An integrated fleet assignment and itinerary choice model

for a new flexible aircraft

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Clip-Air concept

- Flexible capacity
- Modular-detachable capsules
- Wing and capsule separation
- Multi-modality
- Passenger and cargo
- Sustainability
  - Gas emissions
  - Noise
  - Accident rates

Objectives

- Analyze the potential performance of Clip-Air by developing appropriate models
- Introduce demand notion in optimization models through appropriate demand models
- Develop solution methodologies for the integrated model
- Application of the models and solution methods for Clip-Air.

Integration of demand model

Motivation: Demand responsive transportation systems

- Supply ⇒ Flexibility provided by Clip-Air
- Demand ⇒ integration of appropriate demand models

Demand model

- Simple models (e.g. linear, exp.) fail to represent the reality
- Integrated model becomes very sensitive to demand model parameters
- Appropriate models need to be developed

Itinerary choice model ▸ DCA

- Market segments, s, defined by the class and each OD pair
- Itinerary choice among the set of alternatives, I_s, for each segment s
- For each itinerary $i \in I_s$ the utility is defined by:

$$\mathrm{V}_i = \mathrm{ASC}_i + \beta_p \cdot \ln(p_i) + \beta_{time} \cdot \mathrm{time}_i + \beta_{morning} \cdot \mathrm{morning}_i$$
$$\mathrm{V}_i = \mathrm{V}_i(p_i, z_i, \beta)$$

  - ASC_i : alternative specific constant
  - p is a policy variable and included as log
  - p and time are interacted with non-stop/stop
  - $\mathrm{morning}$ is 1 if the itinerary is a morning itinerary

- *No-revenue* represented by the subset $I_s^{'} \in I_s$ for segment s.

Itinerary choice model

- Demand for class h for each itinerary i in market segment s:

$$\tilde{d}_i = D_s \frac{\exp(V_i(p_i, z_i, \beta))}{\sum_{j \in I_s} \exp(V_j(p_j, z_j, \beta))}$$

  - D_s is the total expected demand for market segment s.

- **Spill and recapture effects**: Capacity shortage $\Rightarrow$ passengers may be recaptured by other itineraries (instead of their desired itineraries)
- Recapture ratio is given by:

$$b_{i,j} = \frac{\exp(V_j(p_j, z_j, \beta))}{\sum_{k \in I_s \setminus \{i\}} \exp(V_k(p_k, z_k, \beta))}$$

Estimation

- **Revealed preferences (RP) data:** Booking data from a major European airline
  - Lack of variability
  - Price inelastic demand
- RP data is combined with a **stated preferences (SP) data**
- Time, cost and morning parameters are **fixed** to be the same for the two datasets.
- A **scale** parameter is introduced for SP to capture the differences in variance.

Estimation results

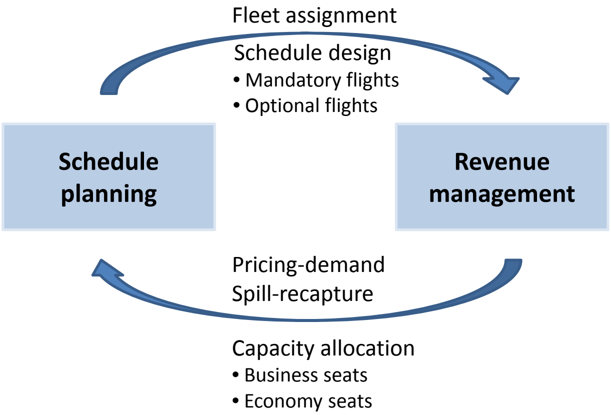
	β_{fare}		β_{time}		
	non-stop	one-stop	non-stop	one-stop	$\beta_{morning}$
economy	-2.23	-2.17	-0.102	-0.0762	0.0283
business	-1.97	-1.97	-0.104	-0.0821	0.079

