A demand-based optimization approach to model oligopolistic competition

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Introduction

Demand-based optimization

Oligopolistic market equilibrium

Algorithmic framework

Numerical experiments and case study
Oligopolistic competition

- **Demand:** consumers as utility maximizers.
- **Supply:** producers as profit maximizers.

- **Market power:** suppliers make strategic decisions which take into account interactions between actors.

- **Interactions:**
  - Supply-demand
  - Supply-supply
Oligopolies in transportation

[Images of various airlines]
Oligopolies in transportation
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Demand: discrete choice

- Customers make indivisible and mutually exclusive purchases.
- Customers have different tastes and socioeconomic characteristics that influence their choice.
- Discrete choice models take into account preference heterogeneity and model individual decisions.
Demand: discrete choice

Nonlinear formulation:

- The probability of customer $n \in N$ choosing alternative $i \in I$ depends on the discrete choice model specification.

- For logit models there are closed-form expressions, e.g. for MNL:

  $$P_{in} = \frac{\exp(V_{in})}{\sum_{j \in I} \exp(V_{jn})}$$

- For other discrete choice models, there are no closed-form expressions and numerical approximation is needed.
Demand: discrete choice

**Linearized formulation** [Pacheco Paneque et al., 2017]:

1. A linear formulation can be obtained by relying on simulation to draw from the distribution of the error term of the utility function.

2. For all customers and all alternatives, $R$ draws of are extracted from the error term distribution. Each $\xi_{inr}$ corresponds to a different behavioral scenario.

   $$ U_{inr} = V_{in} + \xi_{inr} $$

3. In each scenario, customers choose the alternative with the highest utility:

   $$ w_{inr} = 1 \text{ if } U_{inr} = \max_{j \in I} U_{jnr}, \text{ and } w_{inr} = 0 \text{ otherwise} $$

4. Over multiple scenarios, the probability of customer $n$ choosing alternative $i$ is given by

   $$ P_{in} = \frac{\sum_{r \in R} W_{inr}}{R}. $$
Supply: optimization

- Suppliers choose the strategy that maximizes their profits.
- Decisions can include the price, but also quantity, quality and availability of the offered products. The related variables can be continuous or discrete.
- Discrete choice models are embedded into the optimization problem of the suppliers.
- Constrained optimization models can describe the supplier problem.
Demand-based optimization: nonlinear model

\[
\begin{align*}
\max_{s=(p,X)} & \quad z_s = \sum_{i \in I_k} \sum_{n \in N} p_{in} P_{in} - \sum_{i \in I_k} c_i(s, w) \\
\text{s.t.} & \quad P_{in} = \frac{\exp(V_{in})}{\sum_{j \in I} \exp(V_{jn})} \\
& \quad V_{in} = \beta_{in} p_{in} + \beta_{in} X_{in} + q_{in}
\end{align*}
\]

Other constraints
Non-concavity of the profit function

From the MOOC Introduction to Discrete Choice Models (Michel Bierlaire and Virginie Lurkin)
Demand-based optimization: linear model

\[
\begin{align*}
\max_{s=(p,X)} & \quad z_s = \sum_{i \in I_k} \sum_{n \in N} p_{in} P_{in} - \sum_{i \in I_k} c_i(s, w) \\
\text{s.t.} & \quad P_{in} = \frac{1}{R} \sum_{r \in R} w_{inr} \\
& \quad U_{inr} = \beta_{in}^p p_{in} + \beta_{in} X_{in} + q_{in} + \xi_{inr} \\
& \quad U_{inr} \leq U_{nr} \\
& \quad U_{nr} \leq U_{inr} + M_{Unr} (1 - w_{inr}) \\
& \quad \sum_{i \in I} w_{inr} = 1 \\
& \quad \forall i \in I, \forall n \in N, \forall r \in R \\
\end{align*}
\]

Other constraints
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Supply-supply interactions

- We consider non-cooperative games.

- Pure strategy Nash equilibrium solutions: stationary states of the system in which no competitor has an incentive to change its decisions.

