A demand-based optimization approach to model oligopolistic competition

Stefano Bortolomiol

Michel Bierlaire   Virginie Lurkin

Transport and Mobility Laboratory (TRANSP-OR)
École Polytechnique Fédérale de Lausanne

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Oligopolistic competition

- Demand: consumers as utility maximizers.
  Supply: producers as profit maximizers.
- Market power: suppliers make strategic decisions which take into account interactions between actors.
- Interactions:
  - Supply-demand
  - Supply-supply

Oligopolies in transportation

Demand: discrete choice

- Customers make indivisible and mutually exclusive purchases.
- Customers have different tastes and socioeconomic characteristics that influence their choice.
- Discrete choice models take into account preference heterogeneity and model individual decisions.

Demand: discrete choice

Nonlinear formulation:

- The probability of customer $n \in N$ choosing alternative $i \in I$ depends on the discrete choice model specification.

- For logit models there are closed-form expressions, e.g. for MNL:

$$P_{in} = \frac{\exp(V_{in})}{\sum_{j \in I} \exp(V_{jn})}$$

- For other discrete choice models, there are no closed-form expressions and numerical approximation is needed.

Demand: discrete choice

Linearized formulation [Pacheco Paneque et al., 2017]:

- A linear formulation can be obtained by relying on simulation to draw from the distribution of the error term of the utility function.
- For all customers and all alternatives, R draws of are extracted from the error term distribution. Each ξ_{inr} corresponds to a different behavioral scenario.

$$U_{inr} = V_{in} + \xi_{inr}$$

- In each scenario, customers choose the alternative with the highest utility:

$$w_{inr} = 1 \text{ if } U_{inr} = \max_{j \in I} U_{jnr}, \text{ and } w_{inr} = 0 \text{ otherwise}$$

- Over multiple scenarios, the probability of customer n choosing alternative i is given by

$$P_{in} = \frac{\sum_{r \in R} w_{inr}}{R}.$$

Supply: optimization

- Suppliers choose the strategy that maximizes their profits.
- Decisions can include the price, but also quantity, quality and availability of the offered products. The related variables can be continuous or discrete.
- Discrete choice models are embedded into the constrained optimization problem of the suppliers.

Demand-based optimization: linear model

$$\max_{s=(p,X)} \quad z_s = \sum_{i \in I_k} \sum_{n \in N} p_{in} P_{in} - \sum_{i \in I_k} c_i(s, w)$$

$$s.t. \quad P_{in} = \frac{1}{R} \sum_{r \in R} w_{inr} \qquad \forall i \in I, \forall n \in N$$

$$U_{inr} = \beta^p_{in} p_{in} + \beta_{in} X_{in} + q_{in} + \xi_{inr} \qquad \forall i \in I, \forall n \in N, \forall r \in R$$

$$U_{inr} \leq U_{nr} \qquad \forall i \in I, \forall n \in N, \forall r \in R$$

$$U_{nr} \leq U_{inr} + M_{U_{nr}}(1 - w_{inr}) \qquad \forall i \in I, \forall n \in N, \forall r \in R$$

$$\sum_{i \in I} w_{inr} = 1 \qquad \forall n \in N, \forall r \in R$$

Other constraints

Supply-supply interactions

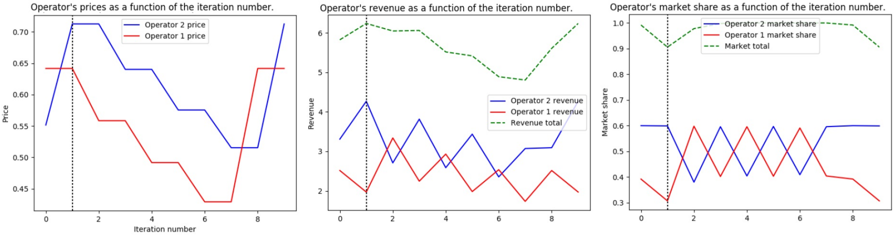
- We consider non-cooperative games.
- Pure strategy Nash equilibrium solutions: stationary states of the system in which no competitor has an incentive to change its decisions.
- Existence, uniqueness, algorithms to find them.

Oligopolistic market equilibrium

- Literature on continuous problems, e.g. electricity markets [Sherali et al., 1983, Pang and Fukushima, 2005, Leyffer and Munson, 2010].
- General assumptions:
  - ✓ continuously differentiable demand curve;
  - ✓/ ✗ continuously differentiable supply curve;
  - ✗ concave profit function.
- We have no proof of existence.
- We can still search for pure strategy equilibria:
  - Fixed-point iteration method
  - Fixed-point MIP model

The fixed-point iteration method

- Sequential algorithm to find an equilibrium solution of a k-player game:
  - Initialization: players start from an initial feasible solution.
  - Iterative phase: players take turns and each plays its best response pure strategy to the current solution.
  - Termination: a Nash equilibrium or a cyclic equilibrium is reached.



