An integrated schedule planning and revenue management model

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Motivation

- Demand responsive transportation systems
  - Better representation of demand ⇒ Appropriate demand models
  - Flexibility in supply ⇒ New concept: Clip-Air (Sponsored by EPFL - Middle East.)
  - Integration of supply-demand interactions in transportation models

Objectives

- Comparative analysis between standard fleet and Clip-Air

  Atasoy, Salani, Bierlaire, and Leonardi, 2012
- Development of appropriate demand models

  Atasoy and Bierlaire, 2012
- Development of integrated schedule design and fleet assignment model and revenue management (supply-demand interactions)

  Atasoy, Salani, and Bierlaire, 2012
- **Solution techniques for the resulting decision problems**

Itinerary choice model

- Market segments, s, defined by the class and each OD pair
- Itinerary choice among the set of alternatives, I_s, for each segment s
- For each itinerary $i \in I_s$ the utility is defined by:

$$\mathrm{V}_i = \mathrm{ASC}_i + \beta_p \cdot \ln(p_i) + \beta_{time} \cdot \mathrm{time}_i + \beta_{morning} \cdot \mathrm{morning}_i$$

$$\mathrm{V}_i = \mathrm{V}_i(p_i, z_i, \beta)$$

  - ASC_i : alternative specific constant
  - p is a policy variable and included as log
  - p and time are interacted with non-stop/stop
  - $\mathrm{morning}$ is 1 if the itinerary is a morning itinerary
- *No-revenue* represented by the subset $I_s' \in I_s$ for segment s.

Itinerary choice model

- Demand for class h for each itinerary i in market segment s:

$$\tilde{d}_i = D_s \frac{\exp(V_i(p_i, z_i, \beta))}{\sum_{j \in I_s} \exp(V_j(p_j, z_j, \beta))}$$

  - D_s is the total expected demand for market segment s.

- **Spill and recapture effects**: Capacity shortage $\Rightarrow$ passengers may be recaptured by other itineraries (instead of their desired itineraries)
- Recapture ratio is given by:

$$b_{i,j} = \frac{\exp(V_j(p_j, z_j, \beta))}{\sum_{k \in I_s \setminus \{i\}} \exp(V_k(p_k, z_k, \beta))}$$

Estimation

- **Revealed preferences (RP) data:** Booking data from a major European airline
    - Lack of variability
    - Price inelastic demand
- RP data is combined with a **stated preferences (SP) data**
- Time, cost and morning parameters are **fixed** to be the same for the two datasets.
- A **scale** parameter is introduced for SP to capture the differences in variance.

Further details in Atasoy and Bierlaire (2012).

Estimation results

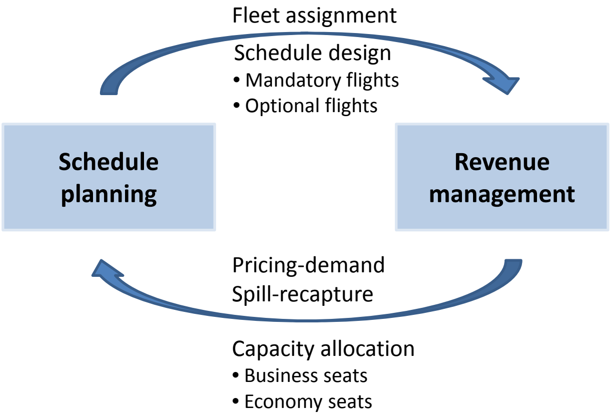
	β_{fare}		β_{time}		
	non-stop	one-stop	non-stop	one-stop	$\beta_{morning}$
economy	-2.23	-2.17	-0.102	-0.0762	0.0283
business	-1.97	-1.97	-0.104	-0.0821	0.079

- **Price elasticity** of demand:

$$E^{P_i}_{price_i} = \frac{\partial P_i}{\partial price_i} \cdot \frac{price_i}{P_i}$$

An example
- for a non-stop itinerary
  - price elasticity for economy is -2.03 and -1.86 for business
- for a one-stop itinerary
  - price elasticity for economy is -2.14 and -1.95 for business

