The Vehicle Routing Problem with Discrete Split Deliveries and Time Windows

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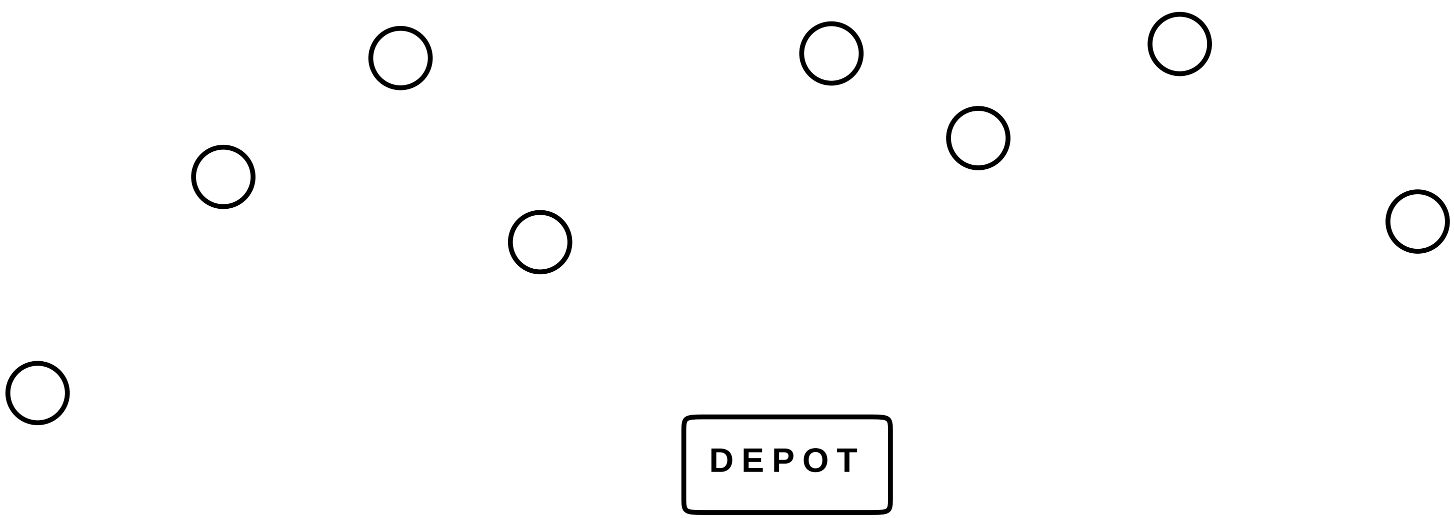
Outline

- Problem description and applications
- MILP formulation and Column generation
- Branch-and-price algorithm
- Computational results
- Conclusions and further research

The Vehicle Routing Problem (VRP)

