zeta_s}}{\sum_q e^{V_q + \sum_s \sqrt{c_{qs}}\sigma_s\zeta_s}} \phi(\zeta) d\zeta$$

Mixture of MNL

$$\Pr(p) = \int_{\zeta} \frac{e^{V_p + \sum_s \sqrt{c_{ps}}\sigma_s \zeta_s}}{\sum_q e^{V_q + \sum_s \sqrt{c_{qs}}\sigma_s \zeta_s}} \phi(\zeta) d\zeta$$

Not a closed form. Simulated Maximum Likelihood is to be used

- Train, K. (2003) Discrete Choice Methods with Simulation, Cambridge University Press

Subnetworks

Advantages

- Rich correlation structure
- Flexibility between complexity and realism

Disadvantages

- Non closed form

Random utility models

Decision-makers with characteristics

- value of time
- access to information
- trip purpose

Alternatives explicit set of paths

Attributes both link-additive and path specific

Decision-rules RUM designed to capture correlations

Note: MNL is a random utility model, but the independence assumption is inappropriate. We must relax it.

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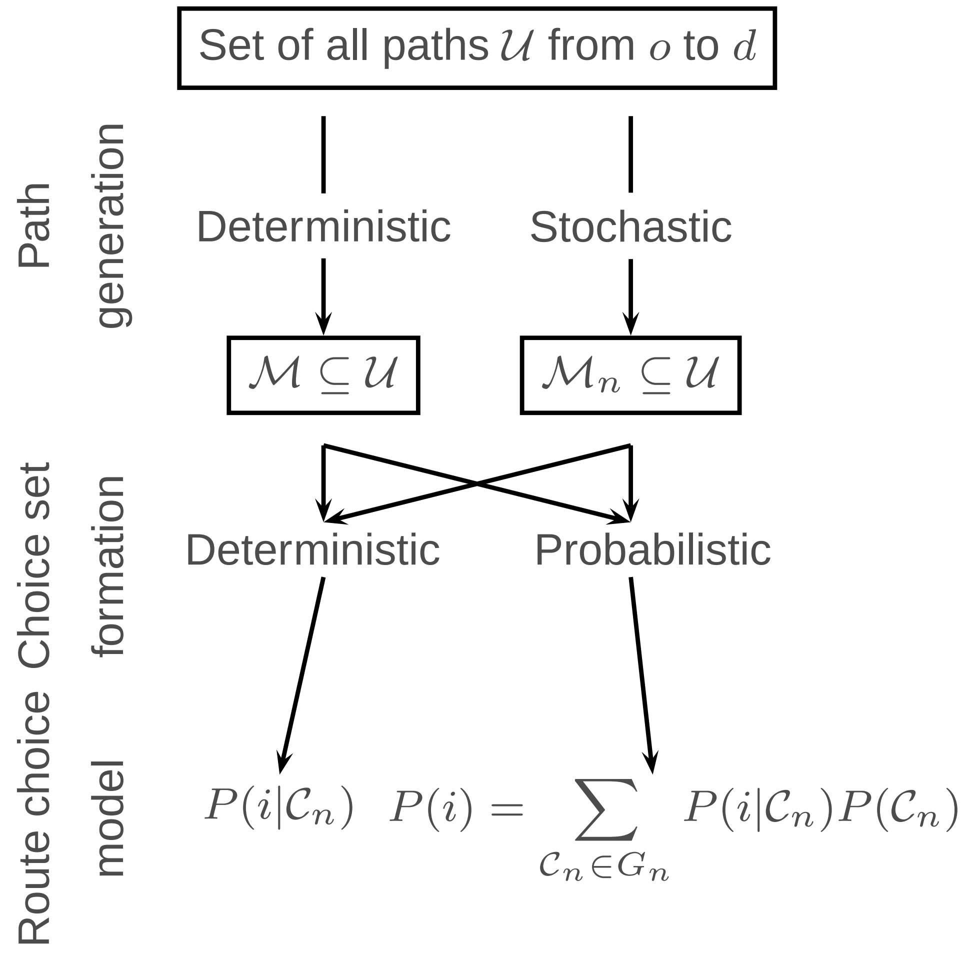
Introduction



