Flexibility in air transportation:

through new technologies
and advanced supply-demand interactions

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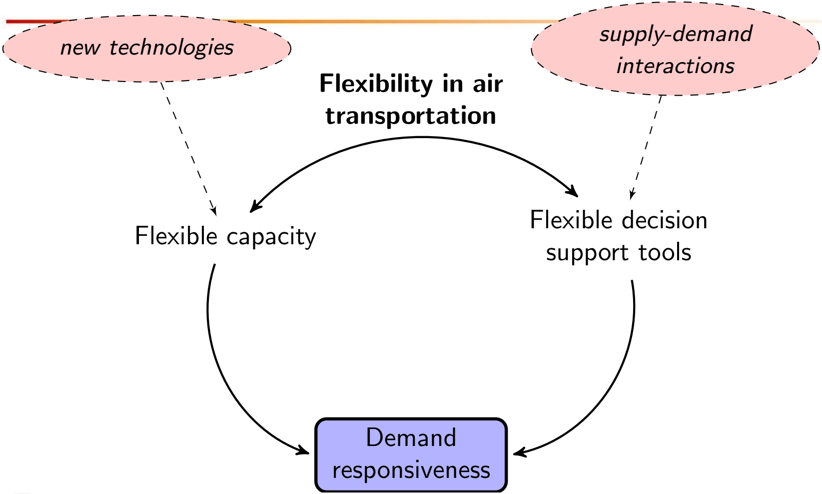
Motivation


new technologies
supply-demand interactions
Flexibility in air transportation
Flexible capacity
Flexible decision support tools
Demand responsiveness

New technology: Clip-Air

- Flexible capacity
- Modular-detachable capsules
- Wing and capsule separation
- Multi-modality
- Passenger and cargo
- Sustainability
  - Gas emissions
  - Noise
  - Accident rates

Clip-Air: Flexible capacity

- A fleet (wing & capsule) assignment model with spill and recapture
- Clip-Air better utilizes the capacity
  - More passengers ...
  - ... with less allocated capacity
- Clip-Air deals better with the insufficient capacity
- Results are robust to the cost values of Clip-Air

- Atasoy, B., Salani, M., Bierlaire, M., and Leonardi, C. (to appear in 2013 April). Impact analysis of a flexible air transportation system, European Journal of Transport and Infrastructure Research 13(2).

Today's talk

- Advance supply-demand interactions
  - Itinerary choice model
  - Integration into the planning model
- Solution methods for the integrated model
  - A local search heuristic
  - Log transformation of the logit model

Itinerary choice model

- Market segments, s, defined by the class and each OD pair
- Itinerary choice among the set of alternatives, I_s, for each segment s
- For each itinerary $i \in I_s$ the utility is defined by:

$$\mathrm{V}_i = \mathrm{ASC}_i + \beta_p \cdot \ln(p_i) + \beta_{time} \cdot \mathrm{time}_i + \beta_{morning} \cdot \mathrm{morning}_i$$
$$\mathrm{V}_i = \mathrm{V}_i(p_i, z_i, \beta)$$

  - ASC_i : alternative specific constant
  - p is the **only policy variable** and included as log
  - p and time are interacted with non-stop/stop
  - $\mathrm{morning}$ is 1 if the itinerary is a morning itinerary
- *No-revenue* represented by the subset $I_s' \in I_s$ for segment s.

Itinerary choice model

- Market share and demand for itinerary i in market segment s:

$$\mathrm{ms}_i = \frac{\exp(V_i(p_i, z_i, \beta))}{\sum_{j \in I_s} \exp(V_j(p_j, z_j, \beta))} \quad \Rightarrow \quad d_i = D_s ms_i$$

  - D_s is the total expected demand for market segment s.

- **Spill and recapture effects**: Capacity shortage $\Rightarrow$ passengers may be recaptured by other itineraries (instead of their desired itineraries)
- **Recapture ratio** is given by:

$$b_{i,j} = \frac{\exp(V_j(p_j, z_j, \beta))}{\sum_{k \in I_s \setminus \{i\}} \exp(V_k(p_k, z_k, \beta))}$$

Estimation

- **Revealed preferences (RP) data:** Booking data from a major European airline
  - Lack of variability
  - Price inelastic demand
- RP data is combined with a **stated preferences (SP) data**
- Time, cost and morning parameters are **fixed** to be the same for the two datasets.
- A **scale** parameter is introduced for SP to capture the differences in variance.

