A demand-based optimization approach to model oligopolistic competition

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Oligopolistic competition

- Demand: consumers as utility maximizers.
  Supply: producers as profit maximizers.

- Market power: suppliers make strategic decisions which take into account interactions between actors.

- Interactions:
  - Supply-demand
  - Supply-supply
Oligopolies in transportation
1 Demand-based optimization

2 Oligopolistic market equilibrium

3 Algorithmic framework

4 Numerical experiments and case study
Demand: discrete choice

- Customers make indivisible and mutually exclusive purchases.
- Customers have different tastes and socioeconomic characteristics that influence their choice.
- Discrete choice models take into account preference heterogeneity and model individual decisions.
 Demand: discrete choice

**Nonlinear formulation:**

- The probability of customer $n \in N$ choosing alternative $i \in I$ depends on the discrete choice model specification.

- For logit models there are closed-form expressions, e.g. for MNL:

  $$P_{in} = \frac{\exp(V_{in})}{\sum_{j \in I} \exp(V_{jn})}$$

- For other discrete choice models, there are no closed-form expressions and numerical approximation is needed.
Demand: discrete choice

**Linearized formulation** [Pacheco Paneque et al., 2017]:

- A linear formulation can be obtained by relying on simulation to draw from the distribution of the error term of the utility function.

- For all customers and all alternatives, $R$ draws of $\xi$ are extracted from the error term distribution. Each $\xi_{inr}$ corresponds to a different behavioral scenario.

  $$U_{inr} = V_{in} + \xi_{inr}$$

- In each scenario, customers choose the alternative with the highest utility:

  $$w_{inr} = 1 \text{ if } U_{inr} = \max_{j \in I} U_{jnr}, \text{ and } w_{inr} = 0 \text{ otherwise}$$

- Over multiple scenarios, the probability of customer $n$ choosing alternative $i$ is given by

  $$P_{in} = \frac{\sum_{r \in R} w_{inr}}{R}.$$
Supply: optimization

- Suppliers choose the strategy that maximizes their profits.

- Decisions can include the price, but also quantity, quality and availability of the offered products. The related variables can be continuous or discrete.

- Discrete choice models are embedded into the constrained optimization problem of the suppliers.
Demand-based optimization: linear model

\[
\begin{align*}
\max_{s=(p,X)} & \quad z_s = \sum_{i \in I_k} \sum_{n \in N} p_{in} P_{in} - \sum_{i \in I_k} c_i(s, w) \\
\text{s.t.} & \quad P_{in} = \frac{1}{R} \sum_{r \in R} w_{inr} \quad \forall i \in I, \forall n \in N \\
& \quad U_{inr} = \beta_{in}^p p_{in} + \beta_{in}X_{in} + q_{in} + \xi_{inr} \quad \forall i \in I, \forall n \in N, \forall r \in R \\
& \quad U_{inr} \leq U_{nr} \quad \forall i \in I, \forall n \in N, \forall r \in R \\
& \quad U_{nr} \leq U_{inr} + M U_{nr}(1 - w_{inr}) \quad \forall i \in I, \forall n \in N, \forall r \in R \\
& \quad \sum_{i \in I} w_{inr} = 1 \quad \forall n \in N, \forall r \in R \\
\end{align*}
\]

Other constraints
1 Demand-based optimization

2 Oligopolistic market equilibrium

3 Algorithmic framework

4 Numerical experiments and case study
Supply-supply interactions

- We consider non-cooperative games.

- Pure strategy Nash equilibrium solutions: stationary states of the system in which no competitor has an incentive to change its decisions.

- Existence, uniqueness, algorithms to find them.
Oligopolistic market equilibrium

- Literature on continuous problems, e.g. electricity markets [Sherali et al., 1983, Pang and Fukushima, 2005, Leyffer and Munson, 2010].

- General assumptions:
  - ✔ continuously differentiable demand curve;
  - ✔/ ✗ continuously differentiable supply curve;
  - ✗ concave profit function.

- We have no proof of existence.

- We can still search for pure strategy equilibria:
  - Fixed-point iteration method
  - Fixed-point MIP model
The fixed-point iteration method

- Sequential algorithm to find an equilibrium solution of a k-player game:
  - Initialization: players start from an initial feasible solution.
  - Iterative phase: players take turns and each plays its best response pure strategy to the current solution.
  - Termination: a Nash equilibrium or a cyclic equilibrium is reached.

Graphs by Nicolas Pradignac (EPFL)
The fixed-point iteration method: applications

- Used in Adler [2001] and Adler et al. [2010] to study a deregulated air transportation market and multimodal rail-air competition.
- Nested logit to model demand. Due to non-concavity, there can be zero, one or more than one pure strategy equilibria.
- Different initial states lead to different solutions. No discrimination between different equilibrium or cyclic equilibrium solutions.
- Case studies related to strategic level decisions: generalizations and averages are reported.
- Also used in Maskin and Tirole [1988] to model dynamic oligopolies in which firms make short-term commitments.
The fixed-point MIP model

- We can minimize the *distance* between two consecutive iterations.

- A generic solution for an oligopolistic market with *k* players: $s_1', s_2', ..., s_k'$, with $s_k' = (p_k, X_k)$.