- Existence, uniqueness, algorithms to find them.
Oligopolistic market equilibrium

- Literature on continuous problems, e.g. electricity markets [Sherali et al., 1983, Pang and Fukushima, 2005, Leyffer and Munson, 2010].

- General assumptions:
  - ✔ continuously differentiable demand curve;
  - ✔/ ✗ continuously differentiable supply curve;
  - ✗ concave profit function.

- We have no proof of existence.

- We can still search for pure strategy equilibria:
  - Fixed-point iteration method
  - Fixed-point MIP model
The fixed-point iteration method

- Sequential algorithm to find an equilibrium solution of a k-player game:
  - Initialization: players start from an initial feasible solution.
  - Iterative phase: players take turns and each plays its best response pure strategy to the current solution.
  - Termination: a Nash equilibrium or a cyclic equilibrium is reached.

Plots by Nicolas Pradignac (EPFL)
The fixed-point iteration method: applications

- Used in Adler [2001] and Adler et al. [2010] to study a deregulated air transportation market and multimodal rail-air competition.

- Multinomial logit and nested logit to model demand. Due to non-concavity, there can be zero, one or more than one pure strategy equilibria.

- Different initial states lead to different solutions. No discrimination between different equilibrium or cyclic equilibrium solutions.

- Case studies related to strategic level decisions: generalizations and averages are reported.

- Also used in Maskin and Tirole [1988] to model dynamic oligopolies in which firms make short-term commitments.
The fixed-point MIP model

- We can minimize the distance between two consecutive iterations.
- A generic solution for an oligopolistic market with $k$ players: $s'_1, s'_2, \ldots, s'_k$, with $s'_k = (p_k, X_k)$.
- Optimization problems for the suppliers:
  \[
  s''_k = \arg\max_{s_k \in S_k} V_k(s_k, s'_{K\setminus\{k\}})
  \]
- All supplier simultaneously solve a best-response problem to the initial (unknown) solution.
- This approach requires finite sets of strategies.
The fixed-point MIP model

- Minimization problem:

  \[ z^* = \min \sum_{k \in K} |s_k'' - s_k'| \]

- If \( z^* = 0 \), we have an equilibrium solution.
  If \( z^* > 0 \), can we still derive meaningful information?

- The objective function allows to discriminate between different equilibrium or near-equilibrium solutions.
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Transport oligopolies: large problems with heterogeneous demand and many constraints and decision variables on the supply side.

Neither traditional microeconomic nor game-theoretic approaches are applicable as such.

Equilibrium problems $\neq$ optimization problems.

In real-life problems equilibrium is quite loosely defined.
Algorithmic framework: our methodology

1. Identify candidate equilibrium solutions or regions efficiently.
2. Use exact method on restricted strategy sets derived from candidate solutions to find subgame equilibria: fixed-point MIP model, linearized formulation.
3. Verify if best-response conditions are satisfied for the initial problem. If they are not, add strategies to the restricted problem and go to step 2.
4. Compare different equilibrium or near-equilibrium solutions: $\varepsilon$-equilibria [Radner, 1980].
Step 1: identify candidate equilibrium regions

- The sequential game generally converges to an "interesting" region of the solution space within few iterations.

- At this stage any fast heuristic that finds near-optimal solutions of the demand-based optimization model is good.

- Nonlinear formulations are faster than the linear formulation for simple discrete choice models. Their performance rapidly deteriorates in case of more complex choice models or with discrete supply decisions.
Steps 2 and 3: captive customers

- The linearized formulation is combinatorial on the sets $I$, $N$, $R$ and $S$. We need to reduce the dimension of the problem to use the fixed-point MIP model efficiently.

- Optimal strategies at equilibrium are determined by a subset of undecided customers.

- Within a limited range of supply decisions (e.g. prices), most customers are captive.

- The simulation of the error term of the utility function and the use of binary variables allows to precompute choices through lower and upper bounds.

\[
LB(U_{inr}) > \max_{j \in I : j \neq i} UB(U_{jnr}) \implies \begin{cases} 
    w_{inr} = 1 \\
    w_{jnr} = 0 \quad \forall j \in I, j \neq i
\end{cases}
\]
Steps 2 and 3: a column-generation-like approach

- The fixed-point MIP model is used to solve a subgame with restricted strategy sets initially derived from the results of step 1.