Introduction

- Underlying assumption in existing approaches: the actual choice set is generated
- Empirical results suggest that this is not always true
- Our approach:
  - True choice set = universal set $\mathcal{U}$
  - Too large
  - Sampling of alternatives

Sampling of Alternatives

- Multinomial Logit model (e.g. Ben-Akiva and Lerman, 1985):

$$P(i|\mathcal{C}_n) = \frac{q(\mathcal{C}_n|i)P(i)}{\sum_{j\in\mathcal{C}_n} q(\mathcal{C}_n|j)P(j)} = \frac{e^{V_{in}+\ln q(\mathcal{C}_n|i)}}{\sum_{j\in\mathcal{C}_n} e^{V_{jn}+\ln q(\mathcal{C}_n|j)}}$$

$\mathcal{C}_n$: set of sampled alternatives
$q(\mathcal{C}_n|j)$: probability of sampling $\mathcal{C}_n$ given that j is the chosen alternative

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Importance Sampling of Alternatives

- Attractive paths have higher probability of being sampled than unattractive paths
- Path utilities must be corrected in order to obtain unbiased estimation results

Path Sampling

- Idea: random walk, biased toward the shortest path
- Probability of selecting next link ℓ depends on its weight ω_ℓ
- Kumaraswamy distribution,

$$\begin{aligned}\omega(\ell|a,b) &= 1-(1-{x_\ell}^a)^b, \quad x_\ell \in [0,1].\\ x_\ell &= \frac{SP(v,d)}{C(\ell)+SP(w,d)}\end{aligned}$$

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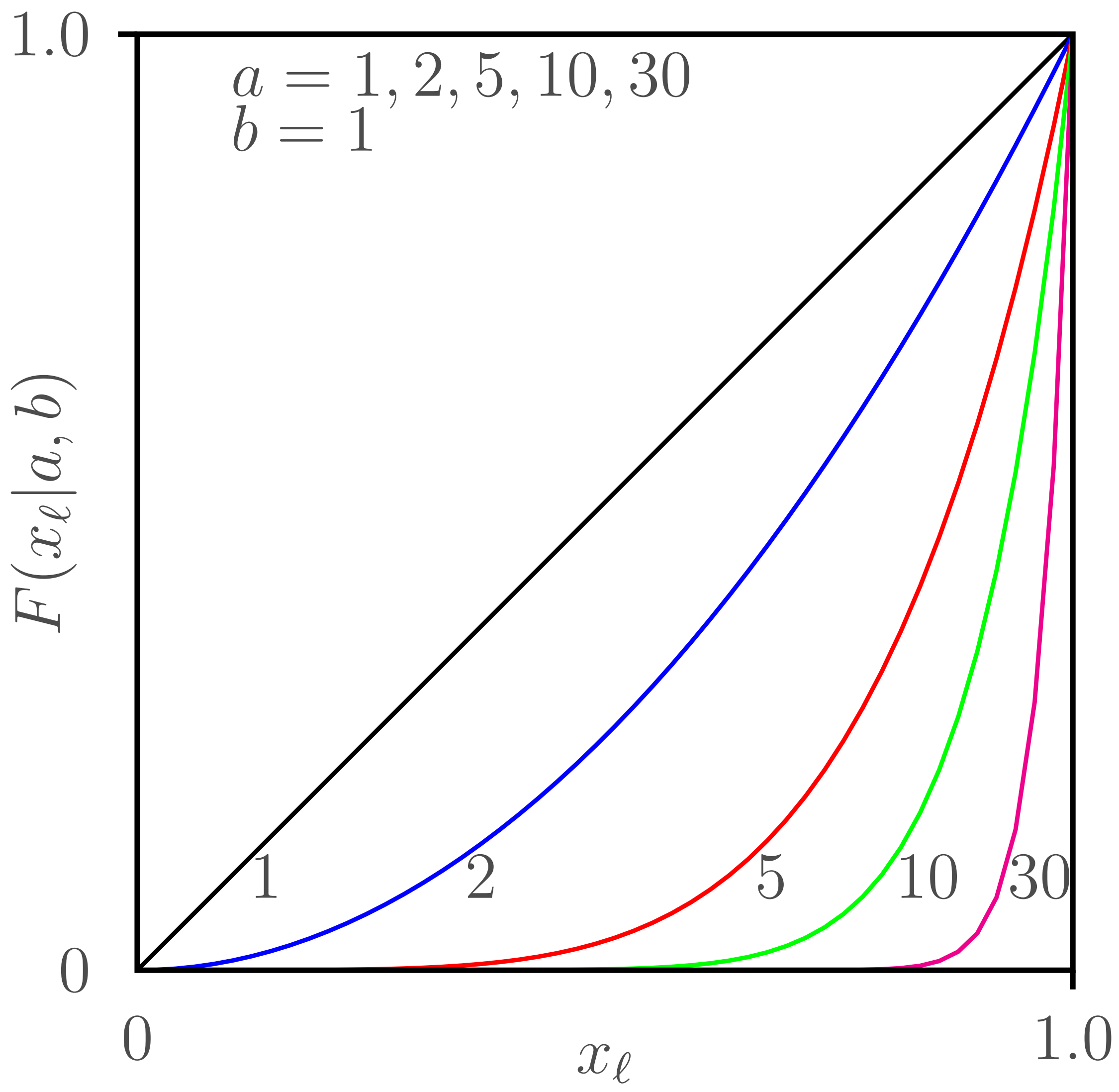
Path Sampling



