Modelling competition in demand-based optimization models

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Outline

Competition in transportation

- Competition is often present in the form of oligopolies (regulations, limited capacity of the infrastructure, barriers to entry, etc.).
- Deregulation has generally led to oligopolistic markets.
  - Airlines: U.S. Deregulation Act (1978), then similar laws in Europe.
  - Railways: directives 91/440/EC and 2012/34/EU give open access to railway lines in the EU to companies other than those that own the infrastructure.
  - Buses: many countries recently opened the market of long-distance buses.

Example

Operators connecting Milan and Rome:

High-speed train operators

TRENITALIA
GRUPPO FERROVIE DELLO STATO

.italo

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Other transport operators

FLiXBUS Alitalia AIRITALY

Modelling demand

- Each customer chooses the alternative that maximizes his/her utility.
- Customers have different tastes and socioeconomic characteristics that influence their choice.

Modelling supply

- Operators take decisions that optimize their objective function (e.g. revenue maximization).
- Decisions can be related to pricing, capacity, frequency, availability, and other variables.
- Decisions are influenced by:
  - The preferences of the customers
  - The decisions of the competitors

Modelling competition

- We consider non-cooperative games.
- We aim at understanding the Nash equilibrium solutions of such games, i.e. stationary states of the system in which no competitor has an incentive to change its decisions.

The framework

Three-level framework: customers, operators and market.

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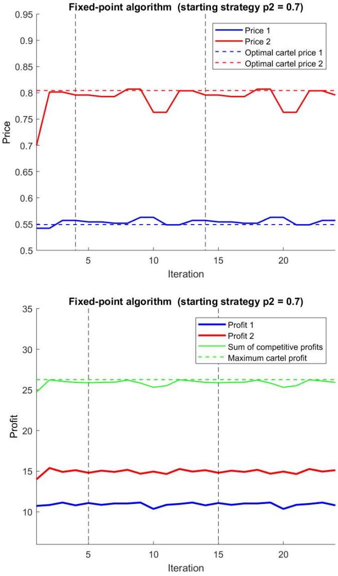
1. **Customer level**: discrete choice models take into account preference heterogeneity and model individual decisions. These can be integrated in a MILP by relying on simulation to draw from the distribution of the error term of the utility function.
2. **Operator level**: a mixed integer linear program can maximize any relevant objective function.
3. **Market level**: Nash equilibrium solutions are found by enforcing best response constraints.

Description

Sequential algorithm to find Nash equilibrium solutions of a two-players game:

- Initialization: definition of the first optimizing operator and of an initial feasible strategy of the competitor.
- Iterative phase: operators take turns and each plays its best response pure strategy to the last strategy played by the competitor.
- Termination criterion: either a Nash equilibrium or a cyclic equilibrium is reached.



Discussion

- The algorithm reproduces the behavior of two or more operators that do not know the competitors' objective function.
- It can be used with both finite and infinite strategy sets.
- Different initial strategies can lead to different equilibria.
- There is no guarantee that a pure strategy Nash equilibrium exists or that it is unique.

Description

- Each operator $k \in K$ can choose a pure strategy from its finite set of strategies S_k.
- Number of pure strategy solutions of the game: $|S| = \prod_{k \in K} S_k$.
- For each solution $s \in S$ we can derive a payoff function V_{ks} for each operator $k \in K$.
- If $s \in S$ includes only best response strategies for all operators, then it is a pure strategy Nash equilibrium for the finite game.

Customer level

Customer constraints:

$$\sum_{i\in I} w_{inrs} = 1 \qquad \forall n \in N, \forall r \in R, \forall s \in S \tag{1}$$

$$w_{inrs} \leq y_{inrs} \qquad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \tag{2}$$

$$y_{inrs} \leq y_{ins} \qquad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \tag{3}$$

$$y_{ins} = 0 \qquad \forall i \in I, \forall n \in N : i \notin C_n, \forall s \in S \tag{4}$$

$$\sum_{n\in N} w_{inrs} \leq C_i \qquad \forall i \in I \setminus \{0\}, \forall r \in R, \forall s \in S \tag{5}$$

$$C_i(y_{ins} - y_{inrs}) \leq \sum_{m\in N: L_{im} < L_{in}} w_{imrs} \qquad \forall i \in I \setminus \{0\}, \forall n \in N, \forall r \in R, \forall s \in S \tag{6}$$

$$\sum_{m\in N: L_{im} < L_{in}} w_{imrs} \leq (C_i - 1)y_{inrs} + (n-1)(1 - y_{inrs}) \qquad \forall i \in I \setminus \{0\}, \forall n \in N, \forall r \in R, \forall s \in S \tag{7}$$

$$U_{inrs} = \beta_{in} p_{ins} + q^d_{in} + \xi_{inr} \qquad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \tag{8}$$

$$lbU_{nr} \leq z_{inrs} \leq lbU_{nr} + M_{U_{nr}} y_{inrs} \qquad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \tag{9}$$

$$U_{inrs} - M_{U_{nr}}(1 - y_{inrs}) \leq z_{inrs} \leq U_{inrs} \qquad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \tag{10}$$

$$z_{inrs} \leq U_{nr} \qquad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \tag{11}$$

$$U_{nr} \leq z_{inrs} + M_{U_{nr}}(1 - w_{inrs}) \qquad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \tag{12}$$

