Robust scheduling and distruption recovery for airlines

Niklaus Eggenberg, **Matteo Salani**

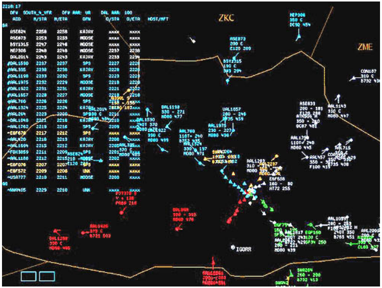
Transport and mobility laboratory
ENAC - EPFL

IDSIA
July 03, 2009

Outline

Outline

Airlines = Complexity

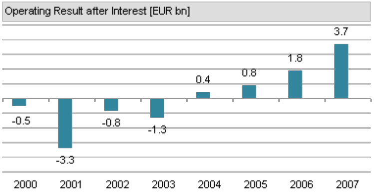


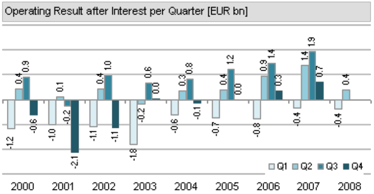
- International / Intercontinental
- Network of flights
- Aircraft types (heterogeneous fleet)
- Airport capacities (gates, slots)
- Air traffic control
- Security/Environmental regulations
- Strict safety requirements (maintenance)
- Infinite workforce rules
- Complicated cost structure/pricing
- High competition/Uncertain demand

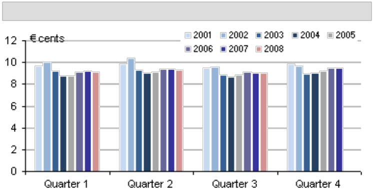
This is a **nightmare** for practitioners and **fun** for researchers.

AEA Financial Report 2000-2008

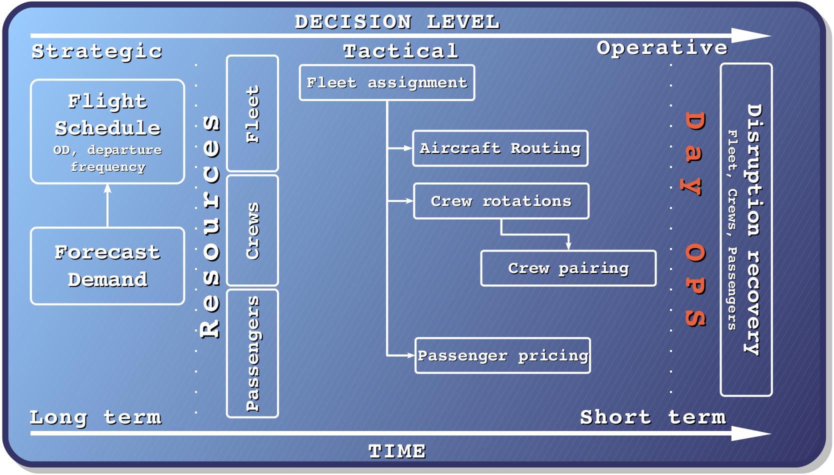
Low margins (around 2-3%)







Airline planning process



Need for robustness

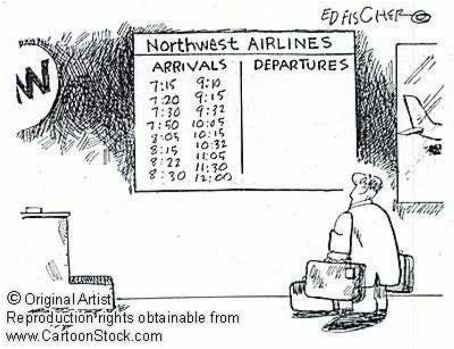
Disruptions are unpredicted events which significantly modify the assumptions (data) used for decision making.

For example, in 2007, 21.1% of departures and 22.3% of arrivals in Europe were delayed by more than 15 minutes. The average cancellation rate is about 1.5% for short haul and 0.6% for long haul.

Robust decision making accounts for noise in the data to obtain a more **stable** system.

The most robust plan

No service is not acceptable!



Recovery strategies

When the system is in a disrupted state (data is revealed or a disruption happens), we refer to a **recovery strategy** as the sequence of actions to restore an operational state of the system.

Often used as an **alternative** to robust planning to cope with noisy data.

Recovery algorithms tend to minimize the total cost of additional operations to restore operational state. Different baseline solutions may incur in different recovery costs given the same disruption.

Outline

Aircraft recovery problem (ARP)

Restore an operational state minimizing **recovery time** and **costs**.

