Cost Optimization for the Capacitated Railroad Blocking and Train Design Problem

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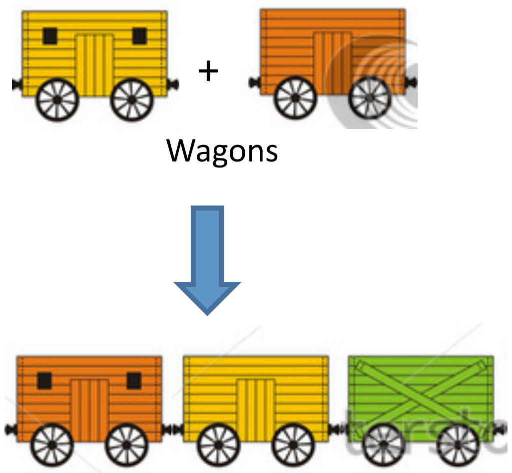
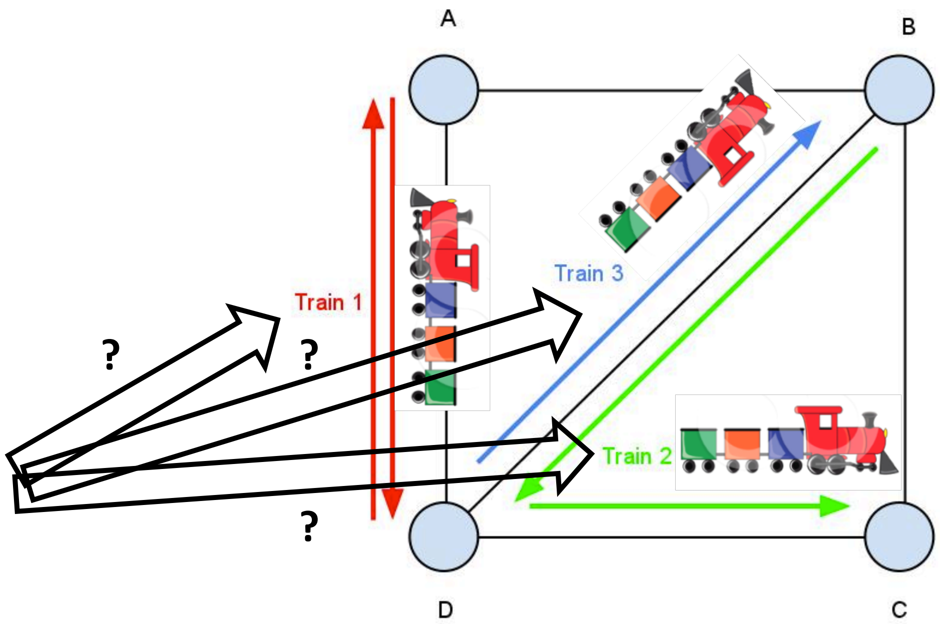
Terminology and Problem Definition



Wagons

Blocks: **same O-D**
Cannot be separated till the block reaches destination



Which trains to run?

Which blocks to be assigned to which trains?

Problem Objectives

- Blocking Problem
  - Combining shipments to form one unit (*block*)
- Train Design Problem
  - Decide trains origins, destinations and paths
  - Crew segment constraint is also considered
- Block-to-Train Assignment (BTA)
  - Determine which block is assigned to which train

- For our problem, the composition of the blocks are known.

Problem Description

- Model: directed graph
  - Nodes -> train stations
  - The graph is not necessarily complete.

