Airline disruption recovery and **robustness**

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Outline

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ROADEF Challenge2009

Forewords:

- We worked on an optimization algorithm for the **Aircraft** recovery problem with **maintenance constraints** in collaboration with an IT company (funded by the swiss government - CTI program).

The problem:

- Recover within a given time horizon an airline schedule in a disrupted state minimizing the recovery costs
- The recent history of the schedule is given to obtain the state of the resources

Data

After data preprocessing, the relevant informations are:

- F: a set of scheduled flights, together with an estimation of cancellation cost c_f
- P: a set of aircrafts
- R: a set of passengers (itineraries)
- I_p, I_r: a set of initial positions for both aircrafts and passengers
- S_p, S_r: a set of required final positions for both aircrafts and passengers
- T: a time horizon
- L: a set of airport slots
- q_l^{Dep}, q_l^{Arr}: slot capacities for take off and landings

Outline

Master problem

We model the recovery problem for aircrafts as:

$$\min z_{MP} = \sum_{r\in\Omega} c_r x_r + \sum_{f\in F} c_f y_f \tag{1}$$

$$\sum_{r\in\Omega} b_r^f x_r + y_f = 1 \qquad \forall f \in F \tag{2}$$

$$\sum_{r\in\Omega} b_r^s x_r = 1 \qquad \forall s \in S_p \tag{3}$$

$$\sum_{r\in\Omega} b_r^p x_r \leq 1 \qquad \forall p \in P \tag{4}$$

$$\sum_{r\in\Omega} b_r^{Dep,l} x_r \leq q_l^{Dep} \qquad \forall l \in L \tag{5}$$

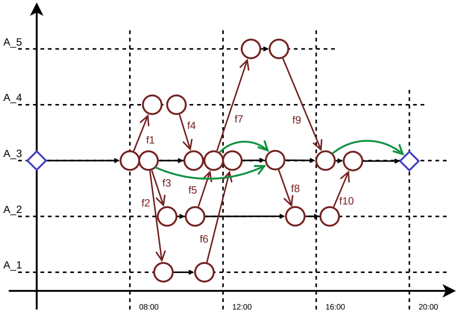
$$\sum_{r\in\Omega} b_r^{Arr,l} x_r \leq q_l^{Arr} \qquad \forall l \in L \tag{6}$$

$$x_r \in \{0,1\}\ \forall r \in \Omega,\ \ y_f \in \{0,1\}\ \forall f \in F \tag{7}$$

Solved by an optimization based heuristic (Column Generation + Dynamic Programming) on a constraint specific recovery network. Eggenberg, S. And Bierlaire (2008a).

Recovery Network

Given T, I_p and S_p the R.N. encodes all possible recovery schemes for plane p.



Scheduled flights, acyclic, polynomial size

Recovery Network

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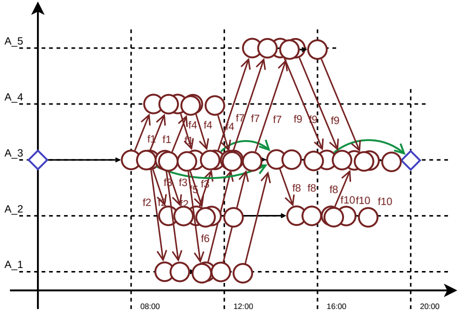

Delay modeling, acyclic network but no more acyclic in terms of flights, exponential size

Recovery Network

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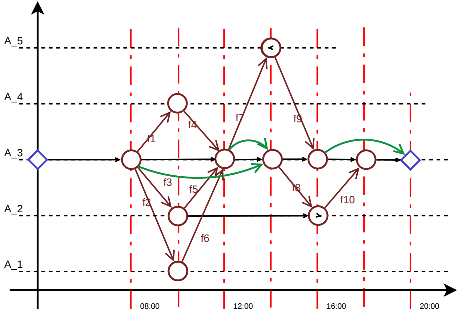


Time band discretization pseudo-polynomial size but unfeasible recovery schemes are encoded

Generating recovery schemes

Given Ω', (x_r^*, y_f^*), $(\lambda_f^*, \eta_s^*, \mu_p^*, \nu_l^*, \rho_l^*)$, new profitable schemes for plane p are computed by solving an ERCSPP on the Recovery Network, minimizing:

$$\tilde{c}_r^p = c_r^p - \sum_{f \in F} b_r^f \lambda_f^* - \sum_{s \in S} b_r^s \eta_s^* - \mu_p^* - \sum_{l \in L} (b_r^{Dep,l} \nu_l^* + b_r^{Arr,l} \rho_l^*) \qquad \forall p \in P$$

Remark: In principle, the R.N. is not necessary (we can use directly the data) but it allows to compute resource bounds and **statically** eliminate most of the unfeasible schemes.
Bi-directional bounded dynamic programming with DSSR. Righini and Salani (2008).

Implementation issues

The algorithm is implemented with BCP framework by COIN-OR.

Speed up, to comply with ROADEF rules:

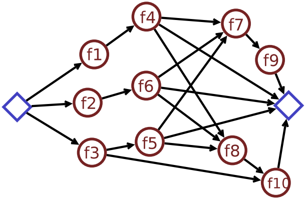
- Network size is reduced by some parameters: permitted delay, permitted plane swaps
- Pricing problem is solved heuristically with relaxed domination criteria and label elimination
- Heuristic search tree exploration

Outline

Passenger routing

An integer solution to z_{MP} gives the aircraft assignment and the flight re-timing or cancellation.
From that solution we build a **unique** connection network which comply with connectivity constraints:

- Arc capacities represent available seats



Passenger itineraries are sorted according to deletion cost and for each itinerary:

- Dummy source and sink connections are the only updated
- Cost of arcs connecting the sink represent the delay cost
- A min-cost flow is solved and decomposed into paths
- Each path is a new itinerary

Conclusions

- The overall recovery procedure is not enough competitive with other methods.