Further details in Atasoy, B., and Bierlaire, M. (2012). An air itinerary choice model based on a mixed RP/SP dataset. Technical report TRANSP-OR 120426. Transport and Mobility Laboratory, ENAC, EPFL.

Estimation results

	β_{fare} non-stop	β_{fare} one-stop	β_{time} non-stop	β_{time} one-stop	$\beta_{morning}$
economy	-2.23	-2.17	-0.102	-0.0762	0.0283
business	-1.97	-1.97	-0.104	-0.0821	0.079

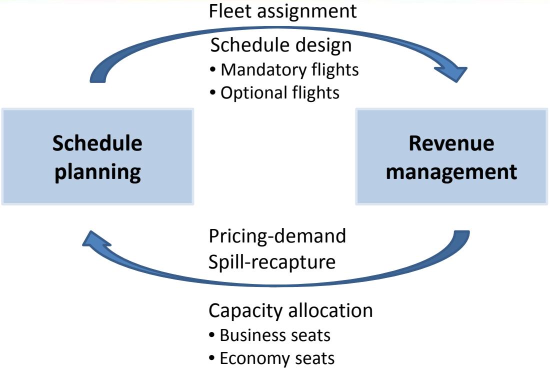
- **Price elasticity** of demand:

$$E^{P_i}_{price_i} = \frac{\partial P_i}{\partial price_i} \cdot \frac{price_i}{P_i}$$

An example

- for a non-stop itinerary
  - price elasticity for economy is -2.03 and -1.86 for business
- for a one-stop itinerary
  - price elasticity for economy is -2.14 and -1.95 for business

Integrated airline scheduling, fleeting and pricing



- *Aim: to take better fleeting decisions with the information provided by the demand model*

Integrated airline scheduling, fleeting and pricing

Decision variables:

- $x_{k,f}$: binary, assignment of aircraft k to flight f
- $\pi^h_{k,f}$: allocated seats for class h on flight f aircraft k
- p_i: price of itinerary i
- d_i: demand of itinerary i
- $t_{i,j}$: spilled passengers from itinerary i to j

Integrated model - Scheduling & fleeting

$$Max \sum_{h\in H}\sum_{s\in S^h}\sum_{i\in(I_s\setminus I_s')}(d_i - \sum_{j\in I_s} t_{i,j} + \sum_{j\in(I_s\setminus I_s')} t_{j,i}b_{j,i})p_i - \sum_{\substack{k\in K\\ f\in F}} C_{k,f}x_{k,f}: \text{ revenue - cost} \quad (1)$$

$$s.t. \sum_{k\in K} x_{k,f} = 1: \text{ mandatory flights} \qquad \forall f \in F^M \quad (2)$$

$$\sum_{k\in K} x_{k,f} \leq 1: \text{ optional flights} \qquad \forall f \in F^O \quad (3)$$

$$y_{k,a,t^-} + \sum_{f\in In(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f\in Out(k,a,t)} x_{k,f}: \text{ flow conservation} \qquad \forall [k,a,t] \in N \quad (4)$$

$$\sum_{a\in A} y_{k,a,minE_a^-} + \sum_{f\in CT} x_{k,f} \leq R_k: \text{ fleet availability} \qquad \forall k \in K \quad (5)$$

$$y_{k,a,minE_a^-} = y_{k,a,maxE_a^+}: \text{ cyclic schedule} \qquad \forall k \in K, a \in A \quad (6)$$

$$\sum_{h\in H} \pi^h_{k,f} = Q_k x_{k,f}: \text{ seat capacity} \qquad \forall f \in F, k \in K \quad (7)$$

$$x_{k,f} \in \{0,1\} \qquad \forall k \in K, f \in F \quad (8)$$

$$y_{k,a,t} \geq 0 \qquad \forall [k,a,t] \in N \quad (9)$$

- *Itinerary-based fleet assignment & Spill and recapture*
- Lohatepanont and Barnhart 2004