Path Sampling

- Probability for path j to be sampled

$$q(j) = \prod_{\ell=(v,w)\in\Gamma_j} q((v,w)|\mathcal{E}_v)$$

- Γ_j: ordered set of all links in j
- v: source node of j
- $\mathcal{E}_v$: set of all outgoing links from v
- In theory, the set of all paths $\mathcal{U}$ may be unbounded. We treat it as bounded with size J

Path Sampling

The correction term can be computed.

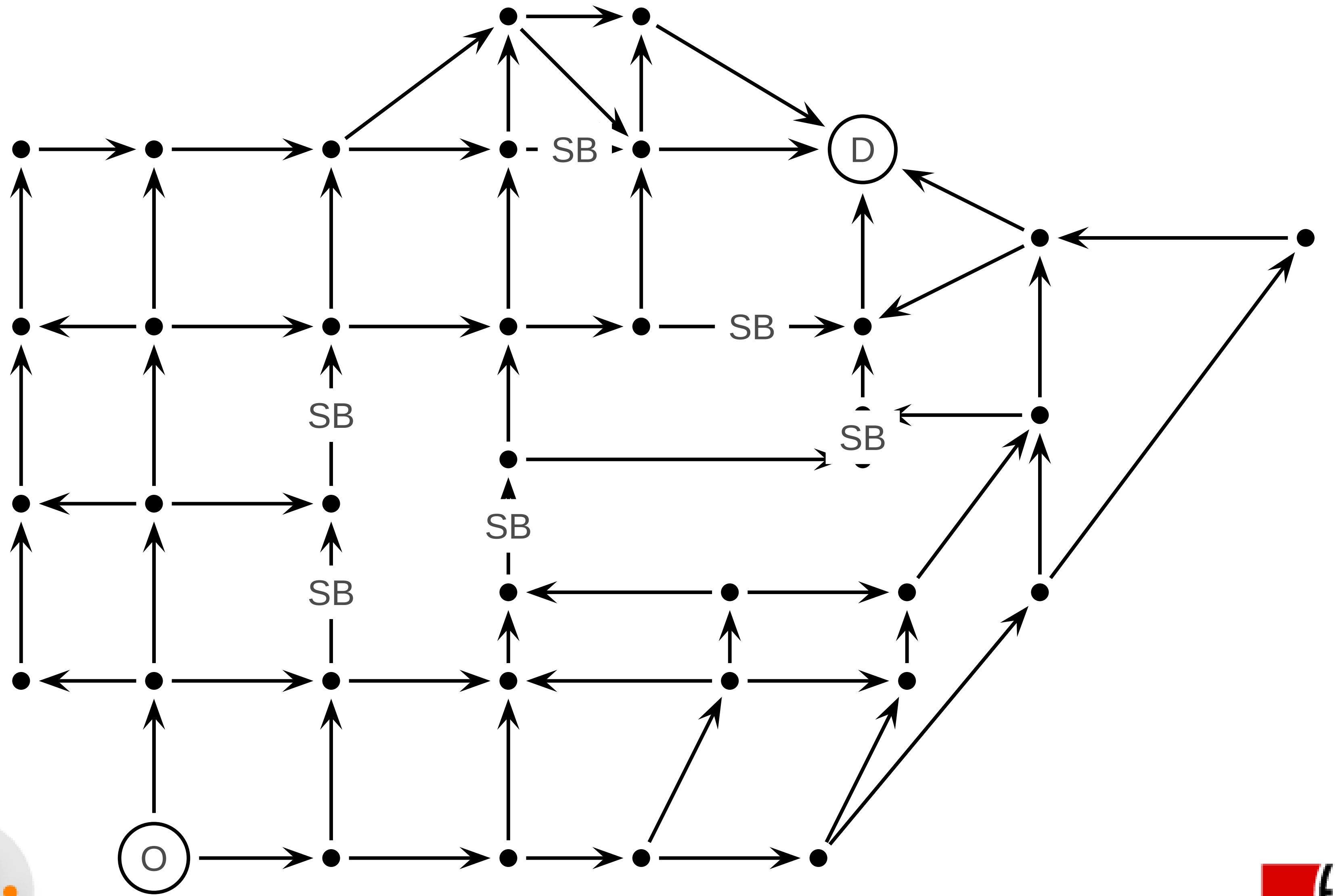
`Frejinger, E., and Bierlaire, M. (2007). Sampling of Alternatives for Route Choice Modeling. Technical report TRANSP-OR 071121. Transport and Mobility Laboratory, ENAC, EPFL. transp-or2.epfl.ch/abstract.php?type=1&id=FrejBier07`