- **Price elasticity** of demand:

$$E^{P_i}_{price_i} = \frac{\partial P_i}{\partial price_i} \cdot \frac{price_i}{P_i}$$

An example
- for a non-stop itinerary
  - price elasticity for economy is -2.03 and -1.86 for business
- for a one-stop itinerary
  - price elasticity for economy is -2.14 and -1.95 for business

Integrated schedule planning and revenue management



Integrated model - Schedule planning

$$Max \sum_{h\in H}\sum_{s\in S^h}\sum_{i\in(I_s\setminus I'_s)}(d_i - \sum_{j\in I_s} t_{i,j} + \sum_{j\in(I_s\setminus I'_s)} t_{j,i} b_{j,i}) p_i - \sum_{\substack{k\in K\\ f\in F}} C_{k,f} x_{k,f} : \text{ revenue - cost} \quad (1)$$

$$s.t. \sum_{k\in K} x_{k,f} = 1 : \text{ mandatory flights} \qquad \forall f\in F^M \quad (2)$$

$$\sum_{k\in K} x_{k,f} \leq 1 : \text{ optional flights} \qquad \forall f\in F^O \quad (3)$$

$$y_{k,a,t^-} + \sum_{f\in In(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f\in Out(k,a,t)} x_{k,f} : \text{ flow conservation} \qquad \forall [k,a,t]\in N \quad (4)$$

$$\sum_{a\in A} y_{k,a,minE_a^-} + \sum_{f\in CT} x_{k,f} \leq R_k : \text{ fleet availability} \qquad \forall k\in K \quad (5)$$

$$y_{k,a,minE_a^-} = y_{k,a,maxE_a^+} : \text{ cyclic schedule} \qquad \forall k\in K, a\in A \quad (6)$$

$$\sum_{h\in H} \pi^h_{k,f} = Q_k x_{k,f} : \text{ seat capacity} \qquad \forall f\in F, k\in K \quad (7)$$

$$x_{k,f}\in\{0,1\} \qquad \forall k\in K, f\in F \quad (8)$$

$$y_{k,a,t} \geq 0 \qquad \forall [k,a,t]\in N \quad (9)$$

Integrated model - Revenue management

$$\sum_{s\in S^h}\sum_{i\in(I_s\setminus I_s^{'})}\delta_{i,f}d_i-\sum_{j\in I_s}\delta_{i,f}t_{i,j}+\sum_{\substack{j\in(I_s\setminus I_s^{'})\\ i\neq j}}\delta_{i,f}t_{j,i}b_{j,i}\leq\sum_{k\in K}\pi_{k,f}:\ \textit{capacity}\qquad \forall h\in H,f\in F \qquad (10)$$

$$\sum_{\substack{j\in I_s\\ i\neq j}}t_{i,j}\leq d_i:\ \textit{total spill}\qquad \forall h\in H,s\in S^h,i\in(I_s\setminus I_s^{'}) \qquad (11)$$

$$\tilde{d}_i=D_s\frac{\exp(V_i(p_i,z_i,\beta))}{\sum_{j\in I_s}\exp(V_j(p_j,z_j,\beta))}:\ \textit{logit demand}\qquad \forall h\in H,s\in S^h,i\in I_s \qquad (12)$$

$$b_{i,j}=\frac{\exp(V_j(p_j,z_j,\beta))}{\sum_{k\in I_s\setminus\{i\}}\exp(V_k(p_k,z_k,\beta))}:\ \textit{recapture ratio}\qquad \forall h\in H,s\in S^h,i\in(I_s\setminus I_s^{'}),j\in I_s \qquad (13)$$

$$d_i\leq\tilde{d}_i:\ \textit{realized demand}\qquad \forall h\in H,s\in S^h,i\in I_s \qquad (14)$$

$$0\leq p_i\leq UB_i:\ \textit{upper bound on price}\qquad \forall h\in H,s\in S^h,i\in I_s \qquad (15)$$