- We easily adapted the code for aircraft disruption recovery with maintenance constraints to comply with ROADEF rules.
- Identified issues: neglected some cost structures, pricing "too heuristic", sequential approach.
- Outlook: solution quality can be improved by a post-processing phase.

Outline

1. Introduction
2. Aircraft Recovery
3. Passenger Recovery
4. Recoverable robustness

Approaches toward robustness

(Airline) schedule disruptions occur because of unpredicted events (noise in the nominal data) which are of stochastic nature.

Reactive and proactive approaches

- Online optimization (Albers (2003))
- Stochastic optimization (with recourse) (Kall and Wallace (1994))
- Worst-case (robust) optimization (Bertsimas and Sim (2004))
- Risk-management/Light robustness (Kall and Mayer (2005), Fischetti and Monaci (2008))

Uncertainty set

Often uncertainty sets (characterization of data fluctuation) are **difficult** to estimate.
Wrong estimation of uncertainty set may lead to **bad** or **too conservative** solutions.

We aim to design an optimization framework which:

- **simple**, has the same complexity as the deterministic problem
- provides solutions with guaranteed deviation from optimum
- does not need for probabilistic uncertainty sets
- **accounts for reactive strategies**

We search a robust **recoverable** solution.

Robustness features

Given a deterministic optimization problem:

$$\begin{aligned} &\min\ f(x) \\ &s.t.\ Ax \leq b \\ &x \in X \end{aligned}$$

Identify **structural properties** $\mu(x)$ of a solution which are exploited by the reactive strategy. Solve a multi-objective optimization problem:

$$\begin{aligned} &\min\ f(x), \max\ \mu(x) \\ &s.t.\ Ax \leq b \\ &x \in X \end{aligned}$$

Relax original objective in a (budget) constraint:

$$\begin{aligned} &\max\ \mu(x) \\ &s.t.\ Ax \leq b \\ &\quad f(x) \leq (1+\rho) f(x^*) \\ &\quad x \in X \end{aligned}$$

Robust recoverable aircraft scheduling

Tactical planning: Re-timing of flights is permitted in the definition of $r \in \Omega$ within a range of 60 minutes.

$$\max z_{RF} = \mu(\mathbf{x}) \tag{8}$$

$$(14)-(15) \tag{9}$$

$$(17)-(21) \tag{10}$$

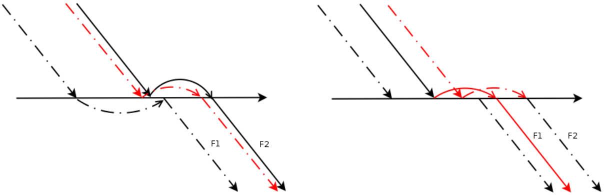
$$\sum_{r \in \Omega} d_r x_r \leq C \tag{11}$$

$$x_r \in \{0,1\} \qquad \forall r \in \Omega \tag{12}$$

$$y_f \in \{0,1\} \qquad \forall f \in F \tag{13}$$

Robust recoverable aircraft scheduling

The recovery algorithm perform better in presence of slack time between flights and effective possibilities of swapping planes.



Increase the minimal idle time of schedule r

$$\mu_{IT}(\mathbf{x}) = \sum_{r \in \Omega} \delta_r^{\mathbf{min}} x_r$$

Quadratic formulation

$$\mu_{CROSS}(\mathbf{x}) = \sum_{r \in \Omega} \sum_{p \in \Omega} b_{rp} x_r x_p$$

Robust recoverable aircraft scheduling

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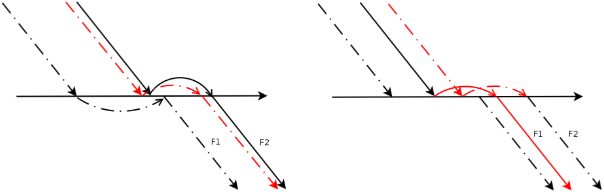

Increase the minimal idle time of schedule r

$$\mu_{IT}(\mathbf{x}) = \sum_{r \in \Omega} \delta_r^{\mathbf{min}} x_r$$

We define meeting points m

$$\sum_{r \in \Omega} b_r^m x_r - y_m \geq 0 \qquad \forall m \in M$$

$$\mu_{CROSS}(\mathbf{x}) = \sum_{m \in M} (y_m - 1)$$

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Robust results

Results on ROADEF09 set A instances (average)

	Original	CROSS	CROSS	IT	IT
BUDGET [min]	0	5000	10000	5000	10000
RECOVERY COST	788775.1	633395.6	555400.3	488701.9	493521.8
# Canceled Flts	6.9	6.9	5.3	5.8	5.9
Total Delay [min]	2142.9	2083.0	2421.8	2214.9	1895.6
Avg Delay[min]	41.0	37.9	42.0	36.9	36.5
# Cancelled Psg	582.8	499.3	420.0	384.5	385.3
# Delayed Psg	553.5	511.1	454.1	501.1	448.1
Avg Psg Delay [min]	34.6	38.7	24.6	29.5	29.8

Eggenberg And S. (2008b).

Thanks

Thanks for your attention

Any question?

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