Integrated airline schedule planning with supply-demand interactions

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3ème cycle romand de Recherche Opérationnelle

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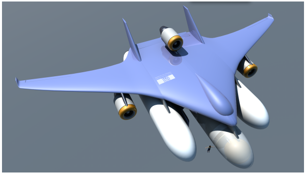
Outline

1. Introduction
2. Integrated Airline Scheduling
3. Results
4. Demand model
5. Future Work

Clip-Air concept

- Flexible capacity with modular-detachable capsules
- Carrier and capsule separation: security, maintenance, storage and crew costs
- Multi-modal transportation for both passenger and cargo
- Sustainable transportation
    - Gas emissions
    - Noise
    - Accident rates



Objectives

- Comparative analysis between standard fleet and Clip-Air
  - profit
  - transported passengers
- Integrated schedule design and fleet assignment models
  - maximize *revenue - operating costs*
  - itinerary-based demand
  - integration of supply and demand interactions

Integrated Airline Scheduling

Considered literature:

- Itinerary based fleet assignment
  - *Itinerary based FAM* - Barnhart, Kniker and Lohatepanont - TS (2002)
  - *Integrated schedule design and FAM* - Barnhart and Lohatepanont - TS (2004)
- Integration of demand modeling
  - *Market-oriented airline service design* - Schön (2008)

Integrated schedule design and fleet assignment model

- Schedule Design: Set of mandatory and optional flights
- Schedule is represented by time-space network
- Cyclic schedule with a period of 1 day
- Single airline
- Supply-demand interaction: demand function as a function of price
  - linear
  - exponential
  - piecewise linear

Model

$$Max \sum_{i \in I} d_i(\mathbf{p})\ p_i - \sum_{k \in K, f \in F} C_{k,f} x_{k,f}$$

$$
\begin{aligned}
s.t. \sum_{k \in K} x_{k,f} &= 1 && \forall f \in F^M \\
\sum_{k \in K} x_{k,f} &\leq 1 && \forall f \in F^O \\
y_{k,a,t^-} + \sum_{f \in I(k,a,t)} x_{k,f} &= y_{k,a,t^+} + \sum_{f \in O(k,a,t)} x_{k,f} && \forall [k,a,t] \in N \\
\sum_{a \in A} y_{k,a,t_n} + \sum_{f \in CT} x_{k,f} &\leq R_k && \forall k \in K \\
y_{k,a,minE_a^-} &= y_{k,a,maxE_a^+} && \forall k \in K, a \in A \\
\sum_{i \in I} \delta_f^i d_i(\mathbf{p}) &\leq \sum_{k \in K} s_k x_{k,f} && \forall f \in F \\
x_{k,f} &\in \{0,1\} && \forall k \in K, f \in F \\
y_{k,a,t} &\geq 0 && \forall (k,a,t) \in N \\
d_i(\mathbf{p}) \geq 0 \quad p_i &\geq 0 && \forall i \in I
\end{aligned}
$$

Model

$$Max \sum_{i \in I} d_i(\mathbf{p})\ p_i - \sum_{k \in K, f \in F} C_{k,f} x_{k,f} : \text{ revenue - operating cost}$$

$$s.t. \sum_{k \in K} x_{k,f} = 1 : \text{ mandatory flights} \qquad \forall f \in F^M$$

$$\sum_{k \in K} x_{k,f} \leq 1 : \text{ optional flights} \qquad \forall f \in F^O$$

$$y_{k,a,t^-} + \sum_{f \in I(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f \in O(k,a,t)} x_{k,f} \ : \text{ flow conservation} \qquad \forall [k,a,t] \in N$$

$$\sum_{a \in A} y_{k,a,t_n} + \sum_{f \in CT} x_{k,f} \leq R_k : \text{ fleet availability} \qquad \forall k \in K$$

$$y_{k,a,minE_a^-} = y_{k,a,maxE_a^+} \ : \text{ cyclic schedule} \qquad \forall k \in K, a \in A$$

$$\sum_{i \in I} \delta_f^i d_i(\mathbf{p}) \leq \sum_{k \in K} s_k x_{k,f} : \text{ fleet capacity} \qquad \forall f \in F$$

$$x_{k,f} \in \{0,1\} \qquad \forall k \in K, f \in F$$

$$y_{k,a,t} \geq 0 \qquad \forall (k,a,t) \in N$$

$$d_i(\mathbf{p}) \geq 0 \quad p_i \geq 0 \qquad \forall i \in I$$

Model - Clip-Air extention

$$x_f^w \in \{0,1\} \qquad : \text{ allocation of wing}$$

$$x_{k,f} \in \{0,1\} \qquad \forall k \in \{1,2,3\}: \text{ allocation of capsules}$$

$$x_f^w = 1 \qquad \forall f \in F^M: \text{ mandatory coverage}$$

$$\sum_k x_{k,f} \leq x_f^w \qquad \forall f \in F: \text{ wing-capsule relation}$$

Realized demand

- Realized demand is limited by both...
  - demand modeling
  - supply decisions
- Embedding the demand model directly into the supply model is not feasible.
- Definition of an additional variable, *realized demand*, is needed which represents the actual number of passengers traveling. Therefore:
  - Demand modeling imposes a demand d_i which is an upper bound for the realized demand.
  - Scheduling model deals with realized demand $\tilde{d}_i$ which is $\leq d_i$

Linear demand function

Objective function becomes quadratic

$$d_i(\mathbf{p}) = a_i + b_i \; p_i \; \forall i \in I$$

Parameters of the demand function are estimated by simple linear regression for the origin-destination pairs.