$$t_{i,j}\geq 0\qquad \forall h\in H,s\in S^h,i\in(I_s\setminus I_s^{'}),j\in I_s \qquad (16)$$

$$b_{i,j}\geq 0\qquad \forall h\in H,s\in S^h,i\in(I_s\setminus I_s^{'}),j\in I_s \qquad (17)$$

$$\pi^h_{k,f}\geq 0\qquad \forall h\in H,k\in K,f\in F \qquad (18)$$

Integrated model

- We consider reference models to evaluate the integrated model
  - **Price-inleastic schedule planning**: M. Lohatepanont and C. Barnhart (2004)
  - **Sequential approach**: Revenue management considers fixed supply capacity

- The resulting model is a mixed integer nonlinear problem
- Nonlinearity is due to the explicit supply-demand interactions
- The model is implemented in AMPL and BONMIN solver is used
- BONMIN does not guarantee optimality

Impact of the integrated model

Number of airports:	3
Number of flights:	26
Average demand:	56.12 passengers per flight
Cabin classes:	Economy and business
Level of service:	All itineraries are nonstop
Available fleet:	3 types of aircraft (100, 50 and 37 seats)

	Price-inelastic schedule planning model	**Integrated model - limited prices**	**Integrated model**
Revenue	204,553	214,380	244,924
Operating costs	150,603	160,003	173,349
Profit	**53,949**	**54,377 (+ 0.8%)**	**71,575 (+ 32.7%)**
Number of flights	**22**	**22**	**24**
Transported passengers	**943**	**1031 (+ 9.3%)**	**1064 (+ 12.7%)**
Economy-Business	882 E - 61 B	970 E - 61 B	997 E - 67 B
Allocated seats	**274**	**324**	**324**

Sequential versus integrated

	Sequential approach				Integrated model - % Improvement			
No	Profit	Pax.	Flights	Seats	Profit	Pax.	Flights	Seats
1	15,091	284	8	124	-	-	-	-
2	35,372	400	8	**150**	**5.55%**	**33.50%**	8	**217**
3	50,149	859	10	300	-	-	-	-
4	69,901	931	**22**	**274**	**1.43%**	**14.18%**	**24**	**324**
5	82,311	1145	16	333	-	-	-	-
6	904,054	1448	10	**1148**	**0.30%**	-	10	**1312**
7	135,656	1814	32	498	-	-	-	-
8	115,983	2236	26	691	-	-	-	-
9	854,902	1270	10	**1016**	**0.43%**	**5.83%**	10	**1090**
10	137,428	1517	34	**391**	**0.83%**	**4.94%**	34	**476**
11	93,347	1144	20	**387**	**3.36%**	**1.40%**	20	**457**
12	49,448	1050	12	370	-	-	-	-
13	27,076	448	10	207	-	-	-	-
14	52,369	599	**10**	267	**1.45%**	**16.69%**	**12**	267
15	26,486	504	6	185	-	-	-	-

Heuristic method

- We are limited in terms of the computational time
- A heuristic based on two simplified versions of the model:
  - FAM^{LS}: price-inelastic schedule planning model
    - Explores new fleet assignment solutions based on a local search
    - Price sampling
    - Variable neighborhood search
  - REV^{LS}: Revenue management with fixed capacity
    - Optimizes the revenue for the explored fleet assignment solution

Heuristic method

Require: $\bar{x}_0$, $\bar{y}_0$, $\bar{d}_0$, $\bar{p}_0$, $\bar{t}_0$, $\bar{b}_0$, $\bar{\pi}_0$, z^*, z_{opt}, k_{max}, ε, n_{min}, n_{max}

$k := 0$, $n_{fixed} := n_{min}$

repeat

$\bar{p}_k$:= Price sampling

$\{\bar{d}_k, \bar{b}_k\}$:= Demand model($\bar{p}_k$)

