Models and algorithms for integrated airline schedule planning and revenue management

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Motivation

- Flexibility in decision support tools,
- demand responsive transportation systems

  ... through ...

- a better understanding of demand behavior,
- integration of explicit supply-demand interactions,
- endogenous demand variables that can be controlled by the optimization models,
- considering demand early in the planning phase.

Related Literature

- Supply-demand interactions in air transport planning
    - Lohatepanont and Barnhart (2004)
    - Wang, Shebalov and Klabjan (2012)
- Exogenous demand models; iterative supply-demand models
    - Jacobs, Smith and Johnson (2008)
    - Dumas, Aithnard and Soumis (2009)
- Endogenous demand models - explicit integration
    - Airlines: Schön (2008)
    - Railways: Cordone and Redaelli (2011)
    - Revenue management: Talluri and van Ryzin (2004)

Itinerary choice model

- Market segments, s, defined by the class and each OD pair
- Itinerary choice among the set of alternatives, I_s, for each segment s
- For each itinerary $i \in I_s$ the utility is defined by:

$$\mathrm{V}_i = \mathrm{ASC}_i + \beta_p \cdot \ln(p_i) + \beta_{time} \cdot \mathrm{time}_i + \beta_{morning} \cdot \mathrm{morning}_i$$
$$\mathrm{V}_i = \mathrm{V}_i(p_i, z_i, \beta)$$

- ASC_i : alternative specific constant
- p is the **only policy variable** and included as log
- p and time are interacted with non-stop/stop
- $\mathrm{morning}$ is 1 if the itinerary is a morning itinerary

Estimation

- **Revealed preferences (RP) data:** Booking data from a major European airline
  - Lack of variability
  - Price inelastic demand
- RP data is combined with a **stated preferences (SP) data**
- Time, cost and morning parameters are **fixed** to be the same for the two datasets.
- A **scale** parameter is introduced for SP to capture the differences in variance.

Market shares

Market share and demand for itinerary i in market segment s:

$$\mathrm{ms}_i = \frac{\exp(V_i(p_i, z_i, \beta))}{\sum_{j \in I_s} \exp(V_j(p_j, z_j, \beta))} \Rightarrow D_s \mathrm{ms}_i$$

Consider a new variable

$$\upsilon_s = \frac{1}{\sum_{j \in I_s} \exp(V_j)}$$

$$\mathrm{ms}_i = \upsilon_s \exp(\beta \ln(p_i) + c_i)$$

$$\sum_{i \in I_s} \mathrm{ms}_i = 1$$

$$\upsilon_s \geq 0$$

TRANSP-OR

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Integrated airline scheduling, fleeting and pricing

Decision variables:

- $x_{k,f}$: binary, assignment of aircraft k to flight f
- $\pi^{h}_{k,f}$: allocated seats for class h on flight f aircraft k
- p_i: price of itinerary i
- ms_i: market share of itinerary i

No-revenue itineraries $I^{'}_{s} \in I_s$ for segment s, no control of airline.

Integrated model - Scheduling & fleeting

$$\max \sum_{h \in H} \sum_{s \in S^h} D_s \sum_{i \in (I_s \setminus I'_s)} \mathrm{ms}_i p_i - \sum_{\substack{k \in K \\ f \in F}} C_{k,f} x_{k,f} : \text{ revenue - cost} \quad (1)$$

$$\text{s.t.} \sum_{k \in K} x_{k,f} = 1 : \text{ mandatory flights} \quad \forall f \in F^M \quad (2)$$

$$\sum_{k \in K} x_{k,f} \leq 1 : \text{ optional flights} \quad \forall f \in F^O \quad (3)$$

$$y_{k,a,t^-} + \sum_{f \in \mathrm{In}(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f \in \mathrm{Out}(k,a,t)} x_{k,f} : \text{ flow conservation} \quad \forall [k,a,t] \in N \quad (4)$$