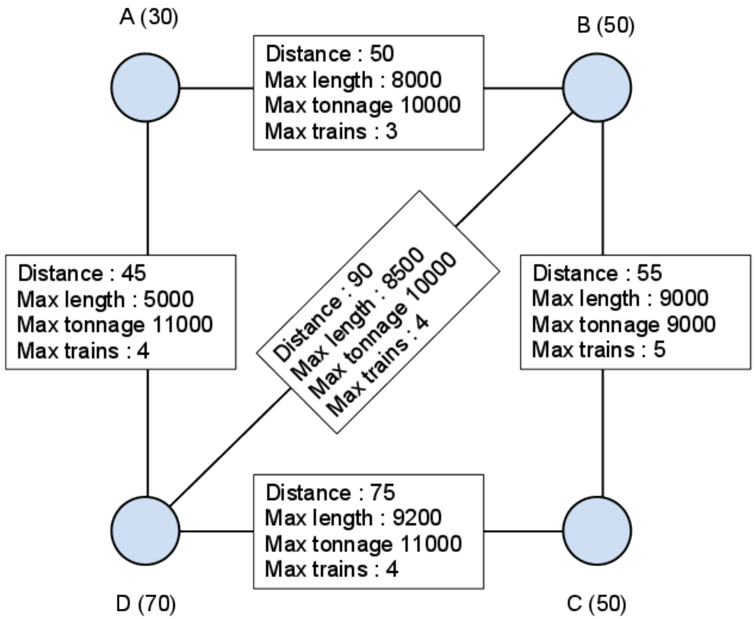
Constraints

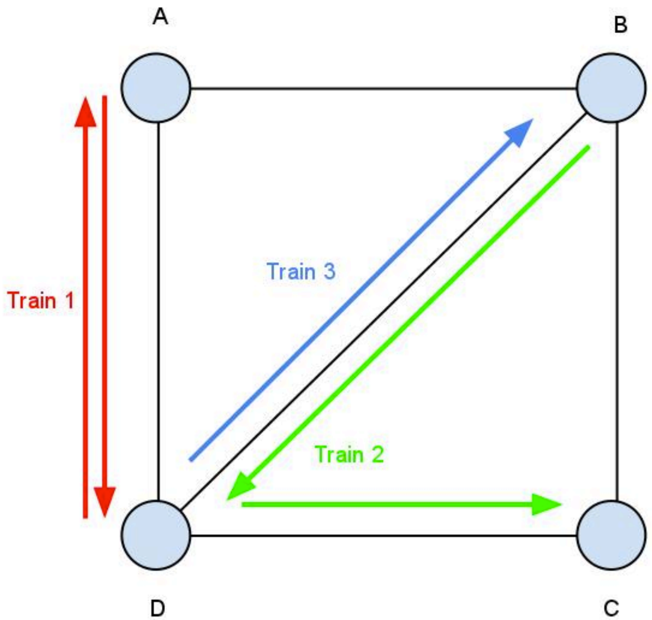
- Number of blocks per train
- Number of block swaps per block
- Number of work events per train
- Length and tonnage restrictions on arcs
- Number of trains per arc
- Crew segments

Cost Components

- Fixed setup and travel costs
- Marginal cost per wagon
- Work event cost
- Block swap cost
- Train imbalance cost
- Crew imbalance cost
- Unsatisfied demand

Toy Problem





Block	Origin	Destination	# of cars	Total length	Total tonnage
1	A	C	50	3000	2500
2	A	D	25	1500	1250
3	B	D	40	2400	2000
4	D	A	28	1680	1400
5	D	B	16	960	800

Literature - Motivation

- R.K. Ahuja, K.C. Jha, and J. Liu (2007). *Solving real-life railroad blocking problems.*
  - MIP formulation of the railroad blocking problem
- K.C. Jha, R.K. Ahuja, and G. Sahin (2008). *New approaches for Solving the Block-to-Train assignment Problem.*
  - Arc-based and path-based formulation of the block-to-train assignment problem

- Literature assumes that train design (with crew constraints) is given and blocking and BTA are solved separately.
- INFORMS RAS 2011 Competition Problem (with real data)

Cost Breakup

Train start cost	$400.00
Train travel cost (per mile)	$10.00
Railcar travel cost (per mile)	$0.75
Work event cost	$350.00
Block swap cost	$40.00 – $100.00
Crew imbalance penalty	$600.00
Train imbalance penalty	$1,000.00
Missed railcar penalty	$5,000.00

Has highest influence on cost, should be minimized with priority

Should be avoided if possible (isolated shipment and/or network capacity)

IDEA:
For each shipment, find path from origin to destination which minimizes travel cost. If no constraints exist, this is the shortest path.

Constraints:

- Length and tonnage restrictions on arcs
- Crew segments

Methodological framework

- Travel cost is the most discriminating cost component
- Crew constraints are the most complex
- Three-step process
  - Identify the shortest path for each block, under constraints
  - Guarantee feasibility wrt crew constraint with pre-preprocessing
  - Solve a MIP