Given

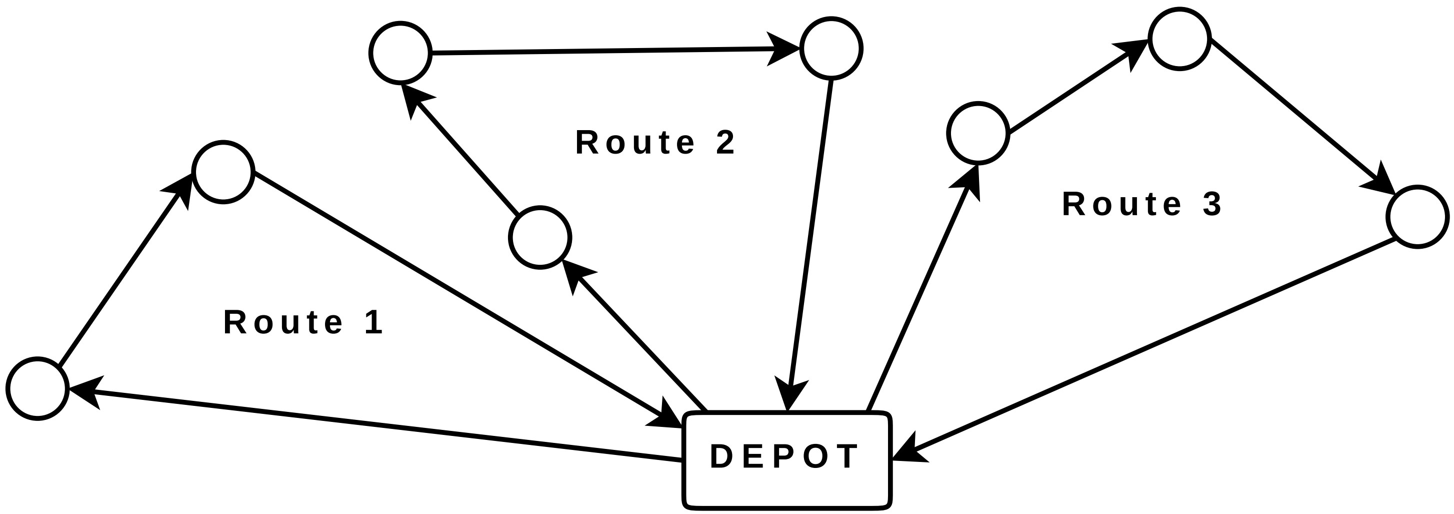
- set of customers with demands to be served (within time windows: VRPTW);
- set of capacitated vehicles available at a single depot;



The Vehicle Routing Problem (VRP)

Objective

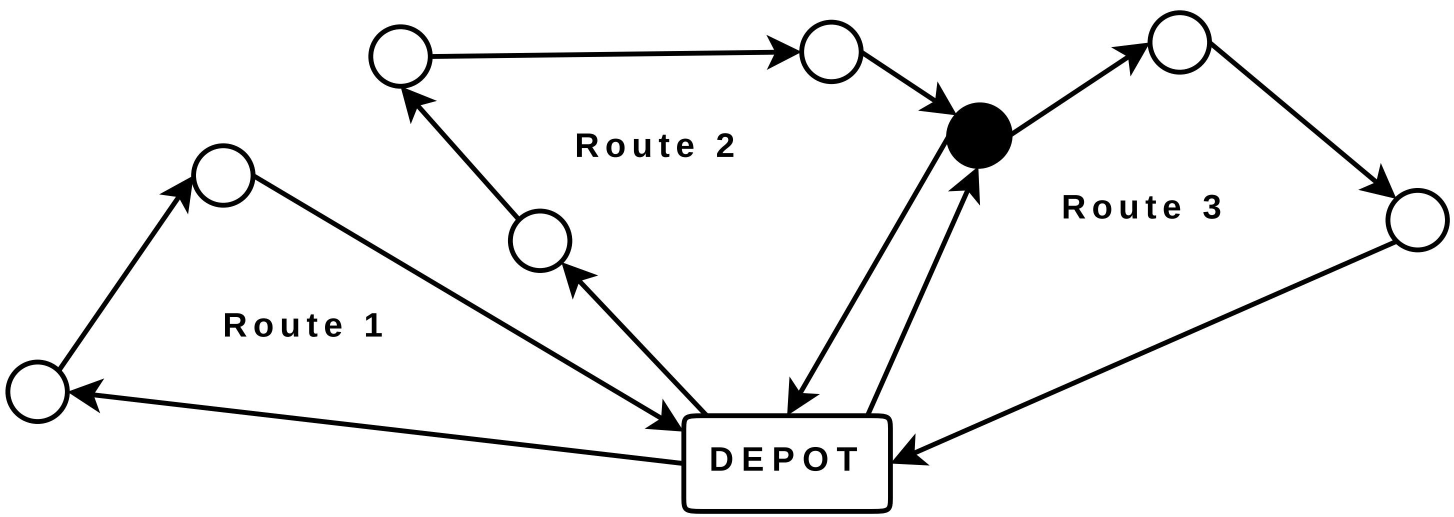
- design the optimal minimum-cost routes for vehicles,
- such that every customer is visited exactly once.



