Uncertainty Feature Optimization for the Airline Scheduling Problem

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Outline

- Uncertainty Feature Optimization (UFO)
- Application to Airline Scheduling
- The ROADEF Challenge 2009
- Computational Results
- Future Research

Optimization with Noisy Data

- Real world problems are due to noisy data
- Noise should not be neglected
- Methods using explicit uncertainty sets:
  - Uncertainty sets are hard to model
  - Methods are computationally hard
  - Solutions are sensitive to errors in noise modeling

=> Uncertainty Features capture noise implicitly

Uncertainty Feature Optimization (UFO) Eggenberg, Salani and Bierlaire (2009b)

Uncertainty Feature (UF): an implicit noise characterization

- ✓ No uncertainty set required
- ✓ Problem Complexity similar to original problem*
- ✓ Not sensitive to modification in noise's nature
- ✓ Models what practitioners do for uncertain problems

➡ Requires a posteriori validation

UFO Framework

Deterministic Problem

$$z^* = \min \quad f(x)$$

$$s.t. \quad a(x) \leq b$$

$$x \in X$$

UFO Formulation
with scalar UF $\mu : X \to \mathbb{R}$

$$\max \quad \mu(x)$$

$$s.t. \quad a(x) \leq b$$

$$f(x) \leq (1+\rho)z^*$$

$$x \in X$$

BUDGET CONSTRAINT

Remarks

- UFs should increase robustness or recoverability
- Using UFs based on uncertainty sets is possible

  $\Rightarrow$ Can express Stochastic Optimization and Robustness of `Bertsimas and Sim (2004)` as UFs
- Can extend any existing model with UFO
- Complexity is similar as long the UF is of same complexity than the deterministic problem

Application to Airline Scheduling

Desired Properties of a Schedule

- Absorb Delays
- Avoid disruption propagation effect
- Easier to recover in case of disruption

Methods used by Practitioners

- Increase idle time
- Increase number of plane crossings

Aircraft Scheduling Problem (ASP)

- A set of flights
- A set of aircraft (fleets)
- A departure time and plane type for each flight (maximizing some potential revenue metric)

→

- One feasible route for each aircraft
- All flights are covered
- Aircraft assignment and departures as close as possible to input

ASP Model Eggenberg, Salani and Bierlaire (2009)

$$\min \sum_{r\in\Omega} c_r x_r$$

$$s.t. \quad \sum_{r\in\Omega} b_r^f x_r = 1 \quad \forall f \in F$$

$$\sum_{r\in\Omega} b_r^s x_r = 1 \quad \forall s \in S$$

$$\sum_{r\in\Omega} b_r^p x_r \leq 1 \quad \forall p \in P$$

$$x_r \in \{0,1\}$$

Column Generation Algorithm

- Use Constraint-Specific Networks for each aircraft
- Pricing is a Resource Constrained Elementary Shortest Path Problem (RCESPP) on the networks

See Eggenberg, Salani and Bierlaire (2009)

ASP: Budget Allocation

Lowest possible deviation of departure times

c_r = total deviation from original schedule of route *r*

Optimum of deterministic problem = 0

Budget Constraint => $(1+\rho)0 = 0$

SOLUTION: Use a constant C for total deviation

$$\sum_{r \in \Omega} c_r x_r \leq C$$

General UFO Formulation

$$
\begin{aligned}
\max \quad & \mu(x) \\
s.t. \quad & \sum_{r\in\Omega} b_r^f x_r = 1 \quad \forall f \in F \\
& \sum_{r\in\Omega} b_r^s x_r = 1 \quad \forall s \in S \\
& \sum_{r\in\Omega} b_r^p x_r \leq 1 \quad \forall p \in P \\
& \sum_{r\in\Omega} c_r x_r \leq C \\
& x_r \in \{0,1\}
\end{aligned}
$$

Used Uncertainty Features

Total Idle Time (IT)	$\mu_{IT}(x) = \sum_{r \in \Omega} \delta_r x_r$
Sum of Minimum Idle Times (MIT)	$\mu_{MIT}(x) = \sum_{r \in \Omega} \delta_r^{MIN} x_r$
Number of Plane Crossings (CROSS)	$\mu_{CROSS}(x)$
Passenger Connection (PCON)	$\mu_{PCON}(x) = \sum_{i \in I} \delta_i$