Step 1: Constrained shortest path

$$\text{Min} \sum_{a \in A} \sum_{b \in B} c_{car} \cdot DistArc_a \cdot NumCar_b \cdot x_{ab} + \sum_{a \in A} (c_{train} \cdot DistArc_a + TrainStartCost) \cdot y_a \quad (1)$$

subject to

$$\sum_{a \in A^+_{O_b}} x_{ab} = 1 \quad \forall b \in B \quad (2)$$

$$\sum_{a \in A^-_{O_b}} x_{ab} = 0 \quad \forall b \in B \quad (3)$$

$$\sum_{a \in A^-_{D_b}} x_{ab} = 1 \quad \forall b \in B \quad (4)$$

$$\sum_{a \in A^+_n} x_{ab} = \sum_{a \in A^-_n} x_{ab} \quad \forall n \in N \setminus \{D_b, O_b\}, \forall b \in B \quad (5)$$

$$\sum_{b \in B} Ton_b \cdot x_{ab} \leq y_a \cdot Max\_Ton_a \quad \forall a \in A \quad (6)$$

$$\sum_{b \in B} Len_b \cdot x_{ab} \leq y_a \cdot Max\_Len_a \quad \forall a \in A \quad (7)$$

$$y_a \leq Max\_Num\_Trains_a \quad \forall a \in A \quad (8)$$

$$x_{ab} \leq y_a \quad \forall a \in A, \forall b \in B \quad (9)$$

$$x_{ab} \in \{0, 1\}, y_a \in \mathbb{N} \quad \forall a \in A, \forall b \in B \quad (10)$$

Step 1: Constrained shortest path

$$\text{Min} \sum_{a\in A}\sum_{b\in B} c_{car} \cdot DistArc_a \cdot NumCar_b \cdot x_{ab} + \sum_{a\in A}(c_{train} \cdot DistArc_a + TrainStartCost) \cdot y_a \quad (1)$$

set of arcs ($a\in A$); set of blocks ($b\in B$)

x_{ab}: if block *b* is carried on arc *a* or not

y_a: number of trains running on arc *a*

(1) railcar travel cost + train travel cost + train start cost

subject to

$$\sum_{a\in A^+_{O_b}} x_{ab} = 1 \qquad \forall b \in B \quad (2)$$

shipment leaves its origin

$A^+_{O_b}$: set of outgoing arcs from the origin of block *b*

$$\sum_{a\in A^-_{O_b}} x_{ab} = 0 \qquad \forall b \in B \quad (3)$$

subtour elimination constraint

$A^-_{O_b}$: set of incoming arcs to the origin of block *b*

$$\sum_{a\in A^-_{D_b}} x_{ab} = 1 \qquad \forall b \in B \quad (4)$$

shipment reaches its destination

$A^-_{D_b}$: set of incoming arcs to the destination of block *b*

$$\sum_{a\in A^+_n} x_{ab} = \sum_{a\in A^-_n} x_{ab} \qquad \forall n \in N \setminus \{D_b, O_b\}, \forall b \in B \quad (5)$$

shipment leaves the node that it enters

A^+_n: set of outgoing arcs from node *n*

A^-_n: set of incoming arcs to node *n*

$$\sum_{b\in B} Ton_b \cdot x_{ab} \le y_a \cdot Max\_Ton_a \qquad \forall a \in A \quad (6)$$

maximum tonnage constraint on arcs

$$\sum_{b\in B} Len_b \cdot x_{ab} \le y_a \cdot Max\_Len_a \qquad \forall a \in A \quad (7)$$

maximum length constraint on arcs

$$y_a \le Max\_Num\_Trains_a \qquad \forall a \in A \quad (8)$$

maximum number of trains constraint on arcs

$$x_{ab} \le y_a \qquad \forall a \in A, \forall b \in B \quad (9)$$

$$x_{ab} \in \{0,1\}, y_a \in \mathbb{N} \qquad \forall a \in A, \forall b \in B \quad (10)$$

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Shipment and Train Path Generation

- Step 1: Resource constraint shortest path problem for shipments
  - Does not take care of crew segments
  - Find shortest crew segment covering for the paths if the shortest path is not on crew segments
- Step 2: Train Path Generation (preprocessing)
  - Assign one train per crew segment
  - It guarantees feasibility wrt crew constraints
  - Assign more trains to sequences of crew segments to increase flexibility
  - We duplicate trains to meet capacity constraints
  - Next, we decide which of these many trains will be actually operated.