$$\sum_{a \in A} y_{k,a,\mathrm{minE}_a^-} + \sum_{f \in CT} x_{k,f} \leq R_k : \text{ fleet size} \quad \forall k \in K \quad (5)$$

$$y_{k,a,\mathrm{minE}_a^-} = y_{k,a,\mathrm{maxE}_a^+} : \text{ cyclic schedule} \quad \forall k \in K, a \in A \quad (6)$$

$$\sum_{h \in H} \pi^h_{k,f} \leq Q_k x_{k,f} : \text{ seat capacity} \quad \forall f \in F, k \in K \quad (7)$$

$$x_{k,f} \in \{0,1\} \quad \forall k \in K, f \in F \quad (8)$$

$$y_{k,a,t} \geq 0 \quad \forall [k,a,t] \in N \quad (9)$$

Integrated model - Revenue management - Pricing

$$\sum_{s\in S^h} D_s \sum_{i\in(I_s\setminus I_s^{'})} \delta_{i,f}\,\mathrm{ms}_i \leq \sum_{k\in K} \pi^h_{k,f}\text{: *demand - capacity*} \qquad \forall h\in H, f\in F \qquad (10)$$

$$\sum_{i\in I_s} \mathrm{ms}_i = 1\text{: *market coverage*} \qquad \forall h\in H, s\in S^h \qquad (11)$$

$$\mathrm{ms}_i \leq \upsilon_s \exp(V_i(p_i, z_i;\beta))\text{: *market share*} \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I_s^{'}) \qquad (12)$$

$$\mathrm{ms}_j = \upsilon_s \exp(V_j(p_j, z_j;\beta))\text{: *market share - competitors*} \qquad \forall h\in H, s\in S^h, j\in I_s^{'} \qquad (13)$$

$$\pi^h_{k,f} \geq 0 \qquad \forall h\in H, k\in K, f\in F \qquad (14)$$

$$\mathrm{LB}_i \leq p_i \leq \mathrm{UB}_i\text{: *bounds on price*} \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I_s^{'}) \qquad (15)$$

$$\mathrm{ms}_i \geq 0 \qquad \forall h\in H, s\in S^h, i\in I_s \qquad (16)$$

$$\upsilon_s \geq 0 \qquad \forall h\in H, s\in S^h \qquad (17)$$

Heuristic method

- Mixed Integer Non-convex Problem
- A heuristic procedure based on two subproblems:
  - FAM^{LS}: price-inelastic schedule planning model $\Rightarrow$ MILP
    - Prices fixed
    - Optimizes the schedule design and fleet assignment
  - REV^{LS}: Revenue management with fixed capacity $\Rightarrow$ NLP
    - Schedule design and fleet assignment fixed
    - Solves pricing, seat allocation
  - Local search based on spill (lost passengers)
    - Price sampling
    - Fixing a subset of FAs & VNS

Data and results

25 data instances are generated from ROADEF 2009 dataset.

Integrated model is solved...

- with BONMIN solver
- as a sequential approach - 1^{st} iteration of the heuristic
- with the heuristic

Up to around 35 flights 3 aircraft types

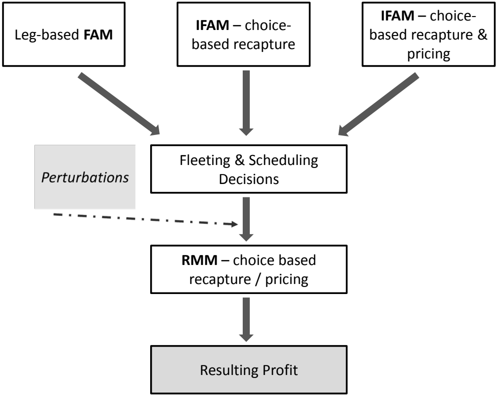
- BONMIN works quite fine.
- Integrated model improves the sequential approach by 2% on the average
- The average demand and capacity of the aircraft types at hand are key factors
- Heuristic finds the solutions at all instances