The problem:

- Given a **baseline** schedule.
- Recover within a given time horizon an airline schedule in a disrupted state minimizing the recovery costs
- Known: ACs' position, ACs' expected position, airports, passengers itineraries
- Recovery cost structure used by a major european airline: linear combination of flight and itinerary cancellation and delay, swapping, up-down grading,...

Data

After data preprocessing, the relevant informations are:

- F: a set of scheduled flights, together with an estimation of cancellation cost c_f
- P: a set of aircrafts
- R: a set of passengers (itineraries)
- I_p, I_r: a set of initial positions for both aircrafts and passengers
- S_p, S_r: a set of required final positions for both aircrafts and passengers
- T: a time horizon
- L: a set of airport slots
- q_l^{Dep}, q_l^{Arr}: slot capacities for take off and landings

Master problem

We model the recovery problem for aircrafts as:

$$\min z_{MP} = \sum_{r\in\Omega} c_r x_r + \sum_{f\in F} c_f y_f \tag{1}$$

$$\sum_{r\in\Omega} b_r^f x_r + y_f = 1 \qquad \forall f \in F \tag{2}$$

$$\sum_{r\in\Omega} b_r^s x_r = 1 \qquad \forall s \in S_p \tag{3}$$

$$\sum_{r\in\Omega} b_r^p x_r \leq 1 \qquad \forall p \in P \tag{4}$$

$$\sum_{r\in\Omega} b_r^{Dep,l} x_r \leq q_l^{Dep} \qquad \forall l \in L \tag{5}$$

$$\sum_{r\in\Omega} b_r^{Arr,l} x_r \leq q_l^{Arr} \qquad \forall l \in L \tag{6}$$

$$x_r \in \{0,1\}\ \forall r \in \Omega,\ \ y_f \in \{0,1\}\ \forall f \in F \tag{7}$$

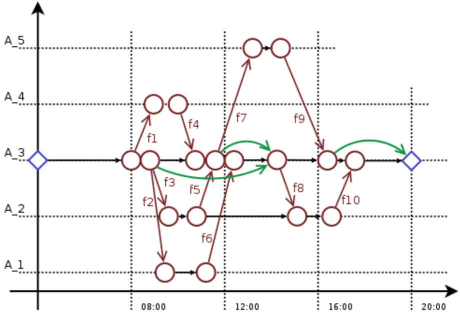
Solved by an optimization based heuristic (Column Generation + Dynamic Programming) on a constraint specific recovery network. Eggenberg, S. And Bierlaire (2008a).

TRANSP-OR

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Recovery Network

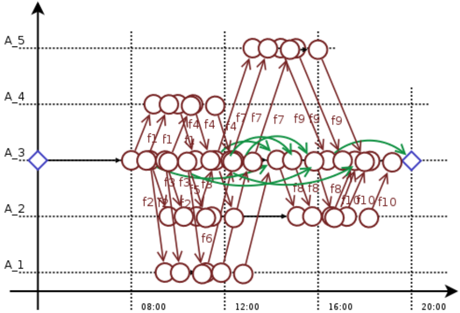
Given T, I_p and S_p the R.N. encodes all possible recovery schemes for plane p.



Scheduled flights, acyclic, polynomial size

Recovery Network

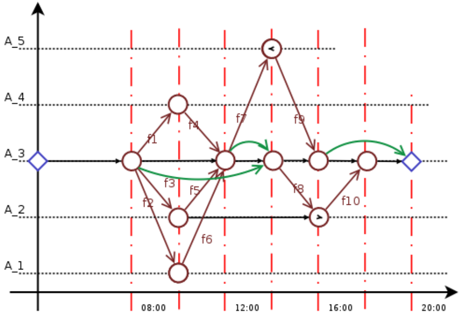
Given T, I_p and S_p the R.N. encodes all possible recovery schemes for plane p.



Delay modeling, acyclic network but no more acyclic in terms of flights, exponential size

Recovery Network

Given T, I_p and S_p the R.N. encodes all possible recovery schemes for plane p.



Time band discretization pseudo-polynomial size but unfeasible recovery schemes are encoded

TRANSP-OR

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Generating recovery schemes

Given Ω', (x^*_r, y^*_f), $(\lambda^*_f, \eta^*_s, \mu^*_p, \nu^*_l, \rho^*_l)$, new profitable schemes for plane p are computed by solving an ERCSPP on the Recovery Network, minimizing:

$$\tilde{c}^p_r = c^p_r - \sum_{f \in F} b^f_r \lambda^*_f - \sum_{s \in S} b^s_r \eta^*_s - \mu^*_p - \sum_{l \in L} (b^{Dep,l}_r \nu^*_l + b^{Arr,l}_r \rho^*_l) \qquad \forall p \in P$$

Bi-directional bounded dynamic programming with DSSR. Righini and S. (2008).