The ROADEF Challenge 2009

- Solve the disrupted airline recovery problem
- Qualification: 8 instances A01-A04 and A06-A09
- 608 flights, 85 aircraft
- Provided solution and cost checkers

Tests Performed

- Compare a priori UF values for original schedule Or and schedules obtained by IT, MIT,CROSS and PCON
- Adapt disruption to schedule
- Compare a posteriori results of our recovery algorithm

A priori results (A01-A04, A06-A09)

MODEL	Or	IT 2500	IT 5000	IT 10000	MIT 2500	MIT 5000	MIT 10000	CROSS 2500	CROSS 5000	CROSS 10000	PCON 1000
IT [k min]	12	14.5	17	19.0	13.5	14.7	16.7	11.4	11.5	11.0	12.8
MIT [min]	790	1025	1150	1230	2210	2835	3330	620	505	460	783
CROSS	3430	3455	3496	3488	3450	3438	3416	3517	3530	3519	3448
PCON [k min]	130.5	135.8	141.1	148.2	132.3	134.6	141.0	127.3	127.7	127.5	134.5
Loss of Revenue [%]	0.0	0.29	0.65	2.42	0.99	1.71	3.51	2.35	3.37	3.65	0.00

Maximum Loss: 22,086€

Maximum Passengers lost: 1.31%

A posteriori results (A01-A04, A06-A09)

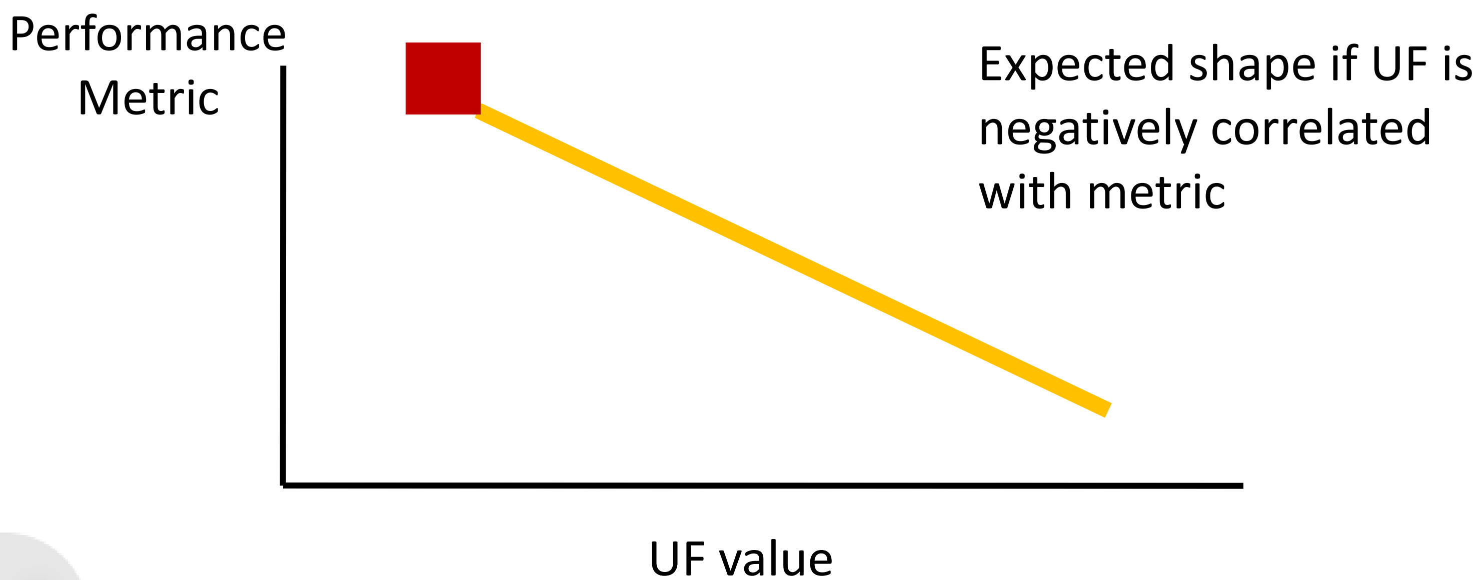
MODEL	Or	IT 2500	IT 5000	IT 10000	MIT 2500	MIT 5000	MIT 10000	CROSS 2500	CROSS 5000	CROSS 10000	PCON 1000
Cost [k€]	697.9	617.5	504.0	474.7	536.8	538.9	321.6	666.3	608.5	576.6	602.7
Savings [%]	0.00	11.52	27.78	31.98	23.08	22.77	53.92	4.52	12.81	17.38	13.64
# Psg Canceled	504	443	359	350	373	372	242	464	418	406	504