VRP with Split Deliveries (SDVRP)

Splittable demand

- demand can be split and thus served by more than one vehicle;
- customers can be visited more than once.



VRP with Discrete Split Deliveries (DSDVRP)

Demand is discretized in items

- variant of VRP with (continuous) split deliveries;
- the demand of each customer is represented by a set of items;
- demand can be split but items cannot.

Demand is delivered in combinations of items

- items are grouped and delivered in combinations (subsets of items);
- for every customer, feasible combinations of items are predefined and known;
- some combinations of items are not allowed.

Sierksma & Tijssen (1998); Nakao & Nagamochi (2007); Ceselli et al. (2009).

DSDVRP with Time Windows (DSDVRPTW)

Service time is quantity-dependent

- we define *service time* the time needed to perform the delivery once the vehicle is arrived at customer's location;
- in our model, service time depends on the quantity delivered;
- in particular, a specific service time is associated to every feasible combination of items;
- we relax the usual assumption of constant service times.

Known properties

Property 1. The SDVRPTW is NP-Hard.

- Property 1 holds for DSDVRPTW.

Property 2. There exists an optimal solution of the SDVRPTW in which no two routes have more than one split demand in common.

- Property 2 doesn't hold for DSDVRPTW (because of discrete demand and quantity-dependant service times).

Field Technician Scheduling Problem

Xu & Chiu (2001)

Problem description

- different types of jobs which require different skills;
- each technician is specialized in a field with certain skills;
- time windows on job starting and completion;
- assignment problem (jobs to technicians) + scheduling problem, where the duration of a job depends on the assignment.

Objective

- maximize the number of jobs completed within a time frame.

TBAP with QC assignment in container terminals

Giallombardo, Moccia, Salani & Vacca (2010)

Problem description

- Tactical Berth Allocation Plan (TBAP): assignment and scheduling of ships to berths;
- Quay-Cranes (QC) assignment: a QC profile (number of QCs per shift) is assigned to each ship;
- feasible profiles can vary in length (number of shifts dedicated to the ship) and in size (number of QCs dedicated to the ship in each active shift);
- time windows on ship arrival and on berth availabilities.

Objective

- maximize the value of chosen profiles.

Modeling the Discrete Split Delivery VRPTW

- $G = (V, E)$ complete graph with $V = \{0\} \cup N$;
- (c_{ij}, t_{ij}) : cost and travel time of arc $(i, j) \in E$;
- N : set of customers $\{1, ..., n\}$; node $\{0\}$ represents the depot;
- K : set of vehicles with identical capacity Q;
- R : set of items; $R = \bigcup_{i \in N} R_i, \;\; R_i \cap R_j = \emptyset \; \forall i \neq j, \;\; i, j \in N$;
- C : set of combinations of items; $C = \bigcup_{i \in N} C_i, \;\; C_i \cap C_j = \emptyset \; \forall i \neq j, \;\; i, j \in N$;
- e_c^r : 1 if item $r \in R$ belongs to combination $c \in C$;
- t_c : service time of combination $c \in C$ such that $\max_{r \in R} e_c^r t^r \leq t_c \leq \sum_{r \in R} e_c^r t_r$;
- q_c : size of combination $c \in C$; $q_c = \sum_{r \in R} e_c^r t^r$;
- $[a_i, b_i]$: time window for customer $i \in N$.

Arc-flow formulation

Decision variables

- x_{ij}^k binary: 1 if arc $(i,j) \in E$ is used by vehicle $k \in K$, 0 otherwise;
- y_c^k binary: 1 if vehicle $k \in K$ delivers combination $c \in C$, 0 otherwise;
- $T_i^k \geq 0$: time when vehicle $k \in K$ arrives at customer $i \in N$.

Objective function

- minimize the total traveling costs: $z^* = \min \sum_{k \in K} \sum_{(i,j) \in E} c_{ij} x_{ij}^k$

Constraints

- flow and precedence constraints;
- demand-satisfaction constraints;
- time-windows constraints;
- capacity constraints.

