A demand-based optimization approach to model oligopolistic competition

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Outline

1. Problem description
2. Oligopolistic market equilibrium
3. Algorithmic framework
Problem description

Oligopolistic market equilibrium

Algorithmic framework
Microeconomic framework

- **Demand**: consumers as utility maximizers.
- **Supply**: producers as profit maximizers.
- **Market**: oligopolistic competition.
Oligopolies

- Market power: suppliers make strategic decisions which take into account interactions between actors.

- Interactions:
  - Supply-demand
  - Supply-supply

- Game theory
Oligopolies in transportation
Oligopolies in transportation
Demand: discrete choice

- Customers make indivisible and mutually exclusive purchases.
- Customers have different tastes and socioeconomic characteristics that influence their choice.
- Discrete choice models take into account preference heterogeneity and model individual decisions.
Demand: discrete choice

Nonlinear formulation:

- The probability of customer $n \in N$ choosing alternative $i \in I$ depends on the discrete choice model specification.

- For logit models there are closed-form expressions, e.g. for MNL:

\[ P_{in} = \frac{\exp(V_{in})}{\sum_{j \in I} \exp(V_{jn})} \]

- For other discrete choice models, there are no closed-form expressions and numerical approximation is needed.
Demand: discrete choice

**Linearized formulation** [Pacheco Paneque et al., 2017]:

- A linear formulation can be obtained by relying on simulation to draw from the distribution of the error term of the utility function.

- For all customers and all alternatives, $R$ draws of are extracted from the error term distribution. Each $\xi_{inr}$ corresponds to a different behavioral scenario.

\[ U_{inr} = \beta_{in} p_{in} + q_{in} + \xi_{inr} \]

- In each scenario, customers choose the alternative with the highest utility:

\[ w_{inr} = 1 \text{ if } U_{inr} = \max_{j \in I} U_{jnr}, \text{ and } w_{inr} = 0 \text{ otherwise} \]

- Over multiple scenarios, the probability of customer $n$ choosing alternative $i$ is given by

\[ P_{in} = \frac{\sum_{r \in R} w_{inr}}{R}. \]
Supply: optimization

- Suppliers choose the strategy that maximizes their profits.
- Strategic decisions include the pricing, quantity and quality of the offered products. The related variables could be continuous or discrete.
- Constrained optimization models can describe the supplier problem.
Demand-based optimization: Stackelberg game

\[
\begin{align*}
\max_s & \quad z_s = \sum_{i \in I_k} \sum_{n \in N} \pi_ip_{in}P_{in} - \sum_{i \in I_k} c_i(s, w) \\
\text{s.t.} & \quad P_{in} = \frac{1}{R} \sum_{r \in R} w_{inr} \\
\end{align*}
\]

\[
\begin{align*}
U_{inr} & = \beta_{p,in}p_{in} + \beta_{in}x_{in} + q_{in} + \xi_{inr} \\
U_{inr} & \leq U_{nr} \\
U_{nr} & \leq U_{inr} + MU_{nr}(1 - w_{inr}) \\
\sum_{i \in I} w_{inr} & = 1 \\
\end{align*}
\]

\[
\forall i \in I, \forall n \in N \\
\forall i \in I, \forall n \in N, \forall r \in R \\
\forall i \in I, \forall n \in N, \forall r \in R \\
\forall n \in N, \forall r \in R
\]
Problem description

Oligopolistic market equilibrium

Algorithmic framework
Supply-supply interactions

- We consider non-cooperative games.
- Pure strategy Nash equilibrium solutions: stationary states of the system in which no competitor has an incentive to change its decisions.
- Existence, uniqueness, algorithms to find them.
Oligopolistic market equilibrium

- Literature on continuous problems, e.g. electricity markets [Sherali et al., 1983, Pang and Fukushima, 2005, Leyffer and Munson, 2010].

- General assumptions:
  - ✔ continuously differentiable demand curve;
  - ✔/ ✗ continuously differentiable supply curve;
  - ✗ concave profit function.

- For the problems we are interested in, we have no proof of existence.