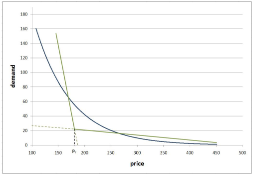
As an another explanatory variable departure time of the itinerary is used.

$$d_i(\mathbf{p}) = a_i + b_i \; p_i + c_i \; t_i \; \forall i \in I$$

Exponential demand function

$$d_i(\mathbf{p}) = \exp\left(a_i + b_i\ p_i\right)\ \forall i \in I$$

Piecewise linear approximation:



$$d_i(\mathbf{p}) = a_i + b_i \min\left(p_i, p_i^r\right) + c_i \max\left(p_i - p_i^r, 0\right)\ \forall i \in I,$$

Linearization:

$$d_i \leq a_i + b_i\ p_i + M\ \lambda_i,$$
$$d_i \leq a_i + p_i^r\ (b_i - c_i) + c_i\ p_i + M\ (1 - \lambda_i).$$

where λ is a binary variable determining which line segment is active.

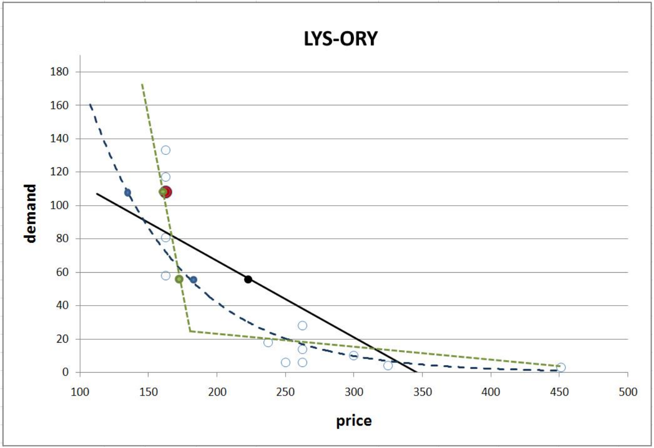
Results

- Input: data from a major European airline company (ROADEF Challenge)
  - set of optional and mandatory flights
  - set of airports
  - set of itinerary demands and fares
  - set of aircrafts for the standard fleet
- Problem resolution with AMPL+BONMIN solver
- Output: an optimized schedule design, fleet assignment and pricing for the given instances

Small data instance

Airports	2
Flights	4
Itineraries	4
Capsule capacity	56
Passengers	454
Total fleet size (seats)	392

Approximations



Results - small data instance

	Base Model		Linear demand fct.		Linear demand - time	
	Std. Fleet	Clip-Air	Std. Fleet	Clip-Air	Std. Fleet	Clip-Air
Operating cost	24,756	47,372	24,756	28,788	24,756	38,079
Revenue	36,288	66,906	47,854	47,854	48,091	57,892
Profit	11,532	19,534	23,098	19,066	23,335	19,813
Total pax	224	413	224	224	224	312
Avg. pax/flight	56	103	56	56	56	78

	Exp. demand fct.		Piecewise linear	
	Std. Fleet	Clip-Air	Std. Fleet	Clip-Air
Operating cost	24,756	47,372	24,756	47,372
Revenue	40,808	63,702	40,701	67,677
Profit	16,052	16,330	15,945	20,305
Total pax	224	413	224	413
Avg. pax/flight	56	103	56	103

Evaluation on results

- Results are very sensitive to the assumptions regarding the demand model
- There is a need for a more reliable demand modeling
- Inclusion of other explanatory variables

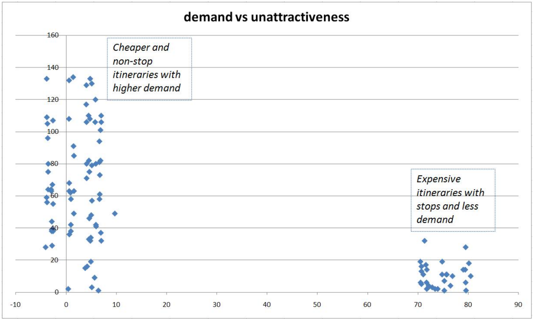
Demand model specification

- Variables
  - fare
  - time of day
  - number of stops
- Type of model
  - linear
  - logit

$$V_i^{OD} = \beta_{OD} + \beta_{fare} fare_i + \beta_{time\ of\ day} + \beta_{stops} stops_i$$

Identification issues

- Aggregate data
- Lack of variability



Use of published models

- The results from models in literature can be used
  - Willingness to pay for time of day, number of stops can be taken
  - Coefficient for fare is estimated with the data
- Validation should be done through sensitivity analysis

Future work

- More reliable demand model
- Time of the itinerary can be a decision variable in the context of re-timing in a time-windows.
- Spill and recapture
  - When there is not enough capacity on the desired itinerary of passengers, they can be redirected to alternative itineraries.
  - The portion of redirected passengers which actually accepted the offer needs to be estimated.
  - To what extend spill is effective

Thanks

Any question?

Results - large data instance

Airports	4
Flights	45
Itineraries	65
Capsule capacity	39
Passengers	3511
Total fleet size (seats)	858

	Base Model		**Linear demand fct.**		**Exp. demand fct.**	
	Std. Fleet	**Clip-Air**	**Std. Fleet**	**Clip-Air**	**Std. Fleet**	**Clip-Air**
Operating cost	357,725	367,621	299,621	320,014	345,341	412,248
Revenue	532,189	558,322	532,799	549,277	503,174	580,487
Profit	174,464	190,701	233,178	229,263	157,833	168,239
Total pax	2,954	3,122	2,226	2,387	2,736	3,401
Avg. pax/flight	74	78	52	57	63	79

Clip-Air: linear and exponential functions take 75 and 19 hours respectively.
Standard fleet: optimality gap is set to 2% and running time is around 10 hours.

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