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Arc-flow formulation

Flow and linking constraints

$$\sum_{j \in V} x_{0j}^k = 1 \qquad \forall k \in K, \tag{1}$$

$$\sum_{j \in V} x_{ij}^k - \sum_{j \in V} x_{ji}^k = 0 \qquad \forall k \in K, \forall i \in V, \tag{2}$$

$$\sum_{j \in V} x_{ij}^k = \sum_{c \in C_i} y_c^k \qquad \forall k \in K, \forall i \in N, \tag{3}$$

Covering constraints

$$\sum_{k \in K} \sum_{c \in C} e_c^r y_c^k = 1 \qquad \forall r \in R, \tag{4}$$

$$\sum_{c \in C_i} y_c^k \leq 1 \qquad \forall k \in K, \forall i \in N, \tag{5}$$

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Arc-flow formulation

Precedence constraints

$$T_i^k + \sum_{c \in C_i} t_c y_c^k + t_{ij} - T_j^k \leq (1 - x_{ij}^k)M \qquad \forall k \in K, \forall i \in N, \forall j \in V, \tag{6}$$

$$T_i^k - t_{0i} \geq (1 - x_{0i}^k)M \qquad \forall k \in K, \forall i \in N, \tag{7}$$

Time windows

$$T_i^k \geq a_i \sum_{j \in V} x_{ij}^k \qquad \forall k \in K, \forall i \in N, \tag{8}$$

$$T_i^k + \sum_{c \in C_i} t_c y_c^k \leq b_i \sum_{j \in V} x_{ij}^k \qquad \forall k \in K, \forall i \in N, \tag{9}$$

Capacity constraints

$$\sum_{c \in C} q_c y_c^k \leq Q \qquad \forall k \in K. \tag{10}$$

Dantzig-Wolfe reformulation

- P : set of feasible routes (constraints (1)-(3) + (5)-(10));
- c_p : cost of route $p \in P$;
- e_p^r : binary parameter equal to 1 if item $r \in R$ is delivered in route $p \in P$;
- λ_p : binary decision variable equal to 1 if route $p \in P$ is chosen;

Master problem (path formulation)

$$\min \sum_{p \in P} c_p \lambda_p \tag{11}$$

$$\sum_{p \in P} e_p^r \lambda_p = 1 \qquad \forall r \in R, \tag{12}$$

$$\sum_{p \in P} \lambda_p \leq |K|, \tag{13}$$

$$\lambda_p \geq 0 \qquad \forall p \in P. \tag{14}$$

Column generation scheme

The so-called **Restricted Master Problem** (RMP) is repeatedly solved on a subset of variables λ, which otherwise would be an exponential number.

At each iteration of column generation we add profitable variables not yet in the formulation by solving the *pricing subproblem*.

Pricing subproblem

Find the route (column) p with the minimum reduced cost $\tilde{c}_p$:

$$p^* = \arg\min_{p \in P}\{\tilde{c}_p\} = \arg\min_{p \in P}\{c_p - \sum_{r \in R} \pi_r e_p^r - \pi_0\} \tag{15}$$

where π_r are the dual variables associated to constraints (12) and π_0 is the dual variable associated to constraint (13).

Column generation scheme

Generation of columns

- solve the pricing subproblem (identify the min-red-cost column);
- if $\tilde{c}_{p^*} < 0$, then column p^* is added to the (restricted) master problem;
- otherwise, the current master problem solution is proven to be optimal.

Remarks

- the pricing subproblem is an Elementary Resource Constrained Shortest Path Problem solved via dynamic programming;
- the underlying network has one node for each combination and transit time equal to $(t_{ij} + t_c)$.

