A demand-based optimization approach to find market equilibria in oligopolies

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Outline

1. Problem description
2. Oligopolistic market equilibrium
3. Current and future work

1 Problem description

2 Oligopolistic market equilibrium

3 Current and future work

Microeconomic theory

- Demand: consumers as utility maximizers.
- Supply: producers as profit maximizers.
- Market: perfect competition vs market power.

Oligopolies

- Market power: suppliers make strategic decisions which take into account interactions between actors.
- Interactions:
  - Supply-demand
  - Supply-supply
- Game theory

Oligopolies in transportation

Oligopolies in transportation

Demand: discrete choice

- Customers make indivisible and mutually exclusive purchases.
- Customers have different tastes and socioeconomic characteristics that influence their choice.
- Discrete choice models take into account preference heterogeneity and model individual decisions.

Demand: discrete choice

Nonlinear formulation:

- The probability of customer $n \in N$ choosing alternative $i \in I$ depends on the discrete choice model specification.
- For the logit model, there is a closed-form expression:

$$P_{in} = \frac{\exp(V_{in})}{\sum_{j \in I} \exp(V_{jn})}$$

- For other discrete choice models, there is no closed-form expression and numerical approximation is needed.

Demand: discrete choice

Linear formulation [Pacheco Paneque et al., 2017]:

- A linear formulation can be obtained by relying on simulation to draw from the distribution of the error term of the utility function.

- For all customers and all alternatives, R draws of are extracted from the error term distribution. Each ξ_{inr} corresponds to a different behavioral scenario.

$$U_{inr} = \beta_{in} p_{in} + q_{in} + \xi_{inr}$$

- In each scenario, customers choose the alternative with the highest utility:

$$w_{inr} = 1 \text{ if } U_{inr} = \max_{j \in I} U_{jnr}, \text{ and } w_{inr} = 0 \text{ otherwise}$$

- Over multiple scenarios, the probability of customer n choosing alternative i is given by

$$P_{in} = \frac{\sum_{r \in R} w_{inr}}{R}.$$

Supply: optimization

- Suppliers choose the strategy that maximizes their profits.
- Strategic decisions include the pricing, quantity and quality of the offered products. The related variables could be continuous or discrete.
- Constrained optimization models can describe the supplier problem.

Demand-based optimization

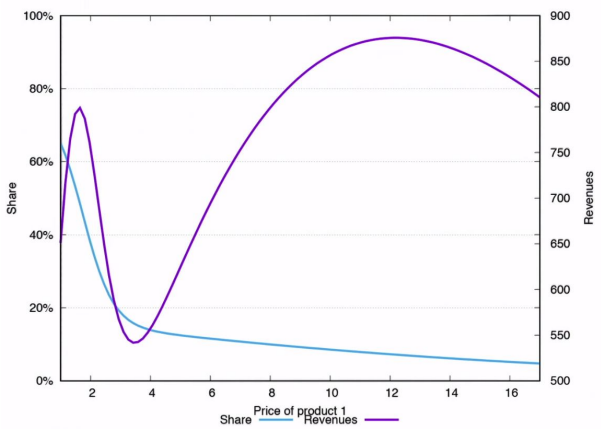
- Stackelberg game:

$$\max_{s} \quad z_s = \sum_{i \in I_k} \sum_{n \in N} p_{in} P_{in} - \sum_{i \in I_k} c_i(s)$$

$$s.t. \quad P_{in} = \frac{\exp(V_{in})}{\sum_{j \in I} \exp(V_{jn})} \qquad \forall i \in I, \forall n \in N$$

$$V_{in} = \beta_{p,in} p_{in} + \beta_{in} X_{in} + q_{in} \qquad \forall i \in I, \forall n \in N$$

- Payoff functions are generally non-convex:



From the MOOC "Introduction to Discrete Choice Models" (Michel Bierlaire and Virginie Lurkin)