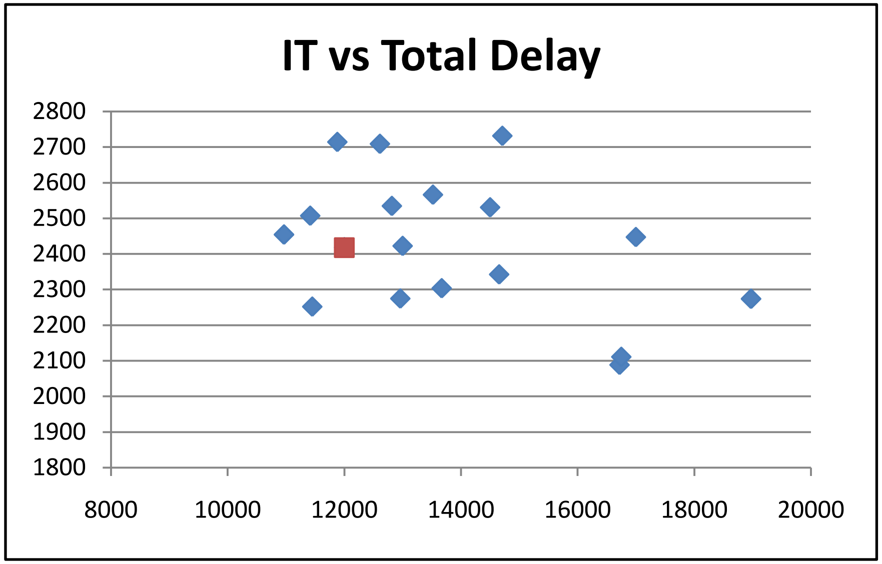
Maximum Savings: 1.32 Mio € (70.6%)