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Branch & Price for the DSDVRPTW

- exact algorithm based on branch-and-bound;
- column generation at each node of the search tree;
- pricing solved using bi-directional dynamic programming;
- branching rules:
  1. total number of vehicles;
  2. number of vehicles visiting a customer;
  3. flow on one arc;
  4. flow on two consecutive arcs.
- cutting: 2-path inequalities at the root node.

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Instances

- derived from Solomon's data sets R1, C1 and RC1 for the VRPTW;
- $N = 25, 50$ customers;
- $Q = 30, 50, 100;$
- the demand of each customer is discretized in 12 items;
- we generated 3 scenarios:

scenario	combinations	description
A	3	full, 50-50%;
B	5	full, 50-50%, 75-25%;
C	7	full, 50-50%, 75-25%, 90-10%;
O	1	full (unsplit case).

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Computational results: $n = 25$ customers

			A		B		C	
class	nb_inst	Q	nb_solved	t	nb_solved	t	nb_solved	t
R1	12	30	12	7	12	75	8	466
		50	12	6	12	60	12	430
		100	12	9	12	41	12	113
C1	9	30	4	1108	0	x	0	x
		50	9	37	4	2137	0	x
		100	7	706	4	705	2	1876
RC1	8	30	2	1988	0	x	0	x
		100	8	3	8	11	8	35

Time limit: 1 hour.

Computational results: $n = 50$ customers

			A		B		C	
class	nb_inst	Q	nb_solved	t	nb_solved	t	nb_solved	t
R1	12	30	1	1010	0	x	0	x
		50	3	1572	1	385	0	x
		100	3	1035	2	167	2	535
RC1	8	50	7	54	6	902	0	x
		100	8	529	6	809	3	2832

Time limit: 1 hour.

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Optimal solutions for class RC1, $n = 50$, $Q = 50$

id	O			A			B			C		
	z_{IP}	veh	t	z_{IP}	veh	t	z_{IP}	veh	t	z_{IP}	veh	t
rc101	1713.2	20	0	**1708.9**	20	13	**1708.3**	20	594	x		
rc102	1704.3	20	0	**1700.5**	20	62	**1700.5**	20	1938	x		
rc103	1703.4	20	1	**1696.8**	20	37	**1696.8**	20	427	x		
rc104	1702.2	20	1	**1696.7**	20	54	**1696.7**	20	677	x		
rc105	1703.9	20	0	**1700.1**	20	73	**1700.1**	20	1132	x		
rc107	1704.1	20	1	**1698.6**	20	58	x			x		
rc108	1702.2	20	2	**1696.7**	20	83	**1696.7**	20	645	x		

Summing up

- 76% (A), 60% (B) and 48% (C) instances solved for 25 customers;
- 25% (A), 17% (B) and 6% (C) instances solved for 50 customers;
- 96% (R1), 37% (C1) and 36% (RC1) instances solved for 25 customers;
- 11% (R1), 0% (C1) and 42% (RC1) instances solved for 50 customers;
- difficulty increases with the number of customers and combinations;
- split deliveries are more frequent with small values of Q;
- in some cases, split deliveries not only decrease the total traveling costs but also allow to save one vehicle.

Conclusions

- finding optimal solutions is difficult, already with a few combinations;
- only a limited number of instances with 50 customers could be solved;
- the bottleneck is the pricing problem:
  - the underlying network is huge (one node per each combination!)
  - how to efficiently handle this feature of the problem?

Ongoing work

New framework: two-stage column generation

- methodology to accelerate an overall B&P algorithm via generation of compact formulation variables;
- useful when the compact formulation exhibits a large number of variables;
- useful to detect sub-optimal compact formulation variables.

Application to the DSDVRPTW

- we start with a subset of combinations (corresponding to a subset of y_c variables);
- we compute a bound of the reduced cost for variables y_c not yet considered;
- we add the most profitable variable and we iterate.

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Thanks for your attention!

More details on **transp-or.epfl.ch** :

M. Salani and I. Vacca, *Branch-and-price for the Vehicle Routing Problem with Discrete Split Deliveries and Time Windows*, Technical report TRANSP-OR 091224.