Step 3: MIP

$$
\begin{aligned}
\text{Min} \sum_{t\in T} & (TrainStartCost + TrainTravelCostPerMile \cdot TrainPathLength_t) \cdot z_t \\
& + \sum_{t\in T} \sum_{a\in A} CostPerWorkEvent \cdot w_t^a + \sum_{b\in B} \sum_{a\in A} BlockSwapCost_a \cdot u_b^a \\
& + \sum_{n\in N} TrainImbalancePenalty \cdot s_n + \sum_{i\in C} CrewImbalancePenalty \cdot c_i \\
& + \sum_{b\in B} (1 - k_b) \cdot MissedRailcarPenalty \cdot NumCar_b
\end{aligned} \tag{1}
$$

subject to

$$\sum_{\{t\in T: a\in I_{bt}\}} x_{bt}^a = k_b \qquad \forall b \in B, \forall a \in R_b \tag{2}$$

$$x_{bt}^a \le y_{bt} \qquad \forall b \in B, \forall t \in T, \forall a \in I_{bt} \tag{3}$$

$$x_{bt}^a \le z_t \qquad \forall b \in B, \forall t \in T, \forall a \in I_{bt} \tag{4}$$

$$x_{bt}^a - x_{bt}^{N_t^a} \le w_t^a, \quad x_{bt}^{N_t^a} - x_{bt}^a \le w_t^a \qquad \forall b \in B, \forall t \in T, \forall a \in I_{bt} \setminus \{a_{t1}, a_{t|P_t|}\} \tag{5}$$

$$\sum_{a\in P_t} w_t^a \le maxNbWorkEvents_t \qquad \forall t \in T \tag{6}$$

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Mathematical Model (cont.)

$$x_{bt}^{a} - x_{bt}^{N_b^a} \leq u_b^a, \quad x_{bt}^{N_b^a} - x_{bt}^{a} \leq u_b^a \qquad \forall b \in B, \forall t \in T, \forall a \in I_{bt} \setminus \{a_{b1}, a_{b|R_b|}\} \qquad (7)$$

$$\sum_{a \in R_b} u_b^a \leq maxNbBlockSwaps_b \qquad \forall b \in B \qquad (8)$$

$$\sum_{\{t \in T : n = O_t\}} z_t - \sum_{\{t \in T : n = D_t\}} z_t \leq s_n \qquad \forall n \in N \qquad (9)$$

$$\sum_{\{t \in T : n = D_t\}} z_t - \sum_{\{t \in T : n = O_t\}} z_t \leq s_n \qquad \forall n \in N \qquad (10)$$

$$\sum_{t \in T} CrSeg_t \cdot z_t - \sum_{t \in T} OpCrSeg_t \cdot z_t \leq c_i \qquad \forall i \in C \qquad (11)$$

$$\sum_{t \in T} OpCrSeg_t \cdot z_t - \sum_{t \in T} CrSeg_t \cdot z_t \leq c_i \qquad \forall i \in C \qquad (12)$$

$$\sum_{b \in B} y_{bt} \leq maxNbBlocksPerTrain \qquad \forall t \in T, \forall b \in B \qquad (13)$$

$$x_{bt}^a, y_{bt}, z_t, w_t^a, u_b^a, k_b \in \{0, 1\} \qquad \forall b \in B, \forall t \in T, \forall a \in A \qquad (14)$$

$$s_n, c_i \in \mathbb{N} \qquad \forall n \in N, \forall i \in C \qquad (15)$$

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Mathematical Model

$$\text{Min} \sum_{t \in T} (TrainStartCost + TrainTravelCostPerMile \cdot TrainPathLength_t) \cdot z_t$$

$t \in T$ → set of trains

$$+ \sum_{t \in T} \sum_{a \in A} CostPerWorkEvent \cdot w_t^a + \sum_{b \in B} \sum_{a \in A} BlockSwapCost_a \cdot u_b^a$$

$a \in A$ → set of arcs; $b \in B$ → set of blocks

$$+ \sum_{n \in N} TrainImbalancePenalty \cdot s_n + \sum_{i \in C} CrewImbalancePenalty \cdot c_i$$

$n \in N$ → set of nodes; $i \in C$ → set of crew segments

$$+ \sum_{b \in B} (1 - k_b) \cdot MissedRailcarPenalty \cdot NumCar_b \quad (1)$$