Testing UF validity

Solutions obtained by UF models

Original Schedule




IT vs Total Delay
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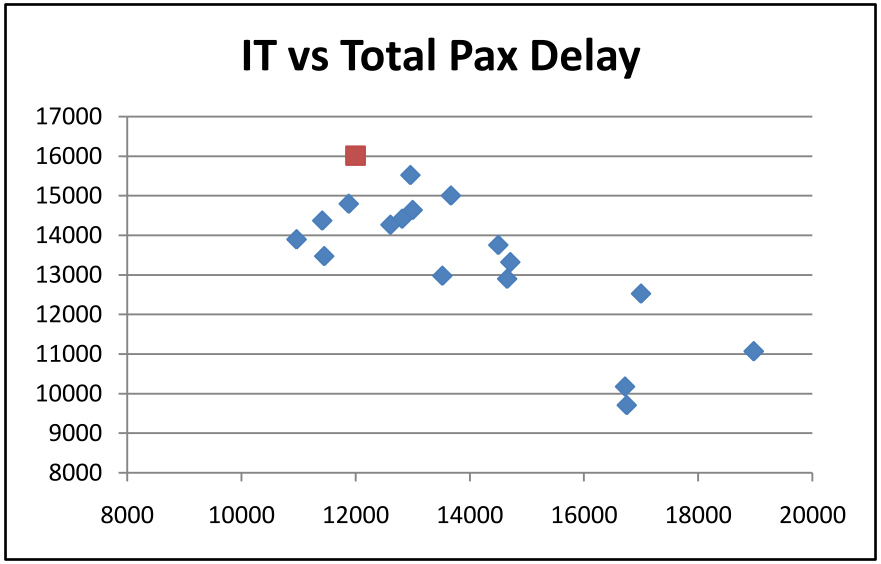
IT vs Total Pax Delay
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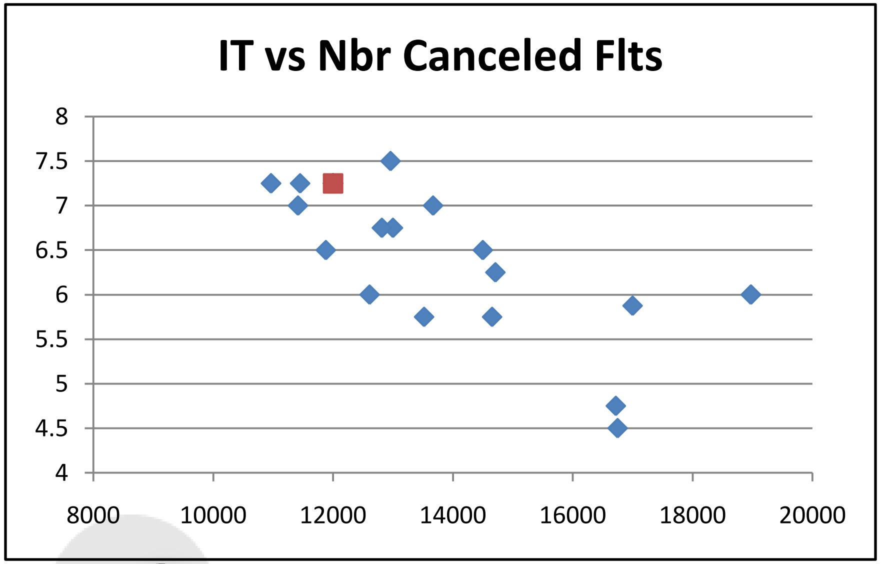
IT vs Nbr Canceled Flts
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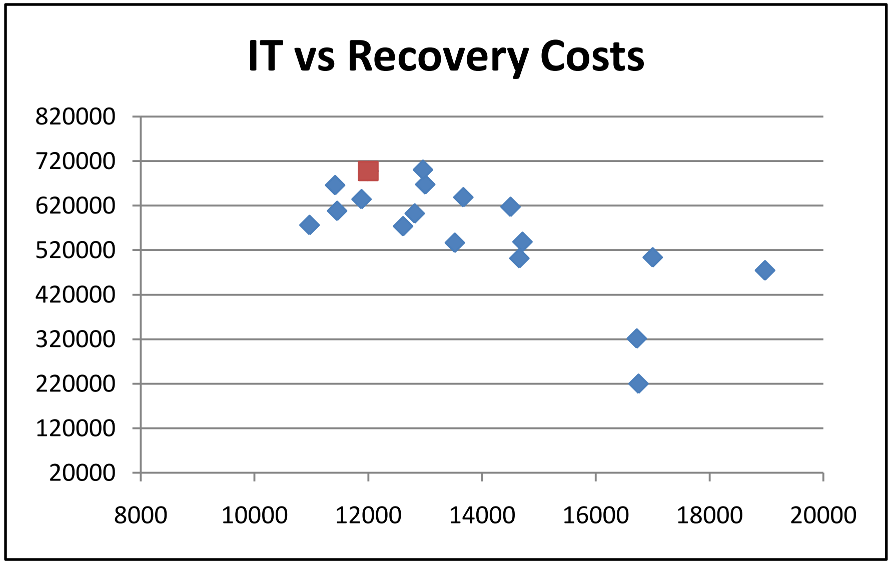
IT vs Recovery Costs
