A demand-based optimization approach to model oligopolistic competition

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Outline

Microeconomic framework

- Demand: consumers as utility maximizers.
- Supply: producers as profit maximizers.
- Market: oligopolistic competition.

Oligopolies

- Market power: suppliers make strategic decisions which take into account interactions between actors.
- Interactions:
  - Supply-demand
  - Supply-supply
- Game theory

Oligopolies in transportation

Oligopolies in transportation

Demand: discrete choice

- Customers make indivisible and mutually exclusive purchases.
- Customers have different tastes and socioeconomic characteristics that influence their choice.
- Discrete choice models take into account preference heterogeneity and model individual decisions.

Demand: discrete choice

Nonlinear formulation:

- The probability of customer $n \in N$ choosing alternative $i \in I$ depends on the discrete choice model specification.
- For logit models there are closed-form expressions, e.g. for MNL:

$$P_{in} = \frac{\exp(V_{in})}{\sum_{j \in I} \exp(V_{jn})}$$

- For other discrete choice models, there are no closed-form expressions and numerical approximation is needed.

Demand: discrete choice

Linearized formulation [Pacheco Paneque et al., 2017]:

- A linear formulation can be obtained by relying on simulation to draw from the distribution of the error term of the utility function.
- For all customers and all alternatives, R draws of are extracted from the error term distribution. Each ξ_{inr} corresponds to a different behavioral scenario.

$$U_{inr} = \beta_{in} p_{in} + q_{in} + \xi_{inr}$$

- In each scenario, customers choose the alternative with the highest utility:

$$w_{inr} = 1 \text{ if } U_{inr} = \max_{j \in I} U_{jnr}, \text{ and } w_{inr} = 0 \text{ otherwise}$$

- Over multiple scenarios, the probability of customer n choosing alternative i is given by

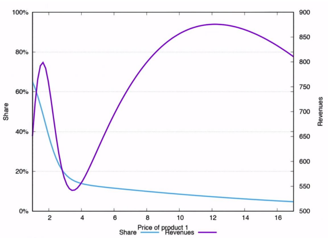
$$P_{in} = \frac{\sum_{r \in R} w_{inr}}{R}.$$

Supply: optimization

- Suppliers choose the strategy that maximizes their profits.
- Strategic decisions include the pricing, quantity and quality of the offered products. The related variables could be continuous or discrete.
- Constrained optimization models can describe the supplier problem.

Demand-based optimization: Stackelberg game

$$
\begin{aligned}
\max_{s} \quad & z_s = \sum_{i \in I_k} \sum_{n \in N} p_{in} P_{in} - \sum_{i \in I_k} c_i(s, w) && \\
s.t. \quad & P_{in} = \frac{1}{R} \sum_{r \in R} w_{inr} && \forall i \in I, \forall n \in N \\
& U_{inr} = \beta_{p,in} p_{in} + \beta_{in} X_{in} + q_{in} + \xi_{inr} && \forall i \in I, \forall n \in N, \forall r \in R \\
& U_{inr} \leq U_{nr} && \forall i \in I, \forall n \in N, \forall r \in R \\
& U_{nr} \leq U_{inr} + M_{U_{nr}} (1 - w_{inr}) && \forall i \in I, \forall n \in N, \forall r \in R \\
& \sum_{i \in I} w_{inr} = 1 && \forall n \in N, \forall r \in R
\end{aligned}
$$



Supply-supply interactions

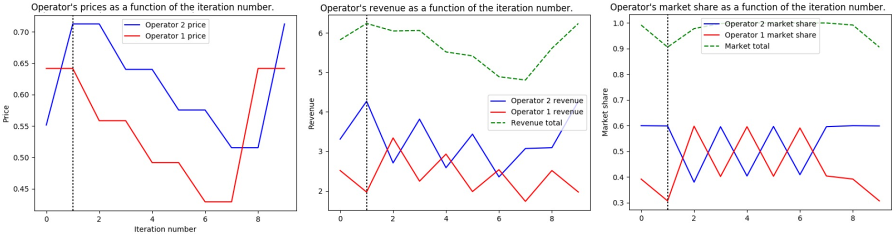
- We consider non-cooperative games.
- Pure strategy Nash equilibrium solutions: stationary states of the system in which no competitor has an incentive to change its decisions.
- Existence, uniqueness, algorithms to find them.