Data and results

no	airports	flights	flights per route	demand per flight	fleet composition	
20	3	33	8.25	71.90	4	85-70-50-35
21	3	46	7.67	86.85	5	128-124-107-100-85
22	7	48	2.29	101.94	4	124-107-100-85
23	3	61	15.25	69.15	4	117-85-50-37
24	8	77	2.08	67.84	4	117-85-50-37
25	8	97	3.46	90.84	5	164-117-100-85-50

	BONMIN Integrated model *max 24 hours*		**Sequential approach (SA)**			**Local search heuristic** *Average over 5 replications max 2 hours*			
	Profit	Time *(sec)*	Profit	% deviation from BONMIN	Time (sec)	Profit	%deviation from BONMIN	%impr. over SA	Time *(sec)*
20	155,772	1,429	154,322	-0.93%	5	155,772	0.00%	0.94%	316
21	303,726	84,872	303,469	-0.08%	28	307,182	1.14%	1.22%	1,819
22	161,197	18,440	163,324	1.32%	11	163,756	1.59%	0.26%	235
23	284,269	971	278,942	-1.87%	51	282,863	-0.49%	1.41%	1,438
24	155,457	79,989	158,106	1.70%	51	165,765	6.63%	4.84%	2,305
25	409,496	85,718	410,632	0.28%	4,278	411,109	0.39%	0.12%	6,832

Sensitivity Analysis



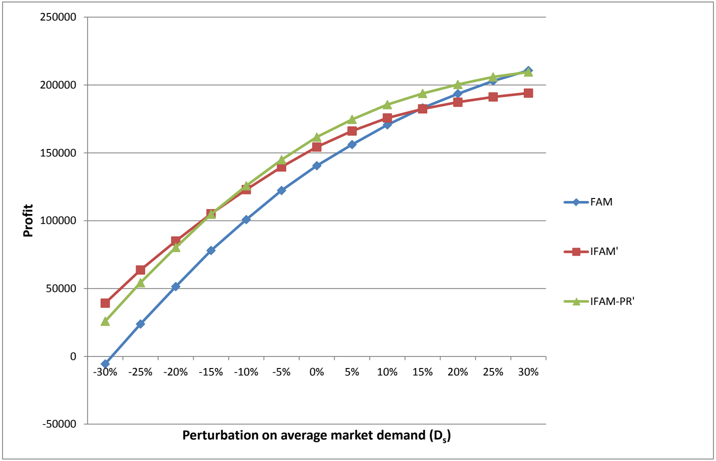
Joint work with Prof. Cynthia Barnhart

Sensitivity to demand fluctuations

- Total market segment demand is assumed to be known
- Fluctuations in reality

- Average demand is perturbed in a range [-30%, +30%]
- For each average demand 500 simulations with Poisson

Sensitivity to demand fluctuations



77 flights 4 aircraft types - heuristic solution

Non-convexity

How to deal with non-convexity ?...

In the literature: inverse-demand function
piecewise linear approximation

A general utility specification...

Transformation of the logit model

$$ms_i = \frac{\exp(V_i)}{\sum_{j \in I_s} \exp(V_j)}, \qquad V_i = \beta \ln(p_i) + c_i$$

A logarithmic transformation:

$$\mathrm{ms}_i = \upsilon_s \exp(\beta \ln(p_i) + c_i)$$
$$\mathrm{ms}_i^{'} = \upsilon_s^{'} + \beta p_i^{'} + c_i$$

$\mathrm{ms}_i^{'} \Rightarrow \ln(\mathrm{ms}_i),\ \upsilon_s^{'} \Rightarrow \ln(\upsilon_s),\ p_i^{'} \Rightarrow \ln(p_i).$

Transformation of the logit model

$$ms_i = \frac{\exp(V_i)}{\sum_{j \in I_s} \exp(V_j)}, \qquad V_i = \beta \ln(p_i) + c_i$$

A logarithmic transformation:

$$\mathrm{ms}_i = \upsilon_s \exp(\beta \ln(p_i) + c_i)$$
$$\mathrm{ms}_i' = \upsilon_s' + \beta p_i' + c_i$$

$\mathrm{ms}_i' \Rightarrow \ln(\mathrm{ms}_i)$, $\upsilon_s' \Rightarrow \ln(\upsilon_s)$, $p_i' \Rightarrow \ln(p_i)$.
This is applicable to any utility specification.