(1) all costs and penalties

k_b → if block b is carried properly or not

subject to

$$\sum_{\{t \in T : a \in I_{bt}\}} x_{bt}^a = k_b \qquad \forall b \in B, \forall a \in R_b \quad (2)$$

x_{bt}^a → if block b is carried by train t on arc a or not

I_{bt} → common arcs of path of block b and train t

R_b → ordered path of block b

(2) checks if block is missing or not

$$x_{bt}^a \leq y_{bt} \qquad \forall b \in B, \forall t \in T, \forall a \in I_{bt} \quad (3)$$

y_{bt} → if block b is carried by train t at least once or not

$$x_{bt}^a \leq z_t \qquad \forall b \in B, \forall t \in T, \forall a \in I_{bt} \quad (4)$$

z_t → if train t is used or not

(3), (4) connect variables

$$x_{bt}^a - x_{bt}^{N_t^a} \leq w_t^a, \quad x_{bt}^{N_t^a} - x_{bt}^a \leq w_t^a \qquad \forall b \in B, \forall t \in T, \forall a \in I_{bt} \setminus \{a_{t1}, a_{t|P_t|}\} \quad (5)$$

a_{t1} → first arc of train t; $a_{t|P_t|}$ → last arc of train t

$$\sum_{a \in P_t} w_t^a \leq maxNbWorkEvents_t \qquad \forall t \in T \quad (6)$$

w_t^a → if train t has a work event after arc a or not

(5), (6) work event

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Mathematical Model (cont.)

block swap

$$x_{bt}^{a} - x_{bt}^{N_b^a} \leq u_b^a \quad x_{bt}^{N_b^a} - x_{bt}^{a} \leq u_b^a \qquad \forall b \in B, \forall t \in T, \forall a \in I_{bt} \setminus \{a_{b1}, a_{b|R_b|}\} \qquad (7)$$

$$\sum_{a \in R_b} u_b^a \leq maxNbBlockSwaps_b \qquad \forall b \in B \qquad (8)$$

- u_b^a: if block *b* has a block swap after arc *a* or not
- a_{b1}: first arc of block *b*
- $a_{b|R_b|}$: last arc of block *b*

train imbalance

$$\sum_{\{t \in T: n = O_t\}} z_t - \sum_{\{t \in T: n = D_t\}} z_t \leq s_n \qquad \forall n \in N \qquad (9)$$

$$\sum_{\{t \in T: n = D_t\}} z_t - \sum_{\{t \in T: n = O_t\}} z_t \leq s_n \qquad \forall n \in N \qquad (10)$$

- s_n: train imbalance in node *n*

crew imbalance

$$\sum_{t \in T} CrSeg_t \cdot z_t - \sum_{t \in T} OpCrSeg_t \cdot z_t \leq c_i \qquad \forall i \in C \qquad (11)$$

$$\sum_{t \in T} OpCrSeg_t \cdot z_t - \sum_{t \in T} CrSeg_t \cdot z_t \leq c_i \qquad \forall i \in C \qquad (12)$$

- c_i: imbalance for crew segment *i*

max number of blocks per train

$$\sum_{b \in B} y_{bt} \leq maxNbBlocksPerTrain \qquad \forall t \in T, \forall b \in B \qquad (13)$$

$$x_{bt}^a, y_{bt}, z_t, w_t^a, u_b^a, k_b \in \{0, 1\} \qquad \forall b \in B, \forall t \in T, \forall a \in A \qquad (14)$$

$$s_n, c_i \in \mathbb{N} \qquad \forall n \in N, \forall i \in C \qquad (15)$$

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Data Sets: CSX - railroad company in the US

Data 1

- 134 arcs
- 94 nodes
- 239 shipments (1-12 arcs)
- 154 crew segments (1-4 arcs)

↓

- 534 candidate trains

Data 2

- 294 arcs
- 221 nodes
- 369 shipments (1-17 arcs)
- 154 crew segments (1-7 arcs)

↓

- 575 candidate trains

Results

Objective Function Component	Data 1		Data 2	
	Cost ($)	%	Cost ($)	%
Train Start Cost	23600	1.2	33600	1.1
Train Travel Cost	238692	12.6	267989	8.9
Railcar Travel Cost	1566012.38	82.5	219517.5	72.8
Work Event Cost	47600	2.6	73150	2.4
Block Swap Cost	4050	0.2	5860	0.2
Crew Imbalance Cost	7200	0.4	13800	0.5
Train Imbalance Cost	12000	0.6	4000	0.1
Missed Railcars Cost	0	0	420000	14.0
Total Cost	1899154.38	100	3008556.5	100

Findings

- Integrated problem can be solved on real instances
- Crew constraints are the most complex → processed first
- We exploit the cost structure → shortest paths followed by MIP

Ongoing Research

- Relax the strong relation with the cost structure
- E.g. combining shipment path generation algorithm with the shipment to train assignment
  - Multioptional shipment paths (e.g. k-shortest paths)
  - Online train generation (column generation)
- Include uncertainty in the model (robust optimization, recovery)
- Include a time dimension (scheduling)