Implementation issues

The algorithm is implemented with BCP framework by COIN-OR.

Speed up, to comply with restricted time limitations:

- Network size is reduced by some parameters: permitted delay, permitted plane swaps
- Pricing problem is solved heuristically with relaxed domination criteria and label elimination
- Heuristic search tree exploration
- Primal heuristics

Real world instances

Mid-size airline (500 flights/week) - synthetic instances (delays, cancellations, apt closures and maintenance disruptions)

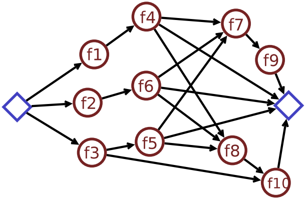
	+ 5%	+ 10%	+ 20%	Heur	Opt
# canceled flts	5	4.7	5.5	2.7	1.5
# delayed flts	52.7	46.7	33.2	2.2	2
# uncovered final states	1.2	0.7	0.3	0.1	0.1
total delay [min]	851.3	635.7	712.5	89.6	52.3
max delay [min]	271.3	251.5	218.2	37.7	37.1

Eggenberg, S. And Bierlaire (2008a).

Outline

Passenger recovery

An integer solution to z_{MP} gives the aircraft assignment and the flight re-timing or cancellation.
From that solution we build a **unique** connection network which comply with connectivity constraints:



- Arc capacities represent available seats

Passenger itineraries are sorted according to deletion cost and for each itinerary:

- Dummy source and sink connections are the only updated
- Cost of arcs connecting the sink represent the delay cost
- A min-cost flow is solved and decomposed into paths
- Each path is a new itinerary

Results for passenger recovery

Instance	A01		A02		A03		A04	
Algorithm	FlowPRP	PRP	FlowPRP	PRP	FlowPRP	PRP	FlowPRP	PRP
# canceled psg	41	33	196	79	499	293	196	116
# rerouted psg	235	2848	587	2468	900	3092	1875	6431
# delayed psg	8664	7852	10430	9969	8798	8569	15612	14365
total delay [min]	280312	259133	581312	557593	511026	523042	1004023	841422
average delay [min]	32.3	33.0	55.7	55.9	58.1	61.0	64.3	58.6
recovery costs	89477	111351	342267	219789	703928	451378	289384	185004
run time [s]	0.66	3155	0.95	1806	1.17	2425	1.84	3755

Instance	A06		A07		A08		A09	
Algorithm	FlowPRP	PRP	FlowPRP	PRP	FlowPRP	PRP	FlowPRP	PRP
# canceled psg	44	10	441	148	954	579	1161	334
# rerouted psg	243	2779	445	2462	843	3167	1206	7427
# delayed psg	11293	10469	13007	12997	10898	11323	18892	19367
total delay [min]	350257	332786	700504	715910	653078	678746	1154229	1171478
average delay [min]	31.0	31.8	53.9	55.1	59.9	59.9	61.1	60.5
recovery costs	99893	64859	632736	268534	1363047	775285	1459473	330232
run time [s]	0.47	3781	0.72	2562	0.97	3363	1.20	3973

Table: Results for the PRP - ROADEF dataset.

Outline

Approaches toward robustness

(Airline) schedule disruptions occur because of unpredicted events (noise in the nominal data) which are of stochastic nature.

Reactive and proactive approaches

- Online optimization (Albers (2003))
- Stochastic optimization (with recourse) (Kall and Wallace (1994))
- Worst-case (robust) optimization (Bertsimas and Sim (2004))
- Risk-management/Light robustness (Kall and Mayer (2005), Fischetti and Monaci (2008))

Uncertainty set

Often uncertainty sets are **difficult** to estimate.
Wrong estimation of uncertainty set may lead to even more **unstable** solutions.

Example on cargo loading value maximisation (multi-dimension knapsack), simulation over 16200 scenarios.

$$\max\ c^T x \tag{8}$$

$$s.t.\ (A_i + \varepsilon_i)x \leq b_i \quad \forall i \in I \tag{9}$$

$$x \in \{0,1\} \tag{10}$$

Robust solutions has an average optimality gap of 10%.
When simulated coefficient realization deviates significantly (50-70%) from estimated we obtain more unfeasible solution for the robust approach than the deterministic.