But...

- We need both ms_i and $\mathrm{ms}_i^{'}$

  ... cannot simply include $\mathrm{ms}_i = \exp(\mathrm{ms}_i^{'})$

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  $M(\mathrm{ms}_i - \exp(\mathrm{ms}_i^{'}))^2$

But...

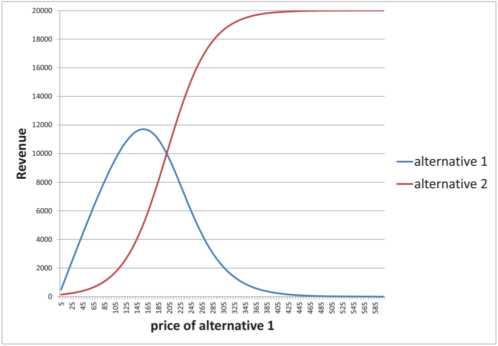
- We need both ms_i and $\mathrm{ms}_i^{'}$
  ... cannot simply include $\mathrm{ms}_i = \exp(\mathrm{ms}_i^{'})$
- We can penalize the deviation
  $M(\mathrm{ms}_i - \exp(\mathrm{ms}_i^{'}))^2$
- The revenue in the objective function
  ... can use similar tricks

Illustrative Example I - Aggregate

$V_1 = \beta p_1$
$V_2 = \beta p_2$
$\mathrm{ms}_1^{'} = \upsilon^{'} + \beta p_1$

demand 100
$p_2 = 200$

optimize p_1
if $\beta = -0.025 \Rightarrow$
$p_1^* = 157$



$$\max \quad 100 ms_1 p_1 \Leftrightarrow \max \quad \exp(\mathrm{ms}_1^{'} + \ln(p1)) \Leftrightarrow \max \quad \mathrm{ms}_1^{'} + \ln(p1)$$

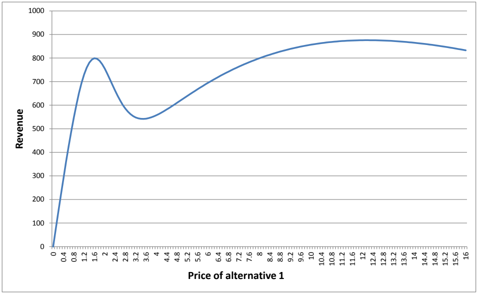
Transformation: $\max \quad \mathrm{ms}_1^{'} + \ln(p1) - M(\mathrm{ms}_1 - \exp(\mathrm{ms}_1^{'}))^2$

Illustrative Example II - Socio-economics

$V_{1,n} = \beta_n p_1 - 0.5$
$V_{2,n} = \beta_n p_2$

Group 1: $N_1 = 600, \beta_1 = -2$
Group 2: $N_2 = 400, \beta_2 = -0.1$

$p_2 = 2$
optimize $p_1 \Rightarrow p_1^* = 12.2$



$$\max \quad R_1 + R_2 \Leftrightarrow \max \quad 600 ms_{1,1} p_1 + 400 ms_{1,2} p_1$$

Transformation: $R_n' = \ln(N_n) + ms_{1,n}' + \ln(p_1)$

$$\max \quad \sum_{n \in N} R_n - M(R_n - \exp(R_n'))^2 - M(\mathrm{ms}_{1,n} - \exp(\mathrm{ms}_{1,n}'))^2$$

Back to the airline case study

980 flights, 2,197 itineraries, all flights have a capacity of 195 seats

Same optimal prices are found for the following set of penalties:

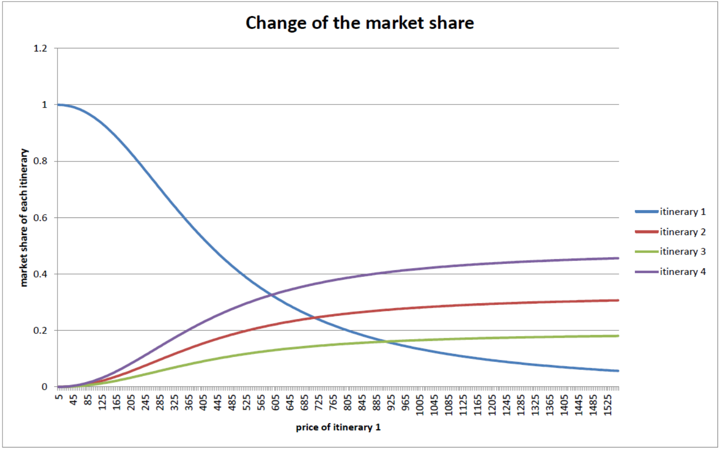
	Revenue	Computational
Reformulated model	(in millions)	time (sec.)
M=(100,000-100,000)	52.398	42.9
M=(10,000-10,000)	52.728	29.5
M=(1,000-10,000)	52.728	17.0
M=(100-10,000)	52.728	11.5
M=(10-10,000)	52.728	9.2
M=(1,000-1,000)	28.870	34.02

Conclusions

- The integrated model has promising results
- ... which motivates the effort in devising solution methodologies
- Logarithmic transformation provides a concave formulation of the revenue problem
- ... is flexible for extensions with socio-economics/more endogenous variables
- ... is expected to facilitate efficient solution methodologies

Thank you for your attention !

Logit behavior



Itinerary choice model

- Market share and demand for itinerary i in market segment s:

$$\mathrm{ms}_i = \frac{\exp(V_i(p_i, z_i, \beta))}{\sum_{j \in I_s} \exp(V_j(p_j, z_j, \beta))} \quad \Rightarrow \quad d_i = D_s ms_i$$

  - D_s is the total expected demand for market segment s.

- **Spill and recapture effects**: Capacity shortage $\Rightarrow$ passengers may be recaptured by other itineraries (instead of their desired itineraries)
- **Recapture ratio** is given by:

$$b_{i,j} = \frac{\exp(V_j(p_j, z_j, \beta))}{\sum_{k \in I_s \setminus \{i\}} \exp(V_k(p_k, z_k, \beta))}$$

Itinerary choice model

- **Value of time (VOT):**

$$VOT_i = \frac{\partial V_i / \partial time_i}{\partial V_i / \partial cost_i} = \frac{\beta_{time} \cdot cost_i}{\beta_{cost}}$$

For the same OD pair...

- VOT for economy, non-stop: 8 €/hour
- VOT for economy, one-stop: 19.8, 11, 9.2 €/hour
- VOT for business, non-stop: 21.7 €/hour

Spill and recapture

▸ model

- Forecasted demand for an itinerary is 120
- Airline considers assigning a capacity of 100 to the associated flight
- Estimated spilled passengers is 20
- If these people are redirected to other itineraries in the market what percantage will accept?