- Optimization problems for the suppliers:

$$s_k'' = \arg \max_{s_k \in S_K} V_k(s_k, s'_K \setminus \{k\})$$

- All supplier simultaneously solve a best-response problem to the initial (unknown) solution.

- This approach requires finite sets of strategies.
The fixed-point MIP model

- Minimization problem:

\[ z^* = \min \sum_{k \in K} |s_k'' - s_k'| \]

- If \( z^* = 0 \), we have an equilibrium solution.
  If \( z^* > 0 \), can we still derive meaningful information?

- The objective function allows to discriminate between different equilibrium or near-equilibrium solutions.
1 Demand-based optimization

2 Oligopolistic market equilibrium

3 Algorithmic framework

4 Numerical experiments and case study
Algorithmic framework: our methodology

1. Identify candidate equilibrium solutions or regions efficiently.

2. Use exact method on restricted strategy sets derived from candidate solutions to find subgame equilibria: fixed-point MIP model, linearized formulation.

3. Verify if best-response conditions are satisfied for the initial problem. If they are not, add strategies to the restricted problem and go to step 2.

4. Compare different equilibrium or near-equilibrium solutions: $\varepsilon$-equilibria [Radner, 1980], supergames.
Step 1: identify candidate equilibrium regions

- The sequential game generally converges to an "interesting” region of the solution space within few iterations.
- At this stage any fast heuristic that finds near-optimal solutions of the demand-based optimization model is good.
- Nonlinear formulations are faster than the linear formulation for simple discrete choice models. Their performance rapidly deteriorates in case of more complex choice models or with discrete supply decisions.
Steps 2 and 3: captive customers

- The linearized formulation is combinatorial on the sets $I$, $N$, $R$ and $S$. We need to reduce the dimension of the problem to use the fixed-point MIP model efficiently.

- Optimal strategies at equilibrium are determined by a subset of undecided customers.

- Within a limited range of supply decisions (e.g. prices), most customers are captive.

- The simulation of the error term of the utility function and the use of binary variables allows to precompute choices through lower and upper bounds:

$$LB(U_{inr}) > \max_{j \in I: j \neq i} UB(U_{jnr}) \implies \begin{cases} w_{inr} = 1 \\ w_{jnr} = 0 \quad \forall j \in I, j \neq i \end{cases}$$
Steps 2 and 3: a column-generation-like approach

- The fixed-point MIP model is used to solve a subgame with restricted strategy sets initially derived from the results of step 1.

- The optimal solution of the restricted problem is a subgame equilibrium, which is then verified on the initial game by solving best-response problems.

- If at least one supplier can improve its profits considerably (more than $\varepsilon$), the subgame equilibrium is not accepted as game equilibrium. Best-response strategies are added to the restricted set.
Step 4: $\varepsilon$-equilibrium solutions

- A number of $\varepsilon$-equilibrium with different tolerance factors is provided at the end of the algorithmic framework.

- Ex-post analyses could answer questions about dominance, Pareto optimality or tacit collusion.
1 Demand-based optimization

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4 Numerical experiments and case study
Testing the framework

- We can use existing discrete choice model estimations available in the literature.

- Tests on two transportation datasets:
  - Urban parking choice (proof of concept)
  - High-speed rail competition (case study)
High-speed rail competition

- Single European Railway Directive (2012) and Railway Packages promoting open access operations on European railways.

- In 2012 Italy was the first country to open to high-speed rail competition. Liberalization of passenger railway transport to be effective in the European Union from December 2020.

High-speed rail competition

Demand:

- Testing with MNL model based on SP survey collected in 2010 Ben-Akiva et al. [2010] to forecast demand and market shares in the competitive market.

- Nested logit model parameters presented by Cascetta and Coppola [2012].

Supply:

- Scenarios based on the current market for the Milan-Rome OD pair, with prices as decision variables.

- Mode choice (cost, travel time, reason for travel, income, origin).

- Mode-run choice (previous attributes and socio-economic characteristics plus early/late arrival).
Numerical experiments and case study

Numerical experiments: sequential game

![Graph showing sequential game outcomes with price and profit over iterations.](image-url)
### Numerical experiments: column-generation-like approach

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<th>Alternative</th>
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<th>Best-response</th>
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#### Table: Iteration 1
Numerical experiments and case study

Numerical experiments: column-generation-like approach

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Table: Iteration 2
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**Table:** Iteration 3
Numerical experiments: $\varepsilon$-equilibrium solutions

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Table: List of $\varepsilon$-equilibrium solutions
Summary

- Demand-based optimization: discrete choice models are embedded in the optimization problem of the supplier (nonlinear and linearized formulations).

- Oligopolistic competition: equilibrium solutions, if they exist, can be found by solving a fixed-point problem.

- Algorithmic approach:
  - quickly find candidate equilibrium regions;
  - solve subgames with a fixed-point MIP model and check solution by computing best responses on the original solution space;
  - compare different equilibrium or $\varepsilon$-equilibrium solutions.

- Application to a real-life case study: flexible and scalable framework, interpretable results.
Next steps and open questions

Next steps:

- Test a mode-run choice scenario using a nested logit model and a set of customers having desired arrival times.
- Investigate high-speed rail pricing structures and effects of capacity constraints.

Open questions:

- Is it possible to relax the assumption of full information, which we currently exploit in our simultaneous approach?
- ...

Questions and discussion time