An alternative approach

We aim to design an optimization framework which:

- **simple**, has the same complexity as the deterministic problem
- provides solutions with guaranteed deviation from optimum
- does not need for probabilistic uncertainty sets
- **accounts for reactive strategies**

We search a robust **recoverable** solution.

Robustness features

Given a deterministic optimization problem:

$$\min\ f(x)$$
$$s.t.\ x \in X$$

Identify **structural properties** $\mu(x)$ of a solution which are exploited by the reactive strategy. Solve a multi-objective optimization problem:

$$\min\ f(x), \max\ \mu(x)$$
$$s.t.\ x \in X$$

Relax original objective in a (budget) constraint:

$$\max\ \mu(x)$$
$$s.t\ f(x) \leq (1+\rho) f(x^*)$$
$$x \in X$$

Remark: stochastic and robust optimization can be obtained for specific $\mu(x)$ and ρ.

Stochastic optimization, Birge and Louveaux (1997)

Stochastic optimization:

$$\begin{aligned}\mu_{\texttt{Stoc}}(\mathbf{x}) &= -\mathbb{E}_U(f(\mathbf{x}))\\ z_{\texttt{Stoc}} &= \min \mathbb{E}_U(f(\mathbf{x}))\\ &\alpha(\mathbf{x}) \leq \mathbf{b}\\ &f(\mathbf{x}) \leq (1+\rho)f^*\\ &\mathbf{x} \in X\end{aligned}$$

Stochastic optimization with recourse:

$$\begin{aligned}\mu_{\texttt{Rec}}(\mathbf{x}) &= -[f(\mathbf{x}) + \mathbb{E}_U(g(\mathbf{x},\xi))]\\ z_{\texttt{Rec}} &= \min f(\mathbf{x}) + \mathbb{E}_U(g(\mathbf{x},\xi))\\ &\alpha(\mathbf{x}) \leq \mathbf{b}\\ &f(\mathbf{x}) \leq (1+\rho)f^*\\ &\mathbf{x} \in X\end{aligned}$$

Robust optimization, Bertsimas and Sim (2004)

$$
\begin{aligned}
(F) \quad z_F^* &= \min_{\mathbf{x}\in X} \{f(\mathbf{x})\} \\
&= \min_{\mathbf{x}\in X} \{\max_{i=1,\ldots,n} (f_i(\mathbf{x}))\} \\
&= \min_{\mathbf{x}\in X} \left\{\max_{i=1,\ldots,n} \left(\sum_{j=1}^{m} a_{ij}x_j + \beta_i(\mathbf{x}, J_i) - b_i\right)\right\}
\end{aligned}
$$

$$
\rho = \max_{i=1,\ldots,n} \left\{ \frac{\rho_i f_i(\mathbf{x}^*)}{z_F^*} - 1 \right\},
$$

where ρ_i is defined as the ratio:

$$
\rho_i = \begin{cases} \frac{\overline{\beta}_i(\mathbf{x},\Gamma_i)}{f_i(\mathbf{x}^*)} & \text{if } f_i(\mathbf{x}^*) > 0 \\ 0 & \text{otherwise.} \end{cases}
$$

$$
\mu_{\text{Rob}}(\mathbf{x}) = -\mathbf{c}^{\mathbf{T}}\mathbf{x}
$$

Robust recoverable aircraft scheduling

Tactical planning: Re-timing of flights is permitted in the definition of $r \in \Omega$ within a range of 60 minutes.

$$\max z_{RF} = \mu(\mathbf{x}) \tag{11}$$

$$(14) - (15) \tag{12}$$

$$(17) - (21) \tag{13}$$

$$\sum_{r \in \Omega} c_r x_r + c_f y_f \leq (1+\rho) z_D^* \tag{14}$$

$$x_r \in \{0,1\} \quad \forall r \in \Omega \tag{15}$$

$$y_f \in \{0,1\} \quad \forall f \in F \tag{16}$$

Robust recoverable aircraft scheduling

Tactical planning: Re-timing of flights is permitted in the definition of $r \in \Omega$ within a range of 60 minutes.

$$\max z_{RF} = \mu(\mathbf{x}) \tag{11}$$

$$(14) - (15) \tag{12}$$

$$(17) - (21) \tag{13}$$

$$\sum_{r \in \Omega} d_r x_r \leq C \tag{14}$$

$$x_r \in \{0,1\} \qquad \forall r \in \Omega \tag{15}$$

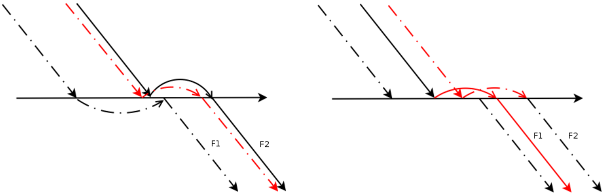
Outline

TRANSP-OR

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Robust recoverable aircraft scheduling

The recovery algorithm perform better in presence of slack time between flights and effective possibilities of swapping planes.