Integrated model - Revenue management - Pricing

$$\sum_{s\in S^h}\sum_{i\in(I_s\setminus I_s')}\delta_{i,f}\Big(d_i-\sum_{j\in I_s}t_{i,j}+\sum_{j\in(I_s\setminus I_s')}t_{j,i}b_{j,i}\Big)\leq\sum_{k\in K}\pi^h_{k,f}\text{: demand-capacity}\qquad \forall h\in H, f\in F \qquad (10)$$

$$\sum_{j\in I_s}t_{i,j}\leq d_i\text{: total spill}\qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I_s') \qquad (11)$$

$$\tilde{d}_i=D_s\frac{\exp(V_i(p_i,z_i,\beta))}{\sum_{j\in I_s}\exp(V_j(p_j,z_j,\beta))}\text{: logit demand}\qquad \forall h\in H, s\in S^h, i\in I_s \qquad (12)$$

$$b_{i,j}=\frac{\exp(V_j(p_j,z_j,\beta))}{\sum_{k\in I_s\setminus\{i\}}\exp(V_k(p_k,z_k,\beta))}\text{: recapture ratio}\qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I_s'), j\in I_s \qquad (13)$$

$$d_i\leq\tilde{d}_i\text{: realized demand}\qquad \forall h\in H, s\in S^h, i\in I_s \qquad (14)$$

$$LB_i\leq p_i\leq UB_i\text{: bounds on price}\qquad \forall h\in H, s\in S^h, i\in I_s \qquad (15)$$

$$t_{i,j}, b_{i,j}\geq 0\qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I_s'), j\in I_s \qquad (16)$$

$$\pi^h_{k,f}\geq 0\qquad \forall h\in H, k\in K, f\in F \qquad (17)$$

- Schön (2008): integration of pricing

Heuristic method

- Mixed Integer Non-convex Problem
- BONMIN solver (Bonami et al. 2008) is able to converge on instances with about 35 flights.
- We devised a heuristic procedure based on two subproblems:
- FAM^{LS}: price-inelastic schedule planning model $\Rightarrow$ MILP
  - Prices fixed
  - Optimizes the schedule design and fleet assignment
- REV^{LS}: Revenue management with fixed capacity $\Rightarrow$ NLP
  - Schedule design and fleet assignment fixed
  - Optimizes the revenue

Procedure

Require: Average prices from the data

Solve FAM^{LS} with given price, obtain initial FA solution

Solve REV^{LS} with the initial FA solution

repeat

- **Price sampling**: Obtain new prices *based on spill*
- New market share and recapture ratios with the new prices
- **VNS -Fixing**: Fix a subset of the FAs *based on spill*
- **Solve** $\mathbf{FAM}^{LS}$ with the sampled price, market share, recapture ratios and fixed assignments
- **Solve** $\mathbf{REV}^{LS}$ with fixed capacity
- **if** Profit improved **then**
  - Update best profit
  - **VNS - Intensification**: fix more FAs
- **else if** No improvement in the profit for the last 3 iterations **then**
  - **VNS - Diversification**: Fix less FAs
- **end if**

until $\text{time} \geq \text{time}_{max}$

Local search based on spill

- Neighborhood solutions are visited based on the spill rather than a fully random search
- Price sampling:
  - A random price is drawn for each itinerary
  - If the spilled passengers are higher than the average ⇒ decrease the price
  - Otherwise ⇒ increase the price
- Fixing FAs & VNS:
  - Low spill value ⇒ associated flights have a higher probability to be fixed to their current aircraft
  - If the solution is improved more assignments are fixed and vice versa.