$$P(i|\mathcal{C}_n) = \frac{e^{V_{in}+\ln\left(\frac{k_i}{q(i)}\right)}}{\sum_{j\in\mathcal{C}_n} e^{V_{jn}+\ln\left(\frac{k_j}{q(j)}\right)}}$$

Numerical Results

- Estimation of models based on synthetic data generated with a postulated model
- Evaluation of
  - Sampling correction
  - Path Size attribute
  - Biased random walk algorithm parameters

Numerical Results



Numerical Results

- True model: Path Size Logit

  $U_j = \beta_{\text{PS}} \ln \text{PS}_j^{\mathcal{U}} + \beta_{\text{L}} \text{Length}_j + \beta_{SB} \text{SpeedBumps}_j + \varepsilon_j$

  $\beta_{\text{PS}} = 1,\ \beta_{\text{L}} = -0.3,\ \beta_{\text{SB}} = -0.1$

  ε_j distributed Extreme Value with scale 1 and location 0

  $\text{PS}_j^{\mathcal{U}} = \sum_{\ell \in \Gamma_j} \frac{L_\ell}{L_j} \frac{1}{\sum_{p \in \mathcal{U}} \delta_{\ell p}}$

- 3000 observations

Numerical Results

- Four model specifications

		Sampling Correction	
		Without	With
Path Size	$\mathcal{C}$	$M_{PS(\mathcal{C})}^{\text{NoCorr}}$	$M_{PS(\mathcal{C})}^{\text{Corr}}$
	$\mathcal{U}$	$M_{PS(\mathcal{U})}^{\text{NoCorr}}$	$M_{PS(\mathcal{U})}^{\text{Corr}}$

$$\text{PS}_i^{\mathcal{U}} = \sum_{\ell \in \Gamma_i} \frac{L_\ell}{L_i} \frac{1}{\sum_{j \in \mathcal{U}} \delta_{\ell j}}$$

$$\text{PS}_{in}^{\mathcal{C}} = \sum_{\ell \in \Gamma_i} \frac{L_\ell}{L_i} \frac{1}{\sum_{j \in \mathcal{C}_n} \delta_{\ell j}}$$

Numerical Results

- Model $M_{PS(\mathcal{C})}^{\text{NoCorr}}$:

$$V_{in} = \mu \left(\beta_{\text{PS}} \ln \text{PS}_{in}^{\mathcal{C}} - 0.3\text{Length}_i + \beta_{SB}\text{SpeedBumps}_i \right)$$