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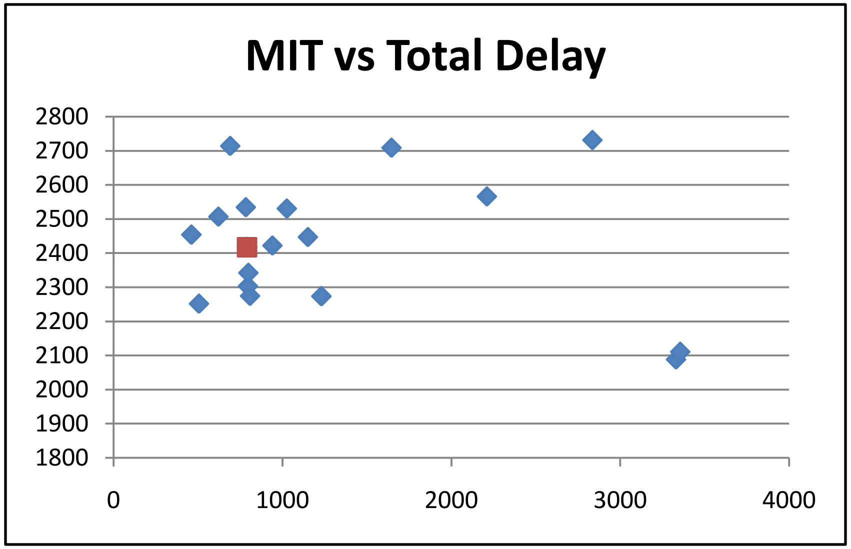
MIT vs Total Delay
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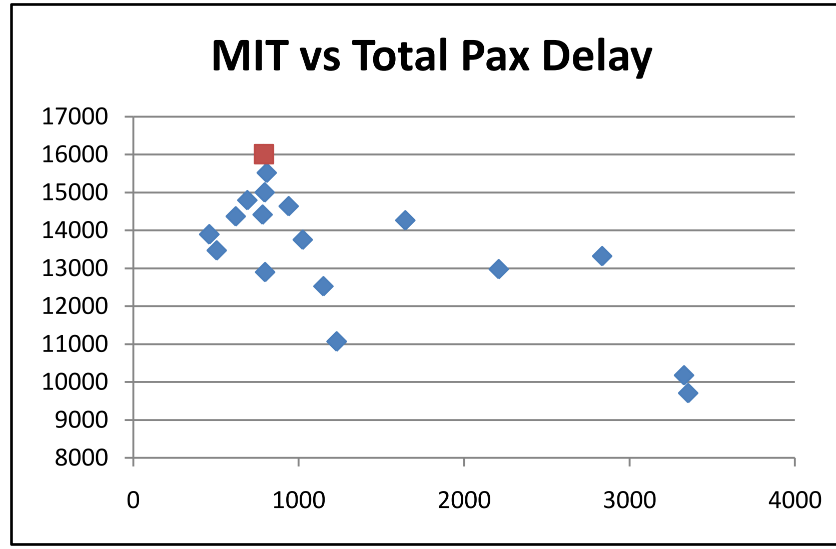
MIT vs Total Pax Delay
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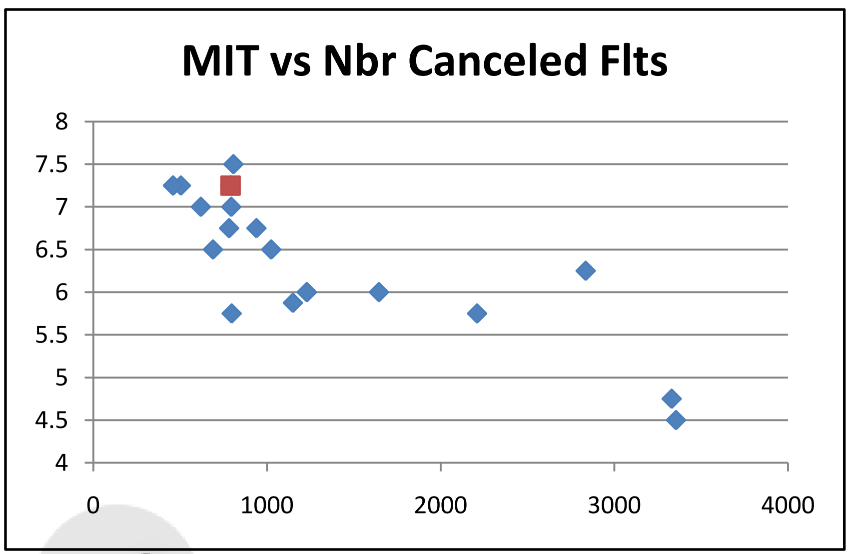
MIT vs Nbr Canceled Flts
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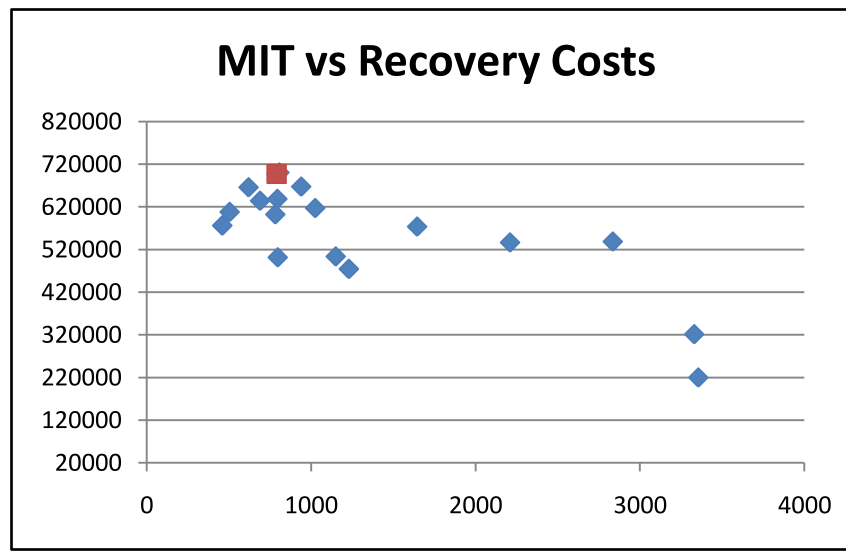
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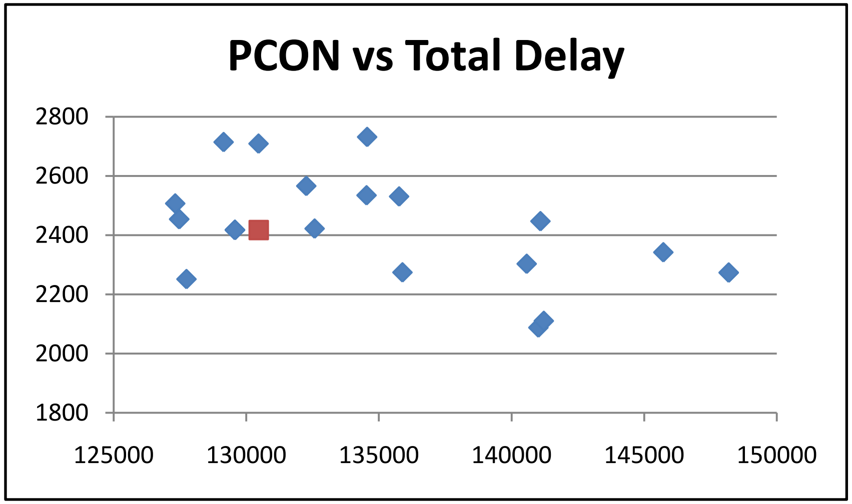