Oligopolistic market equilibrium

- Literature on continuous problems, e.g. electricity markets [Sherali et al., 1983, Pang and Fukushima, 2005, Leyffer and Munson, 2010].
- General assumptions:
  - ✓ continuously differentiable demand curve;
  - ✓/ ✗ continuously differentiable supply curve;
  - ✗ concave profit function.
- For the problems we are interested in, we have no proof of existence.
- We can search for pure strategy equilibria:
  - Fixed-point iteration method
  - Fixed-point MIP model

The fixed-point iteration method

- Sequential algorithm to find an equilibrium solution of a k-player game:
    - Initialization: players start from an initial feasible strategy.
    - Iterative phase: players take turns and each plays its best response pure strategy to the current solution.
    - Termination: a Nash equilibrium or a cyclic equilibrium is reached.



Plots by Nicolas Pradignac (EPFL)

The fixed-point iteration method

- Adler [2001] and Adler et al. [2010] study a deregulated air transportation market and multimodal rail-air competition.
- There can be zero, one or more than one pure strategy equilibria.
- Different initial solutions lead to different results.
- No discrimination between different equilibrium or cyclic equilibrium solutions.

The fixed-point MIP model

- We can minimize the *distance* between two consecutive iterations.
- A generic solution for an oligopolistic market: $s_1^{'}, s_2^{'}, ..., s_k^{'}$, with $s_k^{'} = (p_k, X_k)$.
- Optimization problems for the suppliers:

$$s_k^{''} = \arg\max_{s_k \in S_K} V_k(s_k, s_{K\setminus\{k\}}^{'})$$

- Minimization problem:

$$z^* = \min \quad \sum_{k \in K} |s_k^{''} - s_k^{'}|$$

- If $z^* = 0$, we have an equilibrium solution. What can we say about this equilibrium? If $z^* > 0$, can we still derive meaningful information?

Numerical experiments

- Parking choice case study. Discrete choice parameter estimation from the literature [Ibeas et al., 2014].
- Users choose among 3 options: 2 owned by either the same operator or by 2 different operators, 1 opt-out option.
- Tests: nonlinear and linearized formulations with logit and mixed logit specifications.

Numerical experiments

Instance			MILP				NLP			
DCM	N	R	Time (s)	Obj	p_1	p_2	Time (s)	Obj	p_1	p_2
Logit	10	100	921	6.44	0.67	0.72	0.02	6.36	0.83	0.71
Logit	10	200	7027	6.43	0.66	0.72	0.02	6.36	0.83	0.71
Logit	50	50	7105	32.09	0.68	0.71	0.06	31.93	0.71	0.72
Logit	50	100	55020	32.19	0.68	0.73	0.06	31.93	0.71	0.72
MixL	10	100	2378	5.38	0.55	0.63	0.05	5.31	0.55	0.63
MixL	10	200	3942	5.21	0.54	0.61	0.29	5.22	0.56	0.64
MixL	50	50	13285	27,33	0,58	0.67	0.45	27.20	0.58	0.66
MixL	50	100	72000*	27,00*	0,56*	0.65*	0.70	26.92	0.56	0.66

Table: Leader-follower game

			MILP						MINLP					
DCM	N	R	Time	Obj	p_1	p_2	d_1	d_2	Time	Obj	p_1	p_2	d_1	d_2
Logit	10	100	3679	0	0,05	0,15	2,86	7,14	94	0	0,05	0,15	2,85	7,15
Logit	10	200	5595	0	0,05	0,15	2,84	7,16	94	0	0,05	0,15	2,85	7,15
Logit	50	50	16400	0	0,05	0,15	10,60	39,40	1151	0	0,05	0,15	10,72	39,29
Logit	50	100	6124	0	0,05	0,15	10,81	39,19	1151	0	0,05	0,15	10,72	39,29
MixL	10	100	2204	0	0,15	0,25	3,84	6,16	3499	0	0,10	0,20	3,92	6,08
MixL	10	200	3589	0	0,10	0,20	4,17	5,83	4413	0	0,10	0,20	4,18	5,82
MixL	50	50	13923	0	0,15	0,25	18,28	31,72	16242*	0,19*	0,13*	0,32*	31,42*	18,58*
MixL	50	100	28682	0	0,15	0,25	18,31	31,69	36000*	-	-	-	-	-

