Modelling competition in demand-based optimization models

Stefano Bortolomiol
Virginie Lurkin    Michel Bierlaire

Transport and Mobility Laboratory (TRANSP-OR)
École Polytechnique Fédérale de Lausanne

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Motivation

Competition in transportation

- Competition is often present in the form of oligopolies (regulations, limited capacity of the infrastructure, barriers to entry, etc.).

- Deregulation has generally led to oligopolistic markets.
  - Railways: directives 91/440/EC and 2012/34/EU give open access to railway lines in the EU to companies other than those that own the infrastructure.
  - Buses: many countries recently opened the market of long-distance buses.
Example

Operators connecting Milan and Rome:

High-speed train operators

![Trenitalia](image1.png)  ![Italo](image2.png)
Operators connecting Milan and Rome:

High-speed train operators

Other transport operators

- TRENITALIA
- Italo
- Flixbus
- Alitalia
- Airitaly
Modelling demand

- Each customer chooses the alternative that maximizes his/her utility.
- Customers have different tastes and socioeconomic characteristics that influence their choice.
Modelling supply

- Operators take decisions that optimize their objective function (e.g. revenue maximization).

- Decisions can be related to pricing, capacity, frequency, availability, and other variables.

- Decisions are influenced by:
  - The preferences of the customers
  - The decisions of the competitors
Motivation

Modelling competition

- We consider non-cooperative games.

- We aim at understanding the Nash equilibrium solutions of such games, i.e. stationary states of the system in which no competitor has an incentive to change its decisions.
Modelling the problem

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3 Conclusions
Three-level framework: customers, operators and market.

1. **Customer level:** discrete choice models take into account preference heterogeneity and model individual decisions. These can be integrated in a MILP by relying on simulation to draw from the distribution of the error term of the utility function.
Three-level framework: customers, operators and market.

1. **Customer level**: discrete choice models take into account preference heterogeneity and model individual decisions. These can be integrated in a MILP by relying on simulation to draw from the distribution of the error term of the utility function.

2. **Operator level**: a mixed integer linear program can maximize any relevant objective function.
Three-level framework: customers, operators and market.

1. **Customer level**: discrete choice models take into account preference heterogeneity and model individual decisions. These can be integrated in a MILP by relying on simulation to draw from the distribution of the error term of the utility function.

2. **Operator level**: a mixed integer linear program can maximize any relevant objective function.

3. **Market level**: Nash equilibrium solutions are found by enforcing best response constraints.
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Description

Sequential algorithm to find Nash equilibrium solutions of a two-players game:

- Initialization: definition of the first optimizing operator and of an initial feasible strategy of the competitor.

- Iterative phase: operators take turns and each plays its best response pure strategy to the last strategy played by the competitor.

- Termination criterion: either a Nash equilibrium or a cyclic equilibrium is reached.
Discussion

- The algorithm reproduces the behavior of two or more operators that do not know the competitors’ objective function.

- It can be used with both finite and infinite strategy sets.

- Different initial strategies can lead to different equilibria.

- There is no guarantee that a pure strategy Nash equilibrium exists or that it is unique.
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Each operator $k \in K$ can choose a pure strategy from its finite set of strategies $S_k$.

Number of pure strategy solutions of the game: $|S| = \prod_{k \in K} S_k$.

For each solution $s \in S$ we can derive a payoff function $V_{ks}$ for each operator $k \in K$.