PCON vs Total Delay
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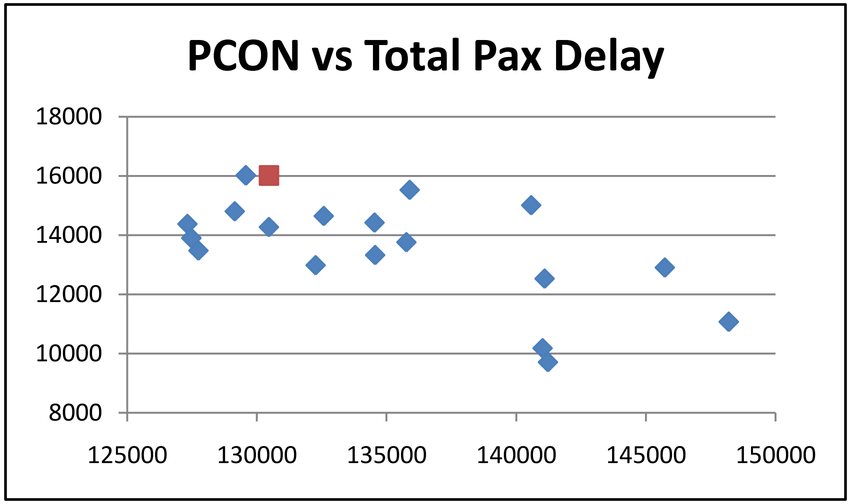
PCON vs Total Pax Delay
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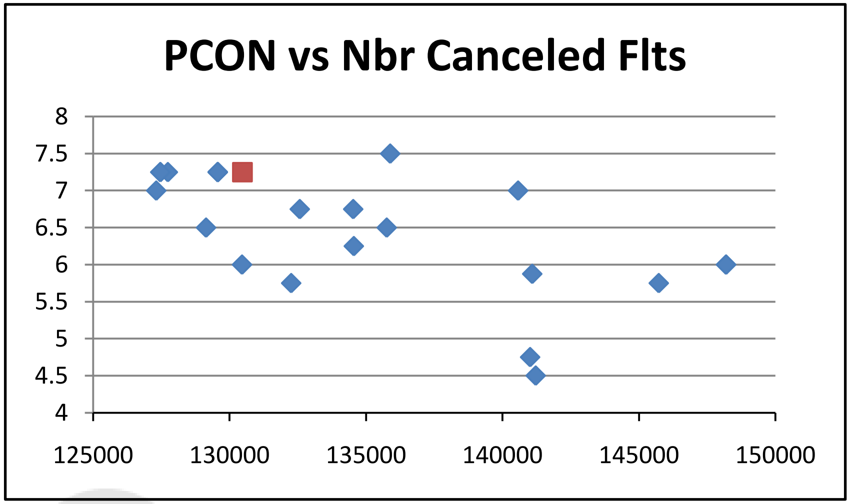
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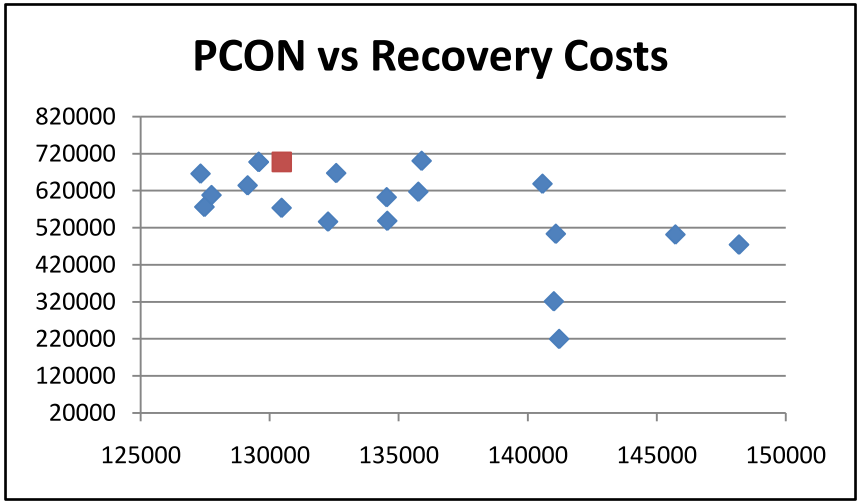
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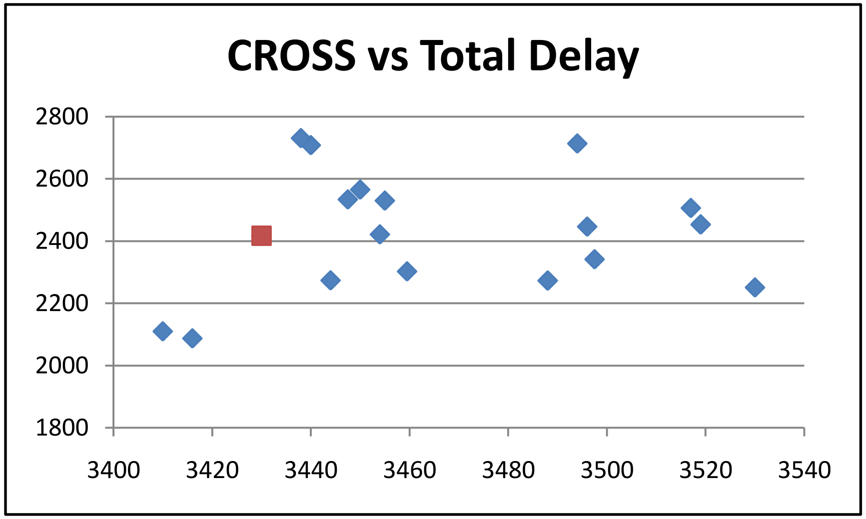