Data

no	airports	flights	flights per route	demand per flight	fleet composition	
1	3	10	1.67	51.90	2	50-37
2	3	11	2.75	83.10	2	117-50
3	3	12	2.00	113.80	2	164-100
4	3	12	2.00	113.80	6	164-146-128-124-107-100
5	3	26	4.33	56.10	3	100-50-37
6	3	19	3.17	96.70	3	164-117-72
7	3	19	3.17	96.70	5	124-107-100-85-72
8	3	12	3.00	193.40	3	293-195-164
9	3	33	8.25	71.90	3	117-70-37
10	3	32	5.33	100.50	3	164-117-85
11	3	32	5.33	100.50	5	128-124-107-100-85
12	2	11	5.50	173.70	3	293-164-127
13	4	39	4.88	64.50	4	117-85-50-37
14	4	23	3.83	86.10	4	117-85-70-50
15	4	19	3.17	101.40	4	134-117-100-85
16	4	19	3.17	101.40	5	128-124-107-100-85
17	4	15	1.88	58.10	5	117-85-70-50-37
18	4	14	2.33	87.60	5	134-117-85-70-50
19	4	13	2.60	100.10	5	164-134-117-100-85
20	3	33	8.25	71.90	4	85-70-50-35
21	3	46	7.67	86.85	5	128-124-107-100-85
22	7	48	2.29	101.94	4	124-107-100-85
23	3	61	15.25	69.15	4	117-85-50-37
24	8	77	2.08	67.84	4	117-85-50-37
25	8	97	3.46	90.84	5	164-117-100-85-50

Data instances are derived from ROADEF 2009 dataset.

Computational results

	BONMIN Integrated model		Sequential approach (SA)			Local search heuristic *Average over 5 replications*			
	Profit	Time(s) *max 12h*	Profit	% dev from BONMIN	Time(s) max 1h	Profit	%dev from BONMIN	%imp. over SA	Time(s) *max 1h*
1	15,091	11	15,091	0.00%	1	15,091	0.00%	0.00%	1
2	37,335	27	35,372	-5.26%	1	37,335	0.00%	5.55%	13
3	50,149	56	50,149	0.00%	1	50,149	0.00%	0.00%	1
4	46,037	2,686	43,990	-4.45%	1	46,037	0.00%	4.66%	3
5	70,904	2,479	69,901	-1.42%	1	70,679	-0.32%	1.11%	6
6	82,311	1,493	82,311	0.00%	1	82,311	0.00%	0.00%	1
7	87,212	42,628	84,186	-3.47%	1	87,212	0.00%	3.59%	60
8	906,791	12,964	904,054	-0.30%	1	906,791	0.00%	0.30%	2
9	135,656	23,662	135,656	0.00%	2	135,656	0.00%	0.00%	2
10	115,983	209	115,983	0.00%	1	115,983	0.00%	0.00%	1
11	94,203	1,724	93,920	-0.30%	3	94,203	0.00%	0.30%	10
12	858,544	7,343	854,902	-0.42%	1	858,545	0.00%	0.43%	1
13	138,575	37,177	137,428	-0.83%	2	138,575	0.00%	0.83%	173
14	96,486	17,142	93,347	-3.25%	1	96,486	0.00%	3.36%	89
15	49,448	32	49,448	0.00%	1	49,448	0.00%	0.00%	1
16	38,205	240	37,100	-2.89%	1	38,205	0.00%	2.98%	1
17	27,076	56	27,076	0.00%	1	27,076	0.00%	0.00%	1
18	53,128	141	52,369	-1.43%	1	53,128	0.00%	1.45%	1
19	26,486	14	26,486	0.00%	1	26,486	0.00%	0.00%	1
20	146,467	31,945	146,464	-0.00%	2	147,506	0.71%	0.71%	380
21	207,434	4,848	217,169	4.69%	9	219,136	5.64%	0.91%	1,395
22	153,789	4,387	163,114	6.06%	4	163,393	6.24%	0.17%	126
23	227,364	22,174	226,615	-0.33%	34	227,284	-0.04%	0.30%	1,283
24	194,598	42,360	208,561	7.18%	19	210,395	8.12%	0.88%	791
25	463,731	31,535	469,136	1.17%	14	470,494	1.46%	0.29%	1,117

Non-convexity

REV^{LS}:non-convex, no information on the quality of the solution

Schön (2008) similar model (based on synthetic data, without spill)

$$ms_i = \frac{\exp(V_i)}{\sum_{j \in I_s} \exp(V_j)}, \qquad V_i = \beta p_i + c_i$$

A new variable υ_s:

$$\upsilon_s = \frac{1}{\sum_{j \in I_s} \exp(V_j)}$$

$$\mathrm{ms}_i = \upsilon_s \exp(\beta p_i + c_i)$$

Inverse price-demand function:

$$p_i = \frac{1}{\beta}(\ln(\frac{\mathrm{ms}_i}{\upsilon_s}) - c_i)$$

$$\sum_{i \in I_s} \mathrm{ms}_i = 1 \quad d_i = D_s \mathrm{ms}_i$$

Revenue $(d_i p_i)$ is convex for $\beta < 0$

Non-convexity

Limiting for advanced demand models:

- more policy variables
  - aircraft type
  - departure time etc.

$$\mathrm{ms}_i = \upsilon_s \exp(\beta_p p_i + \beta_t t_i + c_i)$$

- disaggregate/individual level variables
  - trip purpose
  - income level etc.

$$\mathrm{ms}_{i,n} = \upsilon_{s,n} \exp(\beta_p p_i + \beta_n z_n + c_i)$$
$$d_i = \sum_{n \in N} \mathrm{ms}_{i,n}$$

Log transformation

We propose a logarithmic transformation

$$\begin{aligned} \mathrm{ms}_i &= \upsilon_s \cdot \exp\left(V_i(p_i, z_i, \beta)\right) \qquad \forall h \in H, s \in S^h, i \in I_s \\ &= \upsilon_s \cdot \exp\left(\beta \ln\left(p_i\right) + c_i\right) \end{aligned}$$

$$\begin{aligned} \ln\left(\mathrm{ms}_i\right) &= \ln\left(\upsilon_s\right) + \beta \ln\left(p_i\right) + c_i \\ \mathrm{ms}_i^{'} &= \upsilon_s^{'} + \beta p_i^{'} + c_i, \end{aligned}$$

where $\mathrm{ms}_i > 0, \upsilon_s > 0, p_i > 0$

TRANSP-OR

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Transformed revenue model - FAs fixed, no spill

$$\max \sum_{h \in H} \sum_{s \in S^h} \sum_{i \in (I_s \setminus I'_s)} d_i p_i$$

$$\sum_{h \in H} \pi^h_{k,f} = Q_k X_{k,f} \qquad \forall f \in F, k \in K$$

$$\sum_{s \in S^h} \sum_{i \in (I_s \setminus I'_s)} \delta_{i,f} d_i \leq \sum_{k \in K} \pi^h_{k,f} \qquad \forall h \in H, f \in F$$

$$d_i \leq D_s \mathrm{ms}_i \qquad \forall h \in H, s \in S^h, i \in I_s$$

$$\mathrm{ms}_i = \upsilon_s \exp(\beta \ln(p_i) + c_i) \qquad \forall h \in H, s \in S^h, i \in I_s$$

$$\sum_{i \in I_s} \mathrm{ms}_i = 1 \qquad \forall h \in H, s \in S^h$$

$$\pi^h_{k,f} \geq 0 \qquad \forall h \in H, k \in K, f \in F$$

$$d_i \geq 0 \qquad \forall h \in H, s \in S^h, i \in I_s$$

$$\mathrm{ms}_i \geq 0 \qquad \forall h \in H, s \in S^h, i \in I_s$$

$$LB_i \leq p_i \leq UB_i \qquad \forall h \in H, s \in S^h, i \in I_s$$

$$\upsilon_s \geq 0 \qquad \forall h \in H, s \in S^h$$

Transformed revenue model - FAs fixed, no spill

$$\max \sum_{h\in H}\sum_{s\in S^h}\sum_{i\in(I_s\setminus I_s')} d_i' + p_i'$$

$$\sum_{h\in H} \pi_{k,f}^h = Q_k X_{k,f} \qquad \forall f\in F, k\in K$$

$$\sum_{s\in S^h}\sum_{i\in(I_s\setminus I_s')} \delta_{i,f}\exp(d_i') \le \sum_{k\in K}\pi_{k,f}^h \qquad \forall h\in H, f\in F^*$$

$$d_i' \le \ln(D_s) + \mathrm{ms}_i' \qquad \forall h\in H, s\in S^h, i\in I_s$$

$$\mathrm{ms}_i' = \upsilon_s' + \beta p_i' + c_i \qquad \forall h\in H, s\in S^h, i\in I_s$$