- We can search for pure strategy equilibria:
  - Fixed-point iteration method
  - Fixed-point MIP model
The fixed-point iteration method

- Sequential algorithm to find an equilibrium solution of a k-player game:
  - Initialization: players start from an initial feasible strategy.
  - Iterative phase: players take turns and each plays its best response pure strategy to the current solution.
  - Termination: a Nash equilibrium or a cyclic equilibrium is reached.

Plots by Nicolas Pradignac (EPFL)
The fixed-point iteration method

- Adler [2001] and Adler et al. [2010] study a deregulated air transportation market and multimodal rail-air competition.
- There can be zero, one or more than one pure strategy equilibria.
- Different initial solutions lead to different results.
- No discrimination between different equilibrium or cyclic equilibrium solutions.
The fixed-point MIP model

- We can minimize the *distance* between two consecutive iterations.

- A generic solution for an oligopolistic market: \( s'_1, s'_2, \ldots, s'_k \), with 
  \( s'_k = (p_k, X_k) \).

- Optimization problems for the suppliers:
  \[
  s''_k = \arg \max_{s_k \in S_K} V_k(s_k, s'_{K\setminus\{k\}})
  \]

- Minimization problem:
  \[
  z^* = \min \sum_{k \in K} |s''_k - s'_k|
  \]

- If \( z^* = 0 \), we have an equilibrium solution. What can we say about this equilibrium? If \( z^* > 0 \), can we still derive meaningful information?
Numerical experiments

- Parking choice case study. Discrete choice parameter estimation from the literature [Ibeas et al., 2014].
- Users choose among 3 options: 2 owned by either the same operator or by 2 different operators, 1 opt-out option.
- Tests: nonlinear and linearized formulations with logit and mixed logit specifications.
Oligopolistic market equilibrium

Numerical experiments

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Numerical experiments

- Computational results:
  - Stackelberg game: the nonlinear model is faster on all instances, but no proof of optimality.
  - Fixed-point MIP model: the linear model is more stable with regard to the complexity of the discrete choice model and the supply decision variables.

- Fixed-point MIP model:
  - Finite strategy sets are needed to be able to solve best-response problems as constraints.
  - The size of the strategy sets has an important effect on computational times.
1 Problem description

2 Oligopolistic market equilibrium

3 Algorithmic framework
Algorithmic framework: motivation

- Transport oligopolies: many constraints and many variables (continuous and discrete).
- Neither traditional microeconomic nor game-theoretic approaches are applicable as such.
- Equilibrium problems $\neq$ optimization problems.
- In real-life problems equilibrium is quite loosely defined.
Algorithmic framework: solution approach

- Identify candidate equilibrium solutions or regions efficiently: sequential game, nonlinear or linearized formulation of the Stackelberg game.

- Use exact method on restricted strategy sets derived from candidate solutions to find subgame equilibria: fixed-point optimization model, linearized formulation.

- Verify if best-response conditions are satisfied for the initial problem. If not, add strategies to the restricted problem.

- Compare different equilibrium or near-equilibrium solutions: \(\varepsilon\)-equilibria [Radner, 1980], near-rationality [Akerlof et al., 1985].
Case study: high-speed rail oligopoly

- Single European Railway Directive (2012) and Railway Packages promoting open access operations on European railways.
- In 2012 Italy was the first country to have a real oligopolistic high-speed rail market.
Case study: high-speed rail oligopoly

Demand:

- RP/SP survey collected in 2010 to forecast demand and market shares in the competitive market [Cascetta and Coppola, 2012].
- Discrete choice model estimation using multinomial logit and nested logit.

Supply:

- Realistic competitive scenario based on current market, starting with a single origin-destination pair and with prices as decision variables.
Case study: preliminary tests

- Supply and demand models have not yet been calibrated on the current market.

- Having a well-bounded supply optimization problem is beneficial.
Summary

- Demand-based optimization: discrete choice models can be embedded in the optimization problem of the supplier (nonlinear and linearized formulation).

- Oligopolistic competition: modelled as a sequential game or with a fixed-point MIP model.

- We are working on an algorithmic approach which should:
  - find candidate equilibrium solutions for the initial game;
  - solve restricted equilibrium problems with a demand-based optimization approach;
  - allow comparison between equilibrium or near-equilibrium solutions.

- Application to a real-life case study: flexible and scalable framework, interpretable results.
Questions and discussion time

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References