Results

	BONMIN Integrated model		Sequential approach (SA)			Local search heuristic *Average over 5 replications*			
	Profit	Time *(sec)*	Profit	% deviation from BONMIN	Time (sec)	Profit	%deviation from BONMIN	%impr. over SA	Time *(sec)*
1	15,091	2	15,091	0.00%	1	15,091	0.00%	0.00%	1
2	37,335	22	35,372	-5.26%	1	37,335	0.00%	5.55%	13
3	50,149	62	50,149	0.00%	1	50,149	0.00%	0.00%	1
4	46,037	2,807	43,990	-4.45%	1	46,037	0.00%	4.65%	3
5	70,904	1,580	69,901	-1.41%	1	70,679	-0.32%	1.11%	6
6	82,311	1,351	82,311	0.00%	1	82,311	0.00%	0.00%	1
7	87,212	32,400	84,186	-3.47%	1	87,212	0.00%	3.59%	60
8	779,819	8,137	779,819	0.00%	1	779,819	0.00%	0.00%	1
9	135,656	666	135,656	0.00%	2	135,656	0.00%	0.00%	2
10	107,927	482	107,927	0.00%	1	107,927	0.00%	0.00%	1
11	85,820	31,705	85,535	-0.33%	2	85,820	0.00%	0.33%	88
12	858,544	5,598	854,902	-0.42%	1	858,544	0.00%	0.43%	1
13	112,881	32,713	109,906	-2.64%	1	112,881	0.00%	2.71%	151
14	85,808	10,643	82,440	-3.93%	1	85,808	0.00%	4.09%	9
15	49,448	33	49,448	0.00%	1	49,448	0.00%	0.00%	1
16	38,205	240	37,100	-2.89%	1	38,205	0.00%	2.98%	1
17	27,076	35	27,076	0.00%	1	27,076	0.00%	0.00%	1
18	45,070	78	44,339	-1.62%	1	45,070	0.00%	1.65%	1
19	26,486	13	26,486	0.00%	1	26,486	0.00%	0.00%	1

Improvement due to the local search

	Sequential approach (SA)	Random neighborhood		Neighborhood based on spill		% Improvement	
	Profit	Profit	Time(sec)	Profit	Time(sec)	Quality of the solution	Reduction in time
2	35,372	37,335	116	37,335	13	-	89.10%
4	43,990	44,302	27	46,037	3	3.92%	88.88%
5	69,901	*No imp. over SA*		70,679	6	1.11%	-
7	84,186	85,335	1,649	87,212	60	2.20%	96.36%
8	904,054	906,791	209	906,791	2	-	99.04%
11	93,920	*No imp. over SA*		94,203	10	0.30%	-
12	854,902	*No imp. over SA*		858,545	1	0.43%	-
13	137,428	*No imp. over SA*		138,575	173	0.83%	-
14	93,347	96,365	943	96,486	89	0.13%	90.56%
16	37,100	38,205	6	38,205	1	-	80.65%
18	52,369	53,128	334	53,128	1	-	99.80%
20	146,464	*No imp. over SA*		147,506	380	0.71%	-
21	217,169	*No imp. over SA*		219,136	1,395	0.91%	-
22	163,114	*No imp. over SA*		163,393	126	0.17%	-
23	226,615	*No imp. over SA*		227,284	1,283	0.30%	-
24	208,561	*No imp. over SA*		210,395	791	0.88%	-
25	469,136	*No imp. over SA*		470,494	1,117	0.29%	-

A small example

- 2 airports CDG-MRS
- 4 flights - all are mandatory
- 2 aircraft types: 37-50 seats

We start with an initial FAM solution:

	AC1	AC2
F1	X	
F2	X	
F3	X	
F4	X	

A small example - GBD iterations

Iteration 1		
	Sub	Master
	12522.8	16923.4
	LB	**UB**
	12522.8	16923.4
	AC1	AC2
F1		X
F2		X
F3		X
F4		X

⟹

Iteration 2		
	Sub	Master
	10734.4	14822.8
	LB	**UB**
	12522.8	14822.8
	AC1	AC2
F1		X
F2		X
F3	X	
F4	X	

Iteration 3		
	Sub	Master
	12696.8	14822.8
	LB	**UB**
	12696.8	14822.8
	AC1	AC2
F1	X	
F2		X
F3		X
F4	X	

⟹

Iteration 4		
	Sub	Master
	12474.4	12696.8
	LB	**UB**
	12696.8	**12696.8**
	AC1	AC2
F1		X
F2		X
F3	X	
F4	X	