$$\sum_{i\in I_s}\exp(\mathrm{ms}_i') = 1 \qquad \forall h\in H, s\in S^h$$

$$\pi_{k,f}^h \ge 0 \qquad \forall h\in H, k\in K, f\in F$$

$$d_i' \in \Re \qquad \forall h\in H, s\in S^h, i\in I_s$$

$$\mathrm{ms}_i' \in \Re \qquad \forall h\in H, s\in S^h, i\in I_s$$

$$\ln(LB_i) \le p_i' \le \ln(UB_i) \qquad \forall h\in H, s\in S^h, i\in I_s$$

$$\upsilon_s' \in \Re \qquad \forall h\in H, s\in S^h$$

Transformed revenue model - FAs fixed, no spill

$$\max \sum_{h \in H} \sum_{s \in S^h} \sum_{i \in (I_s \setminus I_s')} d_i' + p_i' \; - \; \text{penalty} \cdot \text{dev}_{s,h}$$

$$\sum_{h \in H} \pi_{k,f}^h = Q_k X_{k,f} \qquad \forall f \in F, k \in K$$

$$\sum_{s \in S^h} \sum_{i \in (I_s \setminus I_s')} \delta_{i,f} \exp(d_i') \leq \sum_{k \in K} \pi_{k,f}^h \qquad \forall h \in H, f \in F^*$$

$$d_i' \leq \ln(D_s) + \text{ms}_i' \qquad \forall h \in H, s \in S^h, i \in I_s$$

$$\text{ms}_i' = \upsilon_s' + \beta p_i' + c_i \qquad \forall h \in H, s \in S^h, i \in I_s$$

$$\sum_{i \in I_s} \exp(\text{ms}_i') \leq 1 \qquad \forall h \in H, s \in S^h$$

$$\text{dev}_{s,h} \geq (1 - \sum_{i \in I_s} \exp(\text{ms}_i'))^2 \qquad \forall h \in H, s \in S^h$$

$$\pi_{k,f}^h \geq 0 \qquad \forall h \in H, k \in K, f \in F$$

$$d_i' \in \Re \qquad \forall h \in H, s \in S^h, i \in I_s$$

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$$\upsilon_s' \in \Re \qquad \forall h \in H, s \in S^h$$

Can be solved with NLP solvers like MOSEK

Transformed revenue model - FAs fixed, no spill

$$\max \sum_{h \in H} \sum_{s \in S^h} \sum_{i \in (I_s \setminus I_s')} d_i' + p_i' \; - \; \text{penalty} \cdot \text{dev}_{s,h}$$

$$\sum_{h \in H} \pi_{k,f}^h = Q_k X_{k,f} \qquad \forall f \in F, k \in K$$

$$\sum_{s \in S^h} \sum_{i \in (I_s \setminus I_s')} \delta_{i,f} \exp(d_i') \leq \sum_{k \in K} \pi_{k,f}^h \qquad \forall h \in H, f \in F^*$$

$$d_i' \leq \ln(D_s) + \text{ms}_i' \qquad \forall h \in H, s \in S^h, i \in I_s$$

$$\text{ms}_i' = \upsilon_s' + \beta p_i' + c_i \qquad \forall h \in H, s \in S^h, i \in I_s$$

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$$\ln(LB_i) \leq p_i' \leq \ln(UB_i) \qquad \forall h \in H, s \in S^h, i \in I_s$$

$$\upsilon_s' \in \Re \qquad \forall h \in H, s \in S^h$$

Can be solved with NLP solvers like MOSEK

Similarly, $\text{ms}_i = \exp(\text{ms}_i')$ could be defined and penalized

What about spill?

Wang, Shebalov and Klabjan 2012, working paper on spill and recapture

- Spill and recapture based on attractiveness
- Attractiveness is fixed, no explicit demand model

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- $\mathrm{ms}_i \leq \frac{\exp(V_i)}{\exp(V_0)} \mathrm{ms}_0,$

  itinerary 0: no-revenue/competing itineraries of segment s
- $\sum_{i \in I_s} \mathrm{ms}_i + \mathrm{ms}_0 = 1$

  $\Rightarrow$ spill is allowed

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- $\sum_{i \in I_s} \mathrm{ms}_i + \mathrm{ms}_0 = 1$

  $\Rightarrow$ spill is allowed

Log transformation is applicable to the new formulation as well.