Operator level

Operator constraints:

$$V_{ks} = \frac{1}{R} \sum_{i \in C_k} \sum_{n \in N} \sum_{r \in R} p_{ins} w_{inrs} \qquad \forall k \in K, \forall s \in S \qquad (13)$$

$$V_{ks} \leq V_{kt}^{max} \qquad \forall k \in K, \forall s \in S_k, \forall t \in S_k^C \qquad (14)$$

$$V_{kt}^{max} \leq V_{ks} + M_r(1 - x_{ks}) \qquad \forall k \in K, \forall s \in S_k, \forall t \in S_k^C \qquad (15)$$

$$\sum_{s \in S} x_{ks} = \left| S_k^C \right| \qquad \forall k \in K \qquad (16)$$

Market level

Find $s \in S$ such that $e_s = 1$

s.t.

Equilibrium constraints:

$$e_s \geq \sum_{k \in K} x_{ks} - (|K| - 1) \qquad \forall s \in S \qquad (17)$$

$$e_s \leq x_{ks} \qquad \forall k \in K, \forall s \in S \qquad (18)$$

Numerical examples

Payoff matrix of player 1

S1 \ S2	*0,70*	*0,73*	*0,76*	*0,79*	*0,82*	*0,85*
0,50	10,00	10,00	10,00	10,00	10,00	10,00
0,53	10,49	**10,60**	**10,60**	10,60	10,60	10,60
0,56	**10,53**	10,42	10,53	**10,86**	**11,20**	11,20
0,59	10,27	10,03	9,80	9,91	10,62	**11,45**
0,62	10,04	9,80	9,42	9,42	9,42	9,92
0,65	9,62	9,36	8,84	8,45	8,71	8,58

Payoff matrix of player 2

S1 \ S2	*0,70*	*0,73*	*0,76*	*0,79*	*0,82*	*0,85*
0,50	14,00	14,45	14,74	14,69	**14,76**	14,62
0,53	14,00	14,45	14,74	**15,01**	14,60	14,45
0,56	14,00	14,60	14,74	**14,85**	14,76	14,28
0,59	14,00	14,60	15,05	**15,48**	15,09	14,45
0,62	14,00	14,60	15,20	15,48	**15,91**	15,81
0,65	14,00	14,60	15,20	15,80	15,91	**16,32**

(a) Game with 1 pure strategy Nash equilibrium

Payoff matrix of player 1

S1 \ S2	*0,75*	*0,77*	*0,79*	*0,81*	*0,83*	*0,85*
0,50	10,00	10,00	10,00	10,00	10,00	10,00
0,52	10,40	10,40	10,40	10,40	10,40	10,40
0,54	**10,80**	**10,80**	10,80	10,80	10,80	10,80
0,56	10,42	10,53	**10,86**	**11,09**	**11,20**	11,20
0,58	9,74	9,86	10,09	10,44	10,67	**11,37**
0,60	9,60	9,60	9,72	10,08	10,44	10,68

Payoff matrix of player 2

S1 \ S2	*0,75*	*0,77*	*0,79*	*0,81*	*0,83*	*0,85*
0,50	14,70	**14,78**	14,69	14,74	14,28	14,62
0,52	14,70	**15,09**	14,85	14,58	14,61	14,45
0,54	14,85	14,94	**15,17**	14,74	14,44	14,45
0,56	14,85	**14,94**	14,85	14,90	14,61	14,28
0,58	15,00	15,09	**15,17**	15,07	15,11	14,45
0,60	15,00	15,25	**15,48**	15,39	15,27	14,30

(b) Game with no pure strategy Nash equilibrium

Payoff matrices for two games with different support strategies. Best response payoffs are in bold. Equilibrium payoffs are in blue.

Discussion

- The model requires finite strategy sets (enumeration).
- All pure strategy Nash equilibria of the game can be found, however the problem is solvable with small solution spaces only.
- The assumption of a finite game requires price discretization.

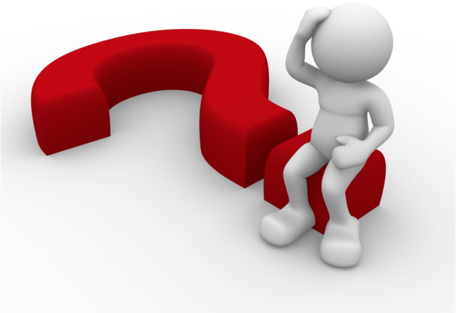
Summary

- We are analyzing oligopolistic markets from three integrated perspectives:
  - Customer level, by using discrete choice models
  - Operator level, by solving a mixed integer program
  - Market level, by including equilibrium constraints
- We presented two different approaches to find Nash equilibrium solutions for the resulting non-cooperative multi-leader-follower game.

Open questions

- Extension of the current MILP to include mixed strategy games (Nash's existence theorem).
- Efficient search for equilibria in the solution space to avoid enumeration.
- Investigation of the concept of Nash equilibrium region for real-life applications.

Questions?



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