$\{\bar{x}_k, \bar{y}_k, \bar{\pi}_k, \bar{t}_k\}$:= solve $z_{\mathrm{FAM}^{\mathrm{LS}}(\bar{d}_k, \bar{b}_k, n_{fixed})}$

$\{\bar{p}_k, \bar{d}_k, \bar{b}_k, \bar{\pi}_k, \bar{t}_k\}$:= solve $z_{\mathrm{REV}^{\mathrm{LS}}(\bar{x}_k, \bar{y}_k)}$

if improvement($z_{\mathrm{REV}^{\mathrm{LS}}}$) **then**

Update z^*

Intensification: $n_{fixed} := n_{fixed} + 1$ when $n_{fixed} < n_{max}$

else

Diversification: $n_{fixed} := n_{fixed} - 1$ when $n_{fixed} > n_{min}$

end if

$k := k + 1$

until $||z_{opt} - z^*||^2 \leq \varepsilon$ **or** $k \geq k_{max}$

Performance of the heuristic

		Best solution reported by BONMIN		Heuristic					
				% deviation			Time(sec)		
Exp.	Flights	Profit	Time (sec)	min	avg.	max	min	avg.	max
1	10	15,091	11	-	0.00%	-	-	1	-
2	11	37,335	27	-	**0.00%**	-	-	2	-
3	12	50,149	56	-	0.00%	-	-	33	-
4	26	70,904	2,479	**1.32%**	1.77%	2.06%	288	1,510	3,129
5	19	82,311	1,493	0.00%	0.13%	0.22%	18	900	3,092
6	12	906,791	12,964	7.37%	7.37%	7.37%	25	279	1,434
7	33	135,656	23,662	13.88%	16.36%	18.84%	74	1,714	3,534
8	32	115,983	209	0.00%	0.01%	0.12%	643	1,955	3,432
9	11	858,544	7,343	3.42%	4.79%	6.92%	1	762	3,322
10	39	138,575	37,177	2.76%	3.94%	4.98%	929	1,775	2,891
11	23	96,486	17,142	**0.00%**	**0.16%**	**0.90%**	236	1,625	3,574
12	19	49,448	32	-	0.00%	-	-	1	-
13	15	27,076	36	-	0.00%	-	-	5	-
14	14	53,128	141	-	**0.00%**	-	-	2	-
15	13	26,486	14	-	0.00%	-	-	4	-
16	77	194,598	42,360	**-5.89%**	**-4.04%**	**-2.41%**	293	1,652	2,990
17	56	191,091	39,447	0.48%	2.13%	4.46%	32	1,646	3,305
18	97	351,655	17,424	4.91%	7.94%	11.22%	840	2099	3331

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Conclusions and future work

- Solution methods for the resulting mixed integer nonlinear problem
  - A Lagrangian relaxation based heuristic
  - Subgradient optimization
  - Performance of the heuristic for larger instances
- Clip-Air
  - Further analysis with the integrated model
  - Multi-modality

Thank you for your attention!

Discrete choice analysis

- Finite and discrete set of alternatives
  - Choice of transportation mode: car, bus, etc.
  - Choice of brand: Leonidas, Lindt, Suchard, Toblerone, etc.
  - Choice of flight: GVA-NCE 10:00, GVA-NCE 06:30, etc.
- Individual n associates a utility to alternative i
- Represented by a random function

$$U_{in} = V_{in} + \varepsilon_{in} = \sum_k \beta_k x_{ink} + \varepsilon_{in}$$

Discrete choice analysis ▸ Choice Model

- Individual n chooses alternative i if $U_{in} \geq U_{jn}$, for all j.
- Utility is random, so we have a probabilistic model

$$P_n(i|C_n) = Pr(U_{in} \geq U_{jn}) = Pr(V_{in} + \varepsilon_{in} \geq V_{jn} + \varepsilon_{jn})$$

- Concrete models require
  - specification of V_{in}
  - assumptions about ε_{in}
  - estimation of the parameters from data