- Model $M_{PS(\mathcal{C})}^{\text{Corr}}$:

$$V_{in} = \mu \left(\beta_{\text{PS}} \ln \text{PS}_{in}^{\mathcal{C}} - 0.3\text{Length}_i + \beta_{SB}\text{SpeedBumps}_i + \ln(\frac{k_i}{q(i)}) \right)$$

- Model $M_{PS(\mathcal{U})}^{\text{NoCorr}}$:

$$V_{in} = \mu \left(\beta_{\text{PS}} \ln \text{PS}_{in}^{\mathcal{U}} - 0.3\text{Length}_i + \beta_{SB}\text{SpeedBumps}_i \right)$$

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Numerical Results

	True PSL	$M^{\text{NoCorr}}_{PS(\mathcal{C})}$ PSL	$M^{\text{Corr}}_{PS(\mathcal{C})}$ PSL	$M^{\text{NoCorr}}_{PS(\mathcal{U})}$ PSL	$M^{\text{Corr}}_{PS(\mathcal{U})}$ PSL
$\widehat{\beta}_{\text{L}}$ fixed	**-0.3**	**-0.3**	**-0.3**	**-0.3**	**-0.3**
$\widehat{\mu}$	**1**	**0.182**	**0.724**	**0.141**	**0.994**
Standard error		0.0277	0.0226	0.0263	0.0286
t-test w.r.t. 1		-29.54	-12.21	-32.64	-0.2
$\widehat{\beta}_{\text{PS}}$	**1**	**1.94**	**0.411**	**-1.02**	**1.04**
Standard error		0.428	0.104	0.383	0.0474
t-test w.r.t. 1		2.20	-5.66	-5.27	0.84
$\widehat{\beta}_{\text{SB}}$	**-0.1**	**-1.91**	**-0.226**	**-2.82**	**-0.0867**
Standard error		0.25	0.0355	0.428	0.0238
t-test w.r.t. -0.1		-7.24	-3.55	-6.36	0.56

Numerical Results

	True PSL	$M^{\text{NoCorr}}_{PS(\mathcal{C})}$ PSL	$M^{\text{Corr}}_{PS(\mathcal{C})}$ PSL	$M^{\text{NoCorr}}_{PS(\mathcal{U})}$ PSL	$M^{\text{Corr}}_{PS(\mathcal{U})}$ PSL
Final Log-likelihood		-6660.45	-6082.53	-6666.82	-5933.98
Adj. Rho-square		0.018	0.103	0.017	0.125

Null Log-likelihood: -6784.96, 3000 observations

Algorithm parameters: 10 draws, $a = 5$, $b = 1$, $C(\ell) = L_\ell$

Average size of sampled choice sets: 9.66

BIOGEME (Bierlaire, 2007 and Bierlaire, 2003) has been used for all model estimations