- The optimal solution of the restricted problem is a subgame equilibrium, which is then verified on the initial game by solving best-response problems.

- If any supplier can improve its profits considerably (more than $\varepsilon$), the subgame equilibrium is not accepted as game equilibrium. Best-response strategies are added to the restricted set.
Step 4: $\varepsilon$-equilibrium solutions

- A number of $\varepsilon$-equilibrium with different tolerance factors is provided at the end of the algorithmic framework.

- Ex-post analyses could answer questions about dominance, Pareto optimality or tacit collusion.
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Testing the framework

- We can use existing discrete choice model estimations available in the literature.
- Tests on two transportation datasets:
  - Urban parking choice (proof of concept)
  - High-speed rail competition (case study)
Urban parking choice

- Mixed multinomial logit model. Parameter estimation taken from the literature [Ibeas et al., 2014].

- Users choose among 3 options: free on-street parking, paid on-street parking, paid underground parking.

- Supply scenario: paid parking options are owned by two different operators, while free parking is considered as the opt-out option.

- Choice model parameters: income, car model and age, trip origin.
High-speed rail competition

- Single European Railway Directive (2012) and Railway Packages promoting open access operations on European railways.

- In 2012 Italy was the first country to open to high-speed rail competition.


- Liberalization of passenger railway transport to be effective all over Europe in December 2020.
High-speed rail competition

Demand:
- RP/SP survey collected in 2010 to forecast demand and market shares in the competitive market [Cascetta and Coppola, 2012].
- Discrete choice model estimation using multinomial logit and nested logit.

Supply:
- Scenarios based on the current market for the Milan-Rome OD pair, with prices as decision variables.
- Mode-class choice (cost, travel time, reason for travel, income, origin).
- Mode-run choice scenario (previous attributes and socio-economic characteristics plus early/late arrival).
High-speed rail fare structures

<table>
<thead>
<tr>
<th>Service</th>
<th>Standard</th>
<th>Premium</th>
<th>Business</th>
<th>Business Area Silenzio</th>
<th>Working Area</th>
<th>Business Salottino</th>
<th>Executive</th>
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<tr>
<td>Base</td>
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<td>112.00 €</td>
<td>129.00 €</td>
<td>129.00 €</td>
<td>129.00 €</td>
<td>159.00 €</td>
<td>245.00 €</td>
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<td>Economy</td>
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<td>89.90 €</td>
<td>97.90 €</td>
<td>102.90 €</td>
<td>97.90 €</td>
<td>107.90 €</td>
<td>195.90 €</td>
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<tr>
<td>Super Economy</td>
<td>59.90 €</td>
<td>62.90 €</td>
<td>85.90 €</td>
<td>90.90 €</td>
<td>85.90 €</td>
<td>95.90 €</td>
<td>154.90 €</td>
</tr>
<tr>
<td>Senior Da 60anni</td>
<td>57.00 €</td>
<td>67.20 €</td>
<td>77.40 €</td>
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<tr>
<td>Young Fino 30anni</td>
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<td>67.20 €</td>
<td>77.40 €</td>
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</tr>
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</table>
High-speed rail fare structures

<table>
<thead>
<tr>
<th>Flex</th>
<th>Smart</th>
<th>Comfort</th>
<th>Prima</th>
<th>Club Executive</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>87.90 €</td>
<td>Sold out</td>
<td>108.90 €</td>
<td>129 €</td>
</tr>
<tr>
<td>Economy</td>
<td>54.90 €</td>
<td>Sold out</td>
<td>59.90 €</td>
<td>Sold out</td>
</tr>
<tr>
<td>Low Cost</td>
<td>Sold out</td>
<td>Sold out</td>
<td>Sold out</td>
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</table>
Numerical experiments and case study