Table: Fixed-point MIP model for the multi-leader-follower game

Numerical experiments

- Computational results:
  - Stackelberg game: the nonlinear model is faster on all instances, but no proof of optimality.
  - Fixed-point MIP model: the linear model is more stable with regard to the complexity of the discrete choice model and the supply decision variables.
- Fixed-point MIP model:
  - Finite strategy sets are needed to be able to solve best-response problems as constraints.
  - The size of the strategy sets has an important effect on computational times.

Algorithmic framework: motivation

- Transport oligopolies: many constraints and many variables (continuous and discrete).
- Neither traditional microeconomic nor game-theoretic approaches are applicable as such.
- Equilibrium problems $\neq$ optimization problems.
- In real-life problems equilibrium is quite loosely defined.

Algorithmic framework: solution approach

- Identify candidate equilibrium solutions or regions efficiently: sequential game, nonlinear or linearized formulation of the Stackelberg game.
- Use exact method on restricted strategy sets derived from candidate solutions to find subgame equilibria: fixed-point optimization model, linearized formulation.
- Verify if best-response conditions are satisfied for the initial problem. If not, add strategies to the restricted problem.
- Compare different equilibrium or near-equilibrium solutions: ε-equilibria [Radner, 1980], near-rationality [Akerlof et al., 1985].

Case study: high-speed rail oligopoly

- Single European Railway Directive (2012) and Railway Packages promoting open access operations on European railways.

- In 2012 Italy was the first country to have a real oligopolistic high-speed rail market.

- Plethora of ex-ante and ex-post research [Ben-Akiva et al., 2010, Cascetta and Coppola, 2012, Valeri, 2013, Mancuso, 2014, Cascetta and Coppola, 2015, Beria et al., 2016].

Case study: high-speed rail oligopoly

Demand:

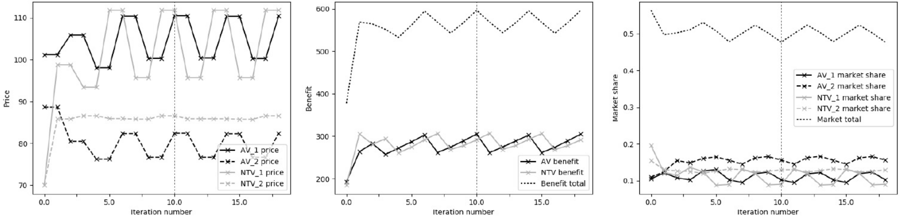
- RP/SP survey collected in 2010 to forecast demand and market shares in the competitive market [Cascetta and Coppola, 2012].
- Discrete choice model estimation using multinomial logit and nested logit.

Supply:

- Realistic competitive scenario based on current market, starting with a single origin-destination pair and with prices as decision variables.

Case study: preliminary tests

- Supply and demand models have not yet been calibrated on the current market.
- Having a well-bounded supply optimization problem is beneficial.



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Summary

- Demand-based optimization: discrete choice models can be embedded in the optimization problem of the supplier (nonlinear and linearized formulation).
- Oligopolistic competition: modelled as a sequential game or with a fixed-point MIP model.
- We are working on an algorithmic approach which should:
  - find candidate equilibrium solutions for the initial game;
  - solve restricted equilibrium problems with a demand-based optimization approach;
  - allow comparison between equilibrium or near-equilibrium solutions.
- Application to a real-life case study: flexible and scalable framework, interpretable results.

Questions and discussion time

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