1 Problem description

2 Oligopolistic market equilibrium

3 Current and future work

Supply-supply interactions

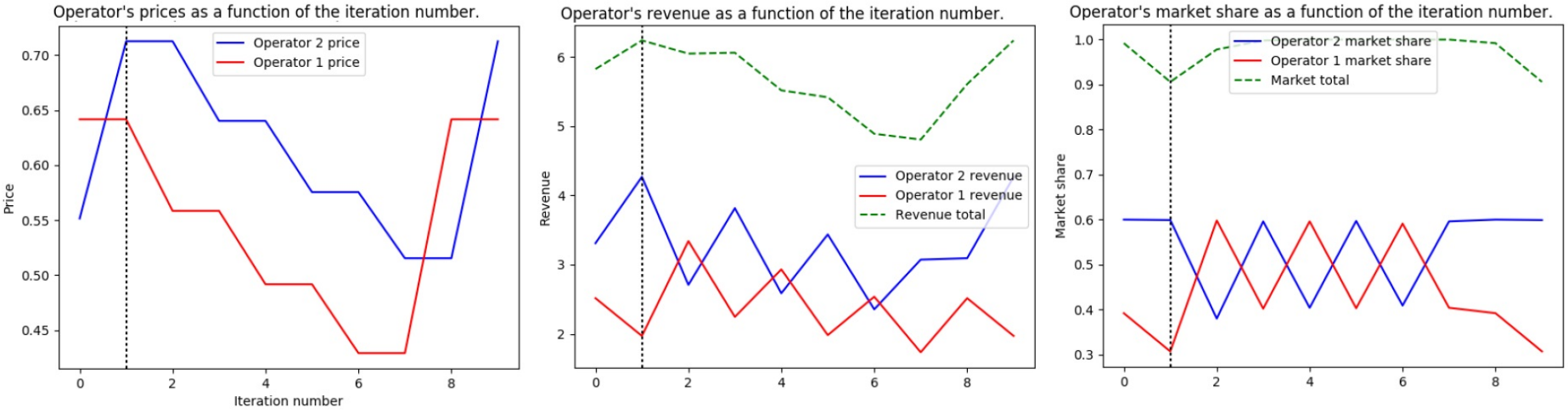
- We consider non-cooperative games.
- Pure strategy Nash equilibrium solutions: stationary states of the system in which no competitor has an incentive to change its decisions.
- Existence, uniqueness, algorithms to find them.

Oligopolistic market equilibria

- Literature on continuous problems, e.g. electricity markets [Sherali et al., 1983, Pang and Fukushima, 2005, Leyffer and Munson, 2010].
- General assumptions:
  - ✓ continuously differentiable demand curve;
  - ✓/ ✗ continuously differentiable supply curve;
  - ✗ concave profit function.
- For the problems we are interested in, we have no existence or uniqueness proof.
- We can search for pure strategy equilibria:
  - Fixed-point iteration method
  - Fixed-point optimization model

The fixed-point iteration method

- Sequential algorithm to find an equilibrium solution of a k-player game:
  - Initialization: players start from an initial feasible strategy.
  - Iterative phase: players take turns and each plays its best response pure strategy to the current solution.
  - Termination: a Nash equilibrium or a cyclic equilibrium is reached.



Plots produced by Nicolas Pradignac (EPFL)

The fixed-point iteration method

- Adler [2001] and Adler et al. [2010] study a deregulated air transportation market and multimodal rail-air competition.
- There can be zero, one or more than one pure strategy equilibria.
- Different initial solutions lead to different results.
- It is not possible to discriminate between different equilibrium or cyclic equilibrium solutions.

The fixed-point optimization model

- We can minimize the *distance* between two consecutive iterations.
- A generic solution for an oligopolistic market: $s_1^{'}, s_2^{'}, ..., s_k^{'}$, with $s_k^{'} = (p_k, X_k)$.
- Optimization problems for the supplier:

$$s_k^{''} = \arg\max_{s_k \in S_K} V_k(s_k, s_{K \setminus \{k\}}^{'})$$

- Minimization problem:

$$z^* = \min \quad \sum_{k \in K} |s_k^{''} - s_k^{'}|$$

- If $z^* = 0$, we have an equilibrium solution. What can we say about this equilibrium? If $z^* > 0$, can we conclude something?

Numerical experiments

- Parking choice case study: users choose among 3 options, 2 owned by 2 different operators and 1 opt-out option.
- Discrete choice parameter estimation available in the literature.
- Tests: nonlinear and linearized formulations with logit and mixed logit specifications.
- Computational results:
  - Stackelberg game: the nonlinear model is faster on all instances and it finds the optimal solution.
  - Fixed-point optimization model: on larger instances and mixed logit specification, the nonlinear model fails to converge.

