A Benders decomposition approach for the choice-based uncapacitated facility location and pricing problem

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Motivation

- In many transportation problems suppliers can benefit from a disaggregate model of demand to capture observed and unobserved heterogeneity.
- Many classes of discrete choice models cannot be easily integrated in mixed integer optimization models.
- The majority of the works in the literature sacrifice complexity either at demand level or at supply level for the sake of tractability.
- Alternative approach: trying to circumvent issues related to non-linearity and non-convexity of the demand function using simulation.

Simulation-based linearization of choice probabilities[1]

- Let I be the universal choice set and N be the set of heterogeneous customers.
- Random utility models:

$$U_{in} = V_{in} + \epsilon_{in} \qquad \forall i \in I, \forall n \in N.$$

- Choice probabilities:

$$P_{in} = \Pr[V_{in} + \epsilon_{in} = \max_{j \in I}(V_{jn} + \varepsilon_{jn})].$$

- Linearization:

$$U_{inr} = V_{in} + \xi_{inr} \qquad \forall i \in I, \forall n \in N, \forall r \in R,$$

$$x_{inr} = \begin{cases} 1 \text{ if } U_{inr} = \max_{j \in I} U_{jnr}, \\ 0 \text{ otherwise} \end{cases} \qquad \forall i \in I, \forall n \in N, \forall r \in R,$$

$$P_{in} = \frac{1}{|R|} \sum_{r \in R} x_{inr} \qquad \forall i \in I, \forall n \in N.$$

[1] Pacheco Paneque et al., "Integrating advanced discrete choice models in mixed integer linear optimization" (2021).

Facility location and pricing with disaggregate demand

- The utility of location $i \in I$ depends on the socioeconomic characteristics of the customer and on the attributes of the alternative (e.g. price, type of service).
- We assume that, if opened, the service is offered at a price chosen among a finite set.
- We consider discrete price levels and "explode" the set I by considering one alternative per price level, all other attributes being the same.
- By doing so, the utilities become parameters of the optimization model:

$$\hat{U}_{inr} = \beta_{p,inr}\hat{p}_i + \beta_{q,inr}\hat{q}_{inr} + \xi_{inr} \quad \forall i \in I, \forall n \in N, \forall r \in R.$$

Notation

Sets

I	Universal choice set.
$I_k \subset I$	Subset of alternatives controlled by the optimizing supplier.
N	Set of heterogeneous (groups of) customers.
R	Set of independent scenarios.

Parameters

θ_n	Number of homogeneous people in group $n \in N$.
$\hat{U}_{inr} \geq 0$	Potential utility of alternative $i \in I$ for $n \in N$ in scenario $r \in R$.
$\hat{p}_i$	Exogenous price of alternative $i \in I$.
F_i	Fixed cost of opening facility $i \in I_k$.
V_i	Variable cost of offering facility $i \in I_k$ to one customer.

Decision variables:

$y_i \in \{0, 1\}$	1 if supplier k offers alternative $i \in I_k$, 0 otherwise.
$x_{inr} \geq 0$	1 if customer $n \in N$ chooses $i \in I$ in scenario $r \in R$, 0 otherwise.

Mathematical model

$$\max_{y} \quad \pi = -\sum_{i \in I_k} F_i y_i + \sum_{i \in I_k} \sum_{n \in N} \sum_{r \in R} \frac{1}{|R|} \theta_n (\hat{p}_i - V_i) x_{inr}, \tag{1}$$

$$s.t. \quad \sum_{i \in I} x_{inr} = 1 \qquad \forall n \in N, \forall r \in R, \tag{2}$$

$$x_{inr} \leq y_i \qquad \forall i \in I, \forall n \in N, \forall r \in R, \tag{3}$$

$$\sum_{j \in I} \hat{U}_{jnr} x_{jnr} \geq \hat{U}_{inr} y_i \qquad \forall i \in I, \forall n \in N, \forall r \in R, \tag{4}$$

$$x_{inr} \geq 0 \qquad \forall i \in I, \forall n \in N, \forall r \in R, \tag{5}$$

$$y_i \in \{0, 1\} \qquad \forall i \in I. \tag{6}$$

Benders decomposition

1. Master problem (MILP):

$$\min_y \quad \pi = \sum_{i \in I_k} F_i y_i - \sum_{n \in N} \sum_{r \in R} z_{nr}, \tag{7}$$

$$s.t. \quad \text{Benders cuts}, \tag{8}$$

$$y_i \in \{0, 1\} \qquad \forall i \in I. \tag{9}$$

2. Independent dual subproblems (LP) can be solved for each $r \in R$ (and each $n \in N$) to obtain the dual variables. Not all dual subproblems need to be solved each time.

3. Benders optimality cuts can be added to the master problem:

$$z_{nr} \geq \sum_{i \in I} (m^*_{inr}) y_i + \sum_{i \in I} c^*_{inr}.$$

(1) - (3) are repeated until convergence.

Branch-and-Benders-cut and enhancements

- Solving the master problem at each iteration is inefficient: Benders cuts can be inserted while processing the branch-and-bound tree.[2]
- Classical Benders cuts provide slow convergence → efficient cuts are key to the success of this approach:
  - Pareto-optimal cuts[3,4],
  - Minimal infeasible subset cuts[5]
  - Partial Benders decomposition[6]

  This is still a work in progress.

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[2] Fischetti, Ljubić, and Sinnl, "Redesigning Benders decomposition for large-scale facility location" (2017).

[3] Magnanti and Wong, "Accelerating Benders decomposition: Algorithmic enhancement and model selection criteria" (1981).

[4] Papadakos, "Practical enhancements to the Magnanti–Wong method" (2008).

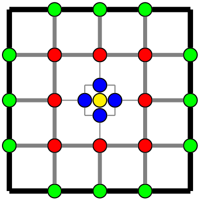
[5] Côté, Dell'Amico, and Iori, "Combinatorial Benders' cuts for the strip packing problem" (2014).

[6] Crainic et al., "Partial Benders decomposition: general methodology and application to stochastic network design" (2021).

Numerical experiments

- Location and price of parking facilities for commuters.
- Mixed logit model taken from the literature[7].
- Demand heterogeneity: 20 origins, 2 income levels, 2 car types. $|N| = 80$.
- 12 candidate locations: 8 on-street + 4 underground.
- 5 possible price levels for each location. $|I_k| = 60$.
- $|R| = 5, 10, 20, 50, 100$ for computational analysis.

$\|I_k\|$	$\|N\|$	$\|R\|$	Optimal solution	CPLEX	Ours
60	80	10	2727.20	166	351
		20	2585.15	566	1744
		50	2493.74	2513	7076
		100	2522.12	9608	28856



[7] Ibeas et al., "Modelling parking choices considering user heterogeneity" (2014).

Summary

- Many supply problems where demand is modeled at a disaggregate level using **advanced discrete choice models** can be written as stochastic optimization problems by relying on **simulation**.
- The resulting formulation exhibit a **block-diagonal structure** which make it particularly suitable to the use of decomposition techniques such as Benders.
- We are working on **efficient enhancements** for our Benders approach and on expanding our analysis beyond the current facility location setting.
- We are planning to investigate data-driven approaches to **generate tighter cuts** and reduce computational times.