CROSS vs Total Delay
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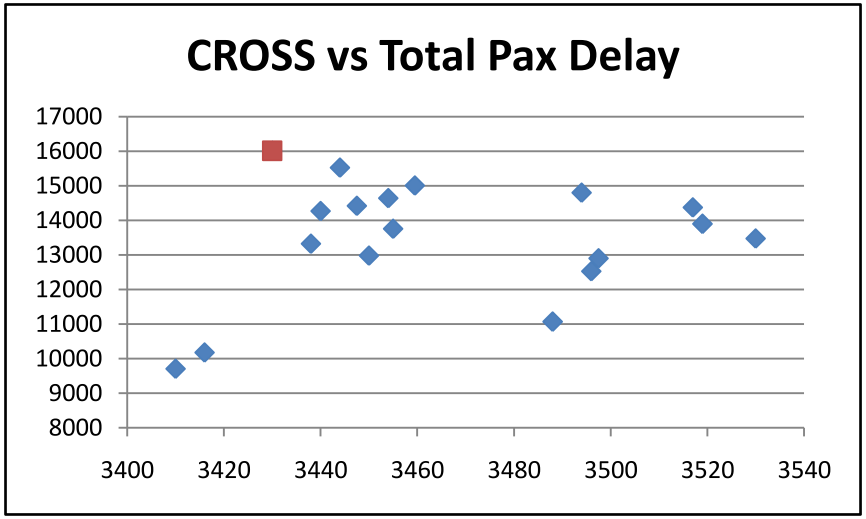
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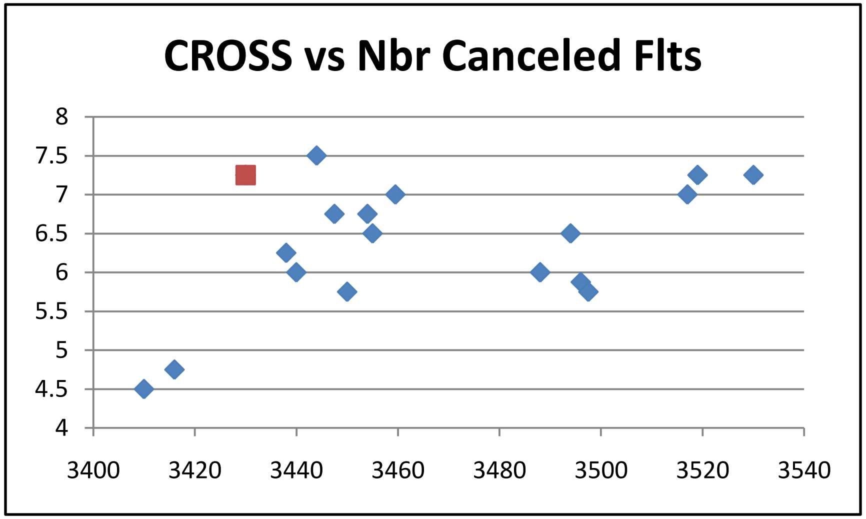
CROSS vs Nbr Canceled Flts
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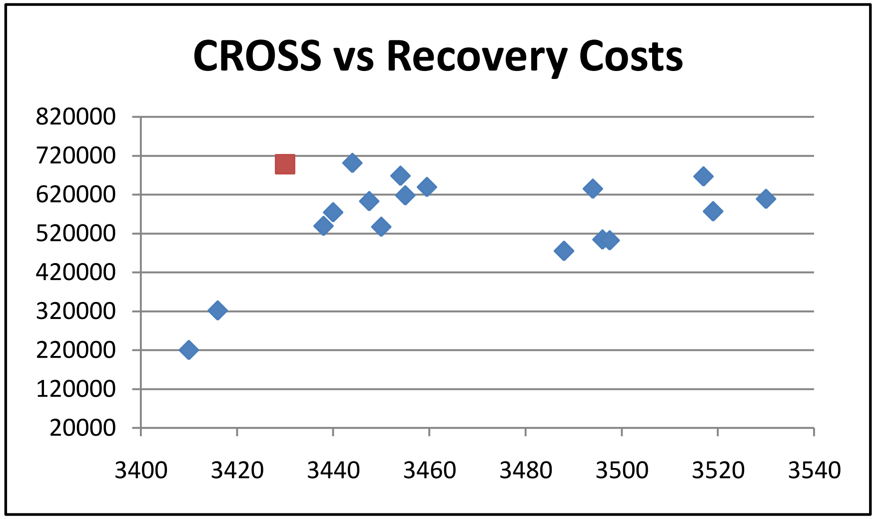
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Conclusions

- UFO leads to *better* (more recoverable) solutions
- MIT 10000: Reduction of recovery costs by 53.9% in average, average revenue loss of 3.51%
- IT, MIT and PCON are correlated with recoverability, CROSS does not work as well

Future Work

- Improve convergence for bigger instances
- Model extensions:
    - Improve CROSS model
    - Improve algorithm for PCON model
    - Include crews
- Application of UFO to other problems

THANKS for your attention! Any Questions?

References

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