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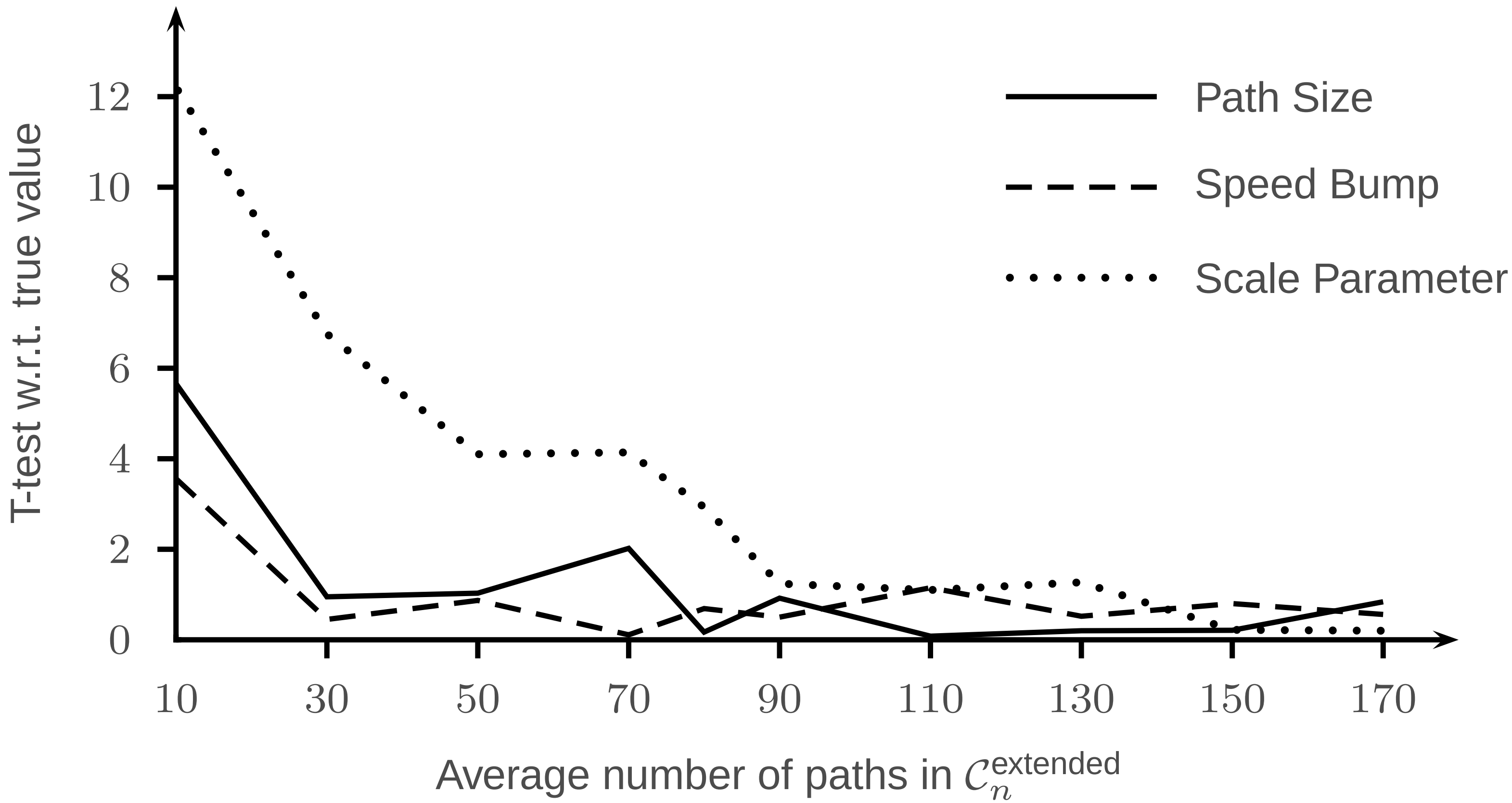
Extended Path Size

- Compute Path Size attribute based on an *extended choice set* $\mathcal{C}_n^{\text{extended}}$
- Simple random draws from $\mathcal{U} \backslash \mathcal{C}_n$ so that $|\mathcal{C}_n| \leq |\mathcal{C}_n^{\text{extended}}| \leq |\mathcal{U}|$

Extended Path Size



Extended Path Size

- Heuristic for finding an extended choice set $\mathcal{C}_n^{\text{extended}}$ (all paths in $\mathcal{C}_n$ are included)
- `Frejinger and Bierlaire (2007)`

Extended Path Size

	True PSL	PS(C^{extended}) PSL	PS(C) PSL
$\widehat{\beta}_{\text{L}}$ fixed	**-0.3**	**-0.3**	**-0.3**
$\widehat{\mu}$	**1**	**0.885**	**0.724**
Standard error		0.0259	0.0266
t-test w.r.t. 1		-4.43	-12.21
$\widehat{\beta}_{\text{PS}}$	**1**	**1.52**	**0.411**
Standard error		0.102	0.104
t-test w.r.t. 1		5.10	-5.66
$\widehat{\beta}_{\text{SB}}$	**-0.1**	**-0.131**	**-0.266**
Standard error		0.0281	0.0355
t-test w.r.t. -0.1		-1.10	-3.55
Adj. Rho-Squared		0.114	0.103
Final Log-likelihood		-6006.96	-6082.53

Conclusions

- Route choice models complicated because:
  1. Complex correlation structure
  2. Large set of alternatives
- Flexible correlation structure: the subnetwork approach
- New point of view on choice set generation and route choice modeling: the path sampling approach

Thank you to Emma Frejinger