Numerical experiments

Stackelberg game:

Instance			MILP				NLP			
DCM	N	R	Time (s)	Obj	p_1	p_2	Time (s)	Obj	p_1	p_2
Logit	10	100	921	6.44	0.67	0.72	0.02	6.36	0.83	0.71
Logit	10	200	7027	6.43	0.66	0.72	0.02	6.36	0.83	0.71
Logit	50	50	7105	32.09	0.68	0.71	0.06	31.93	0.71	0.72
Logit	50	100	55020	32.19	0.68	0.73	0.06	31.93	0.71	0.72
MixL	10	100	2378	5.38	0.55	0.63	0.05	5.31	0.55	0.63
MixL	10	200	3942	5.21	0.54	0.61	0.29	5.22	0.56	0.64
MixL	50	50	13285	27,33	0,58	0.67	0.45	27.20	0.58	0.66
MixL	50	100	72000*	27,00*	0,56*	0.65*	0.70	26.92	0.56	0.66

Table: Uncapacitated case

Instance			MILP				MINLP			
DCM	N	R	Time (s)	Obj	p_1	p_2	Time (s)	Obj	p_1	p_2
MixL	5	100	518	2.28	0.58	0.74	3	1.96	0.70	0.84
MixL	5	200	4428	2.30	0.55	0.70	35	2.04	0.70	0.85
MixL	10	100	4564	4.85	0.58	0.73	12	4.84	0.64	0.78
MixL	10	200	72000*	4.70*	0.58*	0.68*	26	4.75	0.63	0.77
MixL	50	50	72000*	26.09*	0.61*	0.77*	163	26.51	0.60	0.76
MixL	50	100	72000*	25.71*	0.60*	0.74*	661	26.19	0.59	0.75

Table: Capacitated case

Numerical experiments

Multi-leader-follower game:

			MILP						MINLP					
DCM	N	R	Time	Obj	p_1	p_2	d_1	d_2	Time	Obj	p_1	p_2	d_1	d_2
Logit	5	50	68	0	0,05	0,15	1,28	3,72	78	0	0,05	0,15	1,54	3,46
Logit	5	100	203	0	0,05	0,15	1,65	3,35	78	0	0,05	0,15	1,54	3,46
Logit	5	200	818	0	0,05	0,15	1,55	3,45	78	0	0,05	0,15	1,54	3,46
Logit	10	50	208	0	0,05	0,15	2,74	7,26	94	0	0,05	0,15	2,85	7,15
Logit	10	100	3679	0	0,05	0,15	2,86	7,14	94	0	0,05	0,15	2,85	7,15
Logit	10	200	5595	0	0,05	0,15	2,84	7,16	94	0	0,05	0,15	2,85	7,15
Logit	50	25	6894	0	0,05	0,15	11,20	38,80	1151	0	0,05	0,15	10,72	39,29
Logit	50	50	16400	0	0,05	0,15	10,60	39,40	1151	0	0,05	0,15	10,72	39,29
Logit	50	100	6124	0	0,05	0,15	10,81	39,19	1151	0	0,05	0,15	10,72	39,29
MixL	5	50	70	0	0,10	0,20	1,96	3,04	849	0	0,10	0,20	2,05	2,95
MixL	5	100	170	0	0,15	0,20	1,52	3,48	747	0	0,10	0,20	2,22	2,78
MixL	5	200	1013	0	0,10	0,20	2,13	2,87	2962	0	0,10	0,20	2,07	2,93
MixL	10	50	291	0	0,15	0,25	4,16	5,84	2019*	0,09*	0,30*	0,39*	3,95*	6,05*
MixL	10	100	2204	0	0,15	0,25	3,84	6,16	3499	0	0,10	0,20	3,92	6,08
MixL	10	200	3589	0	0,10	0,20	4,17	5,83	4413	0	0,10	0,20	4,18	5,82
MixL	50	25	985	0	0,10	0,20	17,24	32,76	7035	0	0,10	0,20	17,09	32,91
MixL	50	50	13923	0	0,15	0,25	18,28	31,72	16242*	0,19*	0,13*	0,32*	31,42*	18,58*
MixL	50	100	28682	0	0,15	0,25	18,31	31,69	36000*	-	-	-	-	-

Table: Fixed-point MIP model

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Algorithmic framework: motivation

- Transport oligopolies: many strategic variables and many constraints, heterogeneous demand makes the problem even more difficult.
- Neither traditional microeconomic nor game-theoretic approaches are applicable as such.
- Equilibrium problems $\neq$ optimization problems.
- However, in real-life problems equilibrium is quite loosely defined.