GBD framework

Li and Sun (2006) Mixed Integer Nonlinear Programming

- Initial FAM solution ($x_{k,f}$), selection of flights
- Repeat until UB $\leq$ LB + *allowed gap*
    - Solve REV subproblem which is a convex NLP and obtain...
        - Price, market share, allocated seats ($p_i^{'}$, $\mathrm{ms}_i^{'}$, $\pi_{k,f}^h$)
        - Lagrangian multipliers $\Rightarrow$ Benders cuts
          $\Rightarrow$ Information on the potential revenue with capacity change
        - A lower bound (LB) for the problem
    - Solve the FAM master problem (with the cuts) which is a MILP and obtain...
        - An updated FAM solution ($x_{k,f}$)
        - An upper bound (UB) for the problem

Conclusions

- The integrated model has promising results
- ... which motivates the effort in devising solution methodologies
- Logarithmic transformation provides a convex formulation
- ... which is flexible for the integration of advanced demand models
- The model is flexible to include the spill and recapture effects

On-going work

- The GBD will be tested
- Or other bi-level programming tools
- When finalized...
- ... a complete framework for the integration of explicit supply-demand interactions in optimization models
  - scheduling, fleeting, pricing
  - spill and recapture
  - appropriate and flexible solution method

Thank you for your attention!

- **Value of time (VOT):**

$$VOT_i = \frac{\partial V_i / \partial time_i}{\partial V_i / \partial cost_i} = \frac{\beta_{time} \cdot cost_i}{\beta_{cost}}$$

For the same OD pair...

- VOT for economy, non-stop: 8 €/hour
- VOT for economy, one-stop: 19.8, 11, 9.2 €/hour
- VOT for business, non-stop: 21.7 €/hour

Improvement due to the local search

	Sequential approach (SA)	Random neighborhood		Neighborhood based on spill		% Improvement	
	Profit	Profit	Time(sec)	Profit	Time(sec)	Quality of the solution	Reduction in time
2	35,372	37,335	116	37,335	13	-	89.10%
4	43,990	44,302	27	46,037	3	3.92%	88.88%
5	69,901	*No imp. over SA*		70,679	6	1.11%	-
7	84,186	85,335	1,649	87,212	60	2.20%	96.36%
8	904,054	906,791	209	906,791	2	-	99.04%
11	93,920	*No imp. over SA*		94,203	10	0.30%	-
12	854,902	*No imp. over SA*		858,545	1	0.43%	-
13	137,428	*No imp. over SA*		138,575	173	0.83%	-
14	93,347	96,365	943	96,486	89	0.13%	90.56%
16	37,100	38,205	6	38,205	1	-	80.65%
18	52,369	53,128	334	53,128	1	-	99.80%
20	146,464	*No imp. over SA*		147,506	380	0.71%	-
21	217,169	*No imp. over SA*		219,136	1,395	0.91%	-
22	163,114	*No imp. over SA*		163,393	126	0.17%	-
23	226,615	*No imp. over SA*		227,284	1,283	0.30%	-
24	208,561	*No imp. over SA*		210,395	791	0.88%	-
25	469,136	*No imp. over SA*		470,494	1,117	0.29%	-

A small example

- 2 airports CDG-MRS
- 4 flights - all are mandatory
- 2 aircraft types: 37-50 seats

We start with an initial FAM solution:

	AC1	AC2
F1	X	
F2	X	
F3	X	
F4	X	

A small example - GBD iterations

Iteration 1		
	Sub	Master
	12522.8	16923.4
	LB	**UB**
	12522.8	16923.4
	AC1	AC2
F1		X
F2		X
F3		X
F4		X

⟹

Iteration 2		
	Sub	Master
	10734.4	14822.8
	LB	**UB**
	12522.8	14822.8
	AC1	AC2
F1		X
F2		X
F3	X	
F4	X	

Iteration 3		
	Sub	Master
	12696.8	14822.8
	LB	**UB**
	12696.8	14822.8
	AC1	AC2
F1	X	
F2		X
F3		X
F4	X	

⟹

Iteration 4		
	Sub	Master
	12474.4	12696.8
	LB	**UB**
	12696.8	**12696.8**
	AC1	AC2
F1		X
F2		X
F3	X	
F4	X	