Graphs by Nicolas Pradignac (EPFL)

The fixed-point iteration method: applications

- Used in Adler [2001] and Adler et al. [2010] to study a deregulated air transportation market and multimodal rail-air competition.
- Nested logit to model demand. Due to non-concavity, there can be zero, one or more than one pure strategy equilibria.
- Different initial states lead to different solutions. No discrimination between different equilibrium or cyclic equilibrium solutions.
- Case studies related to strategic level decisions: generalizations and averages are reported.
- Also used in Maskin and Tirole [1988] to model dynamic oligopolies in which firms make short-term commitments.

The fixed-point MIP model

- We can minimize the *distance* between two consecutive iterations.
- A generic solution for an oligopolistic market with k players: $s_1^{'}, s_2^{'}, ..., s_k^{'}$, with $s_k^{'} = (p_k, X_k)$.
- Optimization problems for the suppliers:

$$s_k^{''} = \arg\max_{s_k \in S_K} V_k(s_k, s^{'}_{K\setminus\{k\}})$$

- All supplier simultaneously solve a best-response problem to the initial (unknown) solution.
- This approach requires finite sets of strategies.

The fixed-point MIP model

- Minimization problem:

$$z^* = \min \quad \sum_{k \in K} |s_k^{''} - s_k^{'}|$$

- If $z^* = 0$, we have an equilibrium solution.
  If $z^* > 0$, can we still derive meaningful information?

- The objective function allows to discriminate between different equilibrium or near-equilibrium solutions.

Algorithmic framework: our methodology

1. Identify candidate equilibrium solutions or regions efficiently.
2. Use exact method on restricted strategy sets derived from candidate solutions to find subgame equilibria: fixed-point MIP model, linearized formulation.
3. Verify if best-response conditions are satisfied for the initial problem. If they are not, add strategies to the restricted problem and go to step 2.
4. Compare different equilibrium or near-equilibrium solutions: ε-equilibria [Radner, 1980], supergames.

Step 1: identify candidate equilibrium regions

- The sequential game generally converges to an "interesting" region of the solution space within few iterations.
- At this stage any fast heuristic that finds near-optimal solutions of the demand-based optimization model is good.
- Nonlinear formulations are faster than the linear formulation for simple discrete choice models. Their performance rapidly deteriorates in case of more complex choice models or with discrete supply decisions.

Steps 2 and 3: captive customers

- The linearized formulation is combinatorial on the sets I, N, R and S. We need to reduce the dimension of the problem to use the fixed-point MIP model efficiently.
- Optimal strategies at equilibrium are determined by a subset of undecided customers.
- Within a limited range of supply decisions (e.g. prices), most customers are captive.
- The simulation of the error term of the utility function and the use of binary variables allows to precompute choices through lower and upper bounds.

$$LB(U_{inr}) > \max_{j \in I : j \neq i} UB(U_{jnr}) \implies \begin{cases} w_{inr} = 1 \\ w_{jnr} = 0 \quad \forall j \in I, j \neq i \end{cases}$$

Steps 2 and 3: a column-generation-like approach

- The fixed-point MIP model is used to solve a subgame with restricted strategy sets initially derived from the results of step 1.
- The optimal solution of the restricted problem is a subgame equilibrium, which is then verified on the initial game by solving best-response problems.
- If at least one supplier can improve its profits considerably (more than ε), the subgame equilibrium is not accepted as game equilibrium. Best-response strategies are added to the restricted set.

Step 4: ε-equilibrium solutions

- A number of ε-equilibrium with different tolerance factors is provided at the end of the algorithmic framework.
- Ex-post analyses could answer questions about dominance, Pareto optimality or tacit collusion.

Testing the framework

- We can use existing discrete choice model estimations available in the literature.
- Tests on two transportation datasets:
  - Urban parking choice (proof of concept)
  - High-speed rail competition (case study)

High-speed rail competition

- Single European Railway Directive (2012) and Railway Packages promoting open access operations on European railways.
- In 2012 Italy was the first country to open to high-speed rail competition. Liberalization of passenger railway transport to be effective in the European Union from December 2020.
- Plethora of ex-ante and ex-post research [Ben-Akiva et al., 2010, Cascetta and Coppola, 2012, Valeri, 2013, Mancuso, 2014, Cascetta and Coppola, 2015, Beria et al., 2016].