Numerical experiments: sequential game

[Graphs showing price and profit over iterations for different scenarios]
Numerical experiments and case study

Numerical experiments: column-generation-like approach

| Supplier | | Alternative | | Bounds | | MIP | | Best-response | | \( \varepsilon \) |
|---------|---|-----------|-----------|--------|--------|--------|--------|-----------------|-------|
|         | |           | LB | UB | Price | Profit | Price | Profit |  
| Supp1   | 6 | 1st       | 95.11 | 98.10 | 95.11 | 715 | 98.09 | 737 | 0.03 |
|         | | 2nd       | 66.12 | 77.98 | 66.12 | 76.21 | 652 |       |       |      |
| Supp2   | 6 | 1st       | 86.45 | 95.62 | 93.39 | 644 | 90.21 | 652 |       |
|         | | 2nd       | 70.88 | 86.28 | 82.43 |       | 86.34 |       |      |

Table: Iteration 1
Numerical experiments and case study

**Numerical experiments: column-generation-like approach**

| Supplier | $|S|$ | Alternative | Bounds | MIP | Best-response | $\varepsilon$ |
|----------|-----|------------|--------|-----|---------------|-------------|
|          |     |            | LB     | UB  | Price         | Profit      | Price   | Profit |
| Supp1    | 7   | 1st        | 95.11  | 98.10| 96.60         | 733         | 98.03   | 737    |
|          |     | 2nd        | 66.12  | 77.98| 71.55         | 733         | 72.22   |        |
| Supp2    | 7   | 1st        | 86.45  | 95.62| 90.21         | 654         | 93.81   | 670    |
|          |     | 2nd        | 70.88  | 86.34| 86.34         | 654         | 71.67   |        |

**Table: Iteration 2**
### Numerical experiments: column-generation-like approach

| Supplier | $|S|$ | Alternative | Bounds | MIP | Best-response | $\varepsilon$ |
|----------|-----|-------------|--------|-----|---------------|----------------|
|          |     |             | LB     | UB  | Price         | Profit         |
| Supp1    | 8   | 1st         | 95.11  | 98.10 | 95.11         | 702            |
|          |     | 2nd         | 66.12  | 77.98 | 66.12         | 707            |
| Supp2    | 8   | 1st         | 86.45  | 95.62 | 93.81         | 649            |
|          |     | 2nd         | 70.88  | 86.34 | 71.67         | 652            |

**Table:** Iteration 3
## Numerical experiments: $\varepsilon$-equilibrium solutions

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>Supplier 1</th>
<th>Supplier 2</th>
<th>$\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1st 2nd Profit</td>
<td>1st 2nd Profit</td>
<td></td>
</tr>
<tr>
<td>E1</td>
<td>91.71 82.18 719</td>
<td>91.42 72.35 705</td>
<td>0.012</td>
</tr>
<tr>
<td>E2</td>
<td>96.20 85.02 758</td>
<td>99.56 80.65 721</td>
<td>0.029</td>
</tr>
<tr>
<td>E3</td>
<td>93.96 83.60 708</td>
<td>91.42 72.35 714</td>
<td>0.019</td>
</tr>
<tr>
<td>E4</td>
<td>92.64 82.65 722</td>
<td>101.80 70.93 728</td>
<td>0.021</td>
</tr>
<tr>
<td>E5</td>
<td>95.08 84.31 755</td>
<td>100.21 80.52 720</td>
<td>0.020</td>
</tr>
</tbody>
</table>

**Table**: List of $\varepsilon$-equilibrium solutions
Summary

- Demand-based optimization: discrete choice models are embedded in the optimization problem of the supplier (nonlinear and linearized formulations).

- Oligopolistic competition: equilibrium solutions, if they exist, can be found by solving a fixed-point problem.

- Algorithmic approach:
  - quickly find candidate equilibrium regions;
  - solve subgames with a fixed-point MIP model and check solution by computing best responses on the original solution space;
  - compare different equilibrium or $\varepsilon$-equilibrium solutions.

- Application to a real-life case study: flexible and scalable framework, interpretable results.
Questions and discussion time

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References II