Increase the minimal idle time of schedule r

$$\mu_{IT}(\mathbf{x}) = \sum_{r \in \Omega} \delta_r^{\mathbf{min}} x_r$$

Quadratic formulation

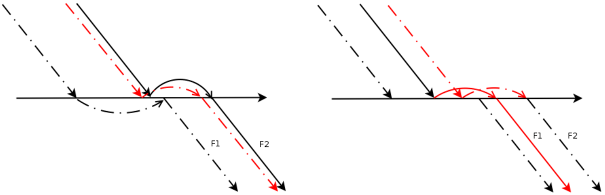
$$\mu_{CROSS}(\mathbf{x}) = \sum_{r \in \Omega} \sum_{p \in \Omega} b_{rp} x_r x_p$$

TRANSP-OR

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Robust recoverable aircraft scheduling

The recovery algorithm perform better in presence of slack time between flights and effective possibilities of swapping planes.



Increase the minimal idle time of schedule r

$$\mu_{IT}(\mathbf{x}) = \sum_{r \in \Omega} \delta_r^{\mathbf{min}} x_r$$

We define meeting points m

$$\sum_{r \in \Omega} b_r^m x_r - y_m \geq 0 \qquad \forall m \in M$$

$$\mu_{CROSS}(\mathbf{x}) = \sum_{m \in M} (y_m - 1)$$

TRANSP-OR

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Robust results

Results on ROADEF09 set A instances (average)

	Original	CROSS	CROSS	IT	IT
BUDGET [min]	0	5000	10000	5000	10000
RECOVERY COST	788775.1	633395.6	555400.3	488701.9	493521.8
# Canceled Flts	6.9	6.9	5.3	5.8	5.9
Total Delay [min]	2142.9	2083.0	2421.8	2214.9	1895.6
Avg Delay[min]	41.0	37.9	42.0	36.9	36.5
# Cancelled Psg	582.8	499.3	420.0	384.5	385.3
# Delayed Psg	553.5	511.1	454.1	501.1	448.1
Avg Psg Delay [min]	34.6	38.7	24.6	29.5	29.8

Eggenberg And S. (2009).

Outline

Conclusions

- Noisy data and primary objectives can lead to unstable solutions.
- Computational tractability of hard combinatorial problems represent an issue for stochastic or robust optimization.
- In several cases, uncertainty set is hardly identifiable.
- Knowledge of reactive strategies can help.
- Robustness features are structural properties of a solution which are computationally tractable.
- Simultaneous robustness and recoverability is a promising approach for reliable operations planning.

Outlook:

- Under final validation on airline scheduling (robust passenger connections).
- To be validated on container terminal optimization.
- Applications to network planning with restoring to be explored.
- Comparison with stochastic optimization with recourse is still weak.

Thanks

Thanks for your attention

Any question?

TRANSP-OR

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

References

Albers, S. (2003). **Online algorithms: A survey**, Mathematical Programming 97: 3-26.
Bertsimas, D. and Sim, M. (2004). **The price of robustness**, Operations Research 52: 35-53.
Eggenberg, N., Salani, M. and Bierlaire, M. (2008a). **Constraint specific recovery networks for solving airline recovery problems**, Technical Report TRANSP-OR 080828, Ecole Polytechnique Fédérale de Lausanne, Switzerland.
Eggenberg, N., Salani, M. and Bierlaire, M. (2008b). **Uncertainty feature optimization: an implicit paradigm for problems with noisy data**, Technical Report TRANSP-OR 080829, Ecole Polytechnique Fédérale de Lausanne, Switzerland.
Eggenberg, N. and Salani, M. (2009). **Uncertainty feature optimization for airline scheduling**, Technical Report TRANSP-OR 090105, Ecole Polytechnique Fédérale de Lausanne, Switzerland.
Fischetti, M. and Monaci, M. (2008). **Light robustness**, Technical report, DEI, Università di Padova, Italy.
Kall, P. and Wallace, S. (eds) (1994). **Stochastic Programming**, John Wiley & Sons, New York, N.Y.
Kall, P. and Mayer, J. (2005). **Stochastic Linear Programming, Models, Theory and Computation**, Springer.
Righini, G. and Salani, M. (2008). **New dynamic programming algorithms for the resource constrained shortest path problem**, Networks 51(3)155-170