If $s \in S$ includes only best response strategies for all operators, then it is a pure strategy Nash equilibrium for the finite game.
Customer level

Customer constraints:

\[ \sum_{i \in I} w_{i\text{in}rs} = 1 \]  \hspace{1cm} (1)

\[ w_{i\text{in}rs} \leq y_{i\text{in}rs} \] \hspace{1cm} (2)

\[ y_{i\text{in}rs} \leq y_{i\text{ins}} \] \hspace{1cm} (3)

\[ y_{i\text{ins}} = 0 \] \hspace{1cm} (4)

\[ \sum_{n \in N} w_{i\text{in}rs} \leq C_i \] \hspace{1cm} (5)

\[ C_i(y_{i\text{ins}} - y_{i\text{in}rs}) \leq \sum_{m \in N: L_{im} \leq L_{in}} w_{imrs} \] \hspace{1cm} (6)

\[ \sum_{m \in N: L_{im} \leq L_{in}} w_{imrs} \leq (C_i - 1)y_{i\text{in}rs} + (n - 1)(1 - y_{i\text{in}rs}) \] \hspace{1cm} (7)

\[ U_{i\text{in}rs} = \beta_{i\text{in}r}p_{i\text{ins}} + q_{i\text{in}}^d + \xi_{i\text{in}r} \] \hspace{1cm} (8)

\[ \text{lb}U_{nr} \leq z_{i\text{in}rs} \leq \text{lb}U_{nr} + M_{nr}y_{i\text{in}rs} \] \hspace{1cm} (9)

\[ U_{i\text{in}rs} - M_{nr}(1 - y_{i\text{in}rs}) \leq z_{i\text{in}rs} \leq U_{i\text{in}rs} \] \hspace{1cm} (10)

\[ z_{i\text{ins}} \leq U_{nr} \] \hspace{1cm} (11)

\[ U_{nr} \leq z_{i\text{ins}} + M_{nr}(1 - w_{i\text{in}rs}) \] \hspace{1cm} (12)
Operator level

Operator constraints:

\[
V_{ks} = \frac{1}{R} \sum_{i \in C_k} \sum_{n \in N} \sum_{r \in R} p_{insw_{inrs}} \\
V_{ks} \leq V_{kt}^{\text{max}} \\
V_{kt}^{\text{max}} \leq V_{ks} + M_r(1 - x_{ks}) \\
\sum_{s \in S} x_{ks} = |S_k| \\
\]

\forall k \in K, \forall s \in S \quad (13) \\
\forall k \in K, \forall s \in S_k, \forall t \in S_k^C \quad (14) \\
\forall k \in K, \forall s \in S_k, \forall t \in S_k^C \quad (15) \\
\forall k \in K \quad (16)
Market level

Find $s \in S$ such that $e_s = 1$

s.t.
Equilibrium constraints:

$$e_s \geq \sum_{k \in K} x_{ks} - (|K| - 1)$$  \hspace{1cm} \forall s \in S \hspace{1cm} (17)$$

$$e_s \leq x_{ks}$$  \hspace{1cm} \forall k \in K, \forall s \in S \hspace{1cm} (18)$$
Numerical examples

<table>
<thead>
<tr>
<th>Payoff matrix of player 1</th>
<th>Payoff matrix of player 1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>S1 \ S2</strong></td>
<td><strong>S1 \ S2</strong></td>
</tr>
<tr>
<td>0.70</td>
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<td>0.50</td>
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<td>10.27</td>
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<tr>
<td>0.65</td>
<td>9.62</td>
</tr>
</tbody>
</table>

(a) Game with 1 pure strategy Nash equilibrium

Payoff matrices for two games with different support strategies. Best response payoffs are in bold. Equilibrium payoffs are in blue.
Discussion

- The model requires finite strategy sets (enumeration).
- All pure strategy Nash equilibria of the game can be found, however the problem is solvable with small solution spaces only.
- The assumption of a finite game requires price discretization.
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Summary

- We are analyzing oligopolistic markets from three integrated perspectives:
  - Customer level, by using discrete choice models
  - Operator level, by solving a mixed integer program
  - Market level, by including equilibrium constraints

- We presented two different approaches to find Nash equilibrium solutions for the resulting non-cooperative multi-leader-follower game.
Open questions

- Extension of the current MILP to include mixed strategy games (Nash’s existence theorem).
- Efficient search for equilibria in the solution space to avoid enumeration.
- Investigation of the concept of Nash equilibrium region for real-life applications.