High-speed rail competition

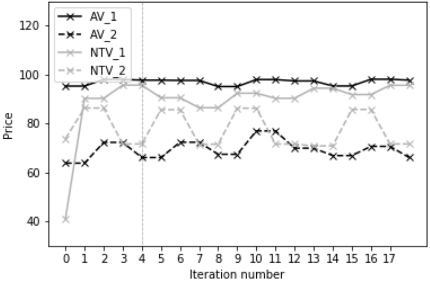
Demand:

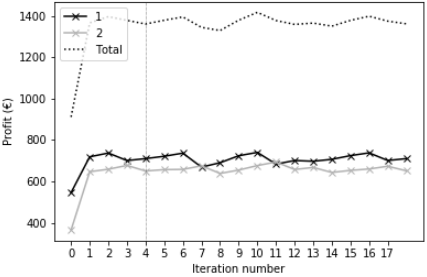
- Testing with MNL model based on SP survey collected in 2010 Ben-Akiva et al. [2010] to forecast demand and market shares in the competitive market.
- Nested logit model parameters presented by Cascetta and Coppola [2012].

Supply:

- Scenarios based on the current market for the Milan-Rome OD pair, with prices as decision variables.
- Mode choice (cost, travel time, reason for travel, income, origin).
- Mode-run choice (previous attributes and socio-economic characteristics plus early/late arrival).

Numerical experiments: sequential game





Numerical experiments: column-generation-like approach

Supplier	$\mid S \mid$	Alternative	Bounds		MIP		Best-response		ε
			LB	UB	Price	Profit	Price	Profit	
Supp1	6	1st 2nd	95.11 66.12	98.10 77.98	95.11 66.12	715	98.09 76.21	737	0.03
Supp2	6	1st 2nd	86.45 70.88	95.62 86.28	93.39 82.43	644	90.21 86.34	652	

Table: Iteration 1

Numerical experiments: column-generation-like approach

Supplier	$\mid S \mid$	Alternative	Bounds		MIP		Best-response		ε
			LB	UB	Price	Profit	Price	Profit	
Supp1	7	1st 2nd	95.11 66.12	98.10 77.98	96.60 71.55	733	98.03 72.22	737	
Supp2	7	1st 2nd	86.45 70.88	95.62 86.34	90.21 86.34	654	93.81 71.67	670	0.023

Table: Iteration 2

Numerical experiments: column-generation-like approach

Supplier	$\vert S \vert$	Alternative	Bounds		MIP		Best-response		ε
			LB	UB	Price	Profit	Price	Profit	
Supp1	8	1st 2nd	95.11 66.12	98.10 77.98	95.11 66.12	702	97.73 66.92	707	0.008
Supp2	8	1st 2nd	86.45 70.88	95.62 86.34	93.81 71.67	649	90.21 86.34	652	

Table: Iteration 3

Numerical experiments: ε-equilibrium solutions

Equilibrium	Supplier 1			Supplier 2			ε
	1st	2nd	Profit	1st	2nd	Profit	
E1	91.71	82.18	719	91.42	72.35	705	0.012
E2	96.20	85.02	758	99.56	80.65	721	0.029
E3	93.96	83.60	708	91.42	72.35	714	0.019
E4	92.64	82.65	722	101.80	70.93	728	0.021
E5	95.08	84.31	755	100.21	80.52	720	0.020

Table: List of ε-equilibrium solutions

Summary

- Demand-based optimization: discrete choice models are embedded in the optimization problem of the supplier (nonlinear and linearized formulations).
- Oligopolistic competition: equilibrium solutions, if they exist, can be found by solving a fixed-point problem.
- Algorithmic approach:
  - quickly find candidate equilibrium regions;
  - solve subgames with a fixed-point MIP model and check solution by computing best responses on the original solution space;
  - compare different equilibrium or ε-equilibrium solutions.
- Application to a real-life case study: flexible and scalable framework, interpretable results.

Next steps and open questions

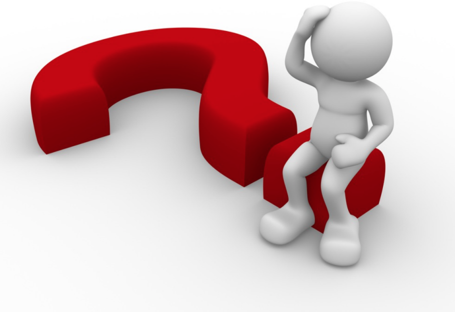
Next steps:

- Test a mode-run choice scenario using a nested logit model and a set of customers having desired arrival times.
- Investigate high-speed rail pricing structures and effects of capacity constraints.

Open questions:

- Is it possible to relax the assumption of full information, which we currently exploit in our simultaneous approach?
- ...

Questions and discussion time



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Transport and Mobility Laboratory (TRANSP-OR)

École Polytechnique Fédérale de Lausanne (EPFL)

Email: stefano.bortolomiol(at)epfl.ch