Algorithmic framework: solution approach

- Identify candidate (sub-game) equilibrium solutions or regions efficiently: sequential game, nonlinear formulation.
- Use exact methods on restricted strategy sets derived from candidate solutions: fixed-point optimization model, linearized formulation.
- Verify if best-response conditions are satisfied for the original game.
- Compare different equilibrium or near-equilibrium solutions: ε-equilibria [Radner, 1980], near-rationality [Akerlof et al., 1985], folk theorem [Friedman, 1971].

Case study: high-speed rail oligopoly

- Single European Railway Directive (2012) and some "Railway packages" promoting open access operations on European railways.
- In 2012 Italy was the first country to have a real oligopolistic high-speed rail market.
- Plethora of ex-ante and ex-post research [Ben-Akiva et al., 2010, Cascetta and Coppola, 2012, Valeri, 2013, Mancuso, 2014, Cascetta and Coppola, 2015, Beria et al., 2016].

Case study: high-speed rail oligopoly

Demand:

- RP/SP survey collected in 2010 to forecast demand and market shares in the competitive market [Cascetta and Coppola, 2012].
- Model estimation using multinomial logit and cross-nested logit.

Supply:

- Starting with a single origin-destination pair and with prices as decision variables.

An applicable framework

Demand:

- Use advanced discrete choice models to better represent the demand.
- Consider endogenous decision variables (continuous or discrete) other than price.

Supply:

- Allow for product and price differentiation across customers.
- Test different cost functions for different operators.

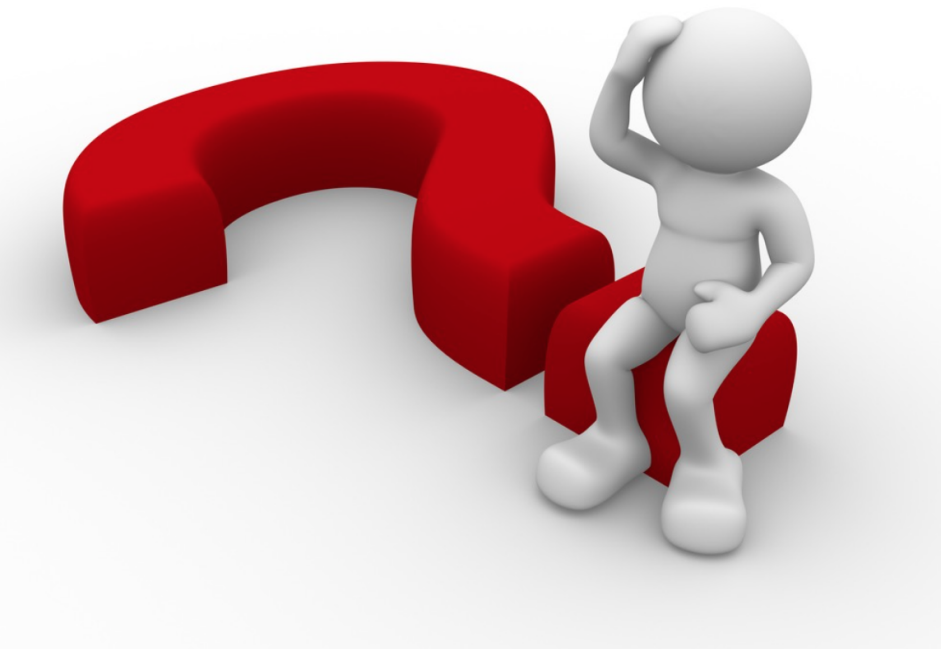
Competition:

- Define equilibrium or near-equilibrium in real-life case studies.

Related fields of work

- Choice-based revenue management [Andersson, 1998, Talluri and Van Ryzin, 2004, Sharif Azadeh et al., 2015].
- Bilevel programming [Labbé et al., 1998, Brotcorne et al., 2001].
- Discrete choice and pricing [Nuzzolo et al., 2000, Aksoy-Pierson et al., 2013, Pels et al., 2017].
- Near-rationality and ε-equilibria [Radner, 1980, Akerlof et al., 1985].
- Dynamic oligopolies and supergames games [Friedman, 1971, Maskin and Tirole, 1988].
- Discrete games [Bresnahan and Reiss, 1991, Aguirregabiria and Mira, 2007].

Questions and discussion time



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