Integrated schedule planning and revenue management



Integrated model - Schedule planning

$$Max \sum_{h\in H}\sum_{s\in S^h}\sum_{i\in(I_s\setminus I_s')}(d_i - \sum_{j\in I_s} t_{i,j} + \sum_{j\in(I_s\setminus I_s')} t_{j,i}b_{j,i})p_i - \sum_{\substack{k\in K\\ f\in F}} C_{k,f}x_{k,f}: \text{ revenue - cost} \quad (1)$$

$$s.t. \sum_{k\in K} x_{k,f} = 1: \text{ mandatory flights} \qquad \forall f \in F^M \quad (2)$$

$$\sum_{k\in K} x_{k,f} \leq 1: \text{ optional flights} \qquad \forall f \in F^O \quad (3)$$

$$y_{k,a,t^-} + \sum_{f\in In(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f\in Out(k,a,t)} x_{k,f}: \text{ flow conservation} \qquad \forall [k,a,t] \in N \quad (4)$$

$$\sum_{a\in A} y_{k,a,minE_a^-} + \sum_{f\in CT} x_{k,f} \leq R_k: \text{ fleet availability} \qquad \forall k \in K \quad (5)$$

$$y_{k,a,minE_a^-} = y_{k,a,maxE_a^+}: \text{ cyclic schedule} \qquad \forall k \in K, a \in A \quad (6)$$

$$\sum_{h\in H} \pi^h_{k,f} = Q_k x_{k,f}: \text{ seat capacity} \qquad \forall f \in F, k \in K \quad (7)$$

$$x_{k,f} \in \{0,1\} \qquad \forall k \in K, f \in F \quad (8)$$

$$y_{k,a,t} \geq 0 \qquad \forall [k,a,t] \in N \quad (9)$$

- *Itinerary-based fleet assignment*
- *Spill and recapture*

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Integrated model - Revenue management

$$\sum_{s\in S^h}\sum_{i\in(I_s\setminus I_s^{'})}\delta_{i,f}d_i-\sum_{j\in I_s}\delta_{i,f}t_{i,j}+\sum_{\substack{j\in(I_s\setminus I_s^{'})\\ i\neq j}}\delta_{i,f}t_{j,i}b_{j,i}\leq\sum_{k\in K}\pi^h_{k,f}:\ \textit{capacity}\qquad \forall h\in H,f\in F \qquad (10)$$

$$\sum_{\substack{j\in I_s\\ i\neq j}}t_{i,j}\leq d_i:\ \textit{total spill}\qquad \forall h\in H,s\in S^h,i\in(I_s\setminus I_s^{'}) \qquad (11)$$

$$\tilde{d}_i=D_s\frac{\exp(V_i(p_i,z_i,\beta))}{\sum_{j\in I_s}\exp(V_j(p_j,z_j,\beta))}:\ \textit{logit demand}\qquad \forall h\in H,s\in S^h,i\in I_s \qquad (12)$$

$$b_{i,j}=\frac{\exp(V_j(p_j,z_j,\beta))}{\sum_{k\in I_s\setminus\{i\}}\exp(V_k(p_k,z_k,\beta))}:\ \textit{recapture ratio}\qquad \forall h\in H,s\in S^h,i\in(I_s\setminus I_s^{'}),j\in I_s \qquad (13)$$

$$d_i\leq\tilde{d}_i:\ \textit{realized demand}\qquad \forall h\in H,s\in S^h,i\in I_s \qquad (14)$$

$$LB_i\leq p_i\leq UB_i:\ \textit{bounds on price}\qquad \forall h\in H,s\in S^h,i\in I_s \qquad (15)$$

$$t_{i,j}\geq 0\qquad \forall h\in H,s\in S^h,i\in(I_s\setminus I_s^{'}),j\in I_s \qquad (16)$$

$$b_{i,j}\geq 0\qquad \forall h\in H,s\in S^h,i\in(I_s\setminus I_s^{'}),j\in I_s \qquad (17)$$

$$\pi^h_{k,f}\geq 0\qquad \forall h\in H,k\in K,f\in F \qquad (18)$$

Integrated model

- The resulting model is a mixed integer nonlinear problem
- Nonlinearity is due to the explicit supply-demand interactions
- The model is implemented in AMPL and BONMIN solver is used
- BONMIN does not guarantee optimality

- We consider a **sequential approach** as a reference model to evaluate the integrated model:
  - Fleet assignment is optimized with estimated demand/price
  - Revenue is optimized with the resulting capacity

Added value of the integration - Sequential vs integrated

	Sequential approach (SA)				Integrated model - % Change			
	Profit	Pax.	Flights	Seats	Profit	Pax.	Flights	Seats
1	15,091	284	8	124	-	-	8	124
2	35,372	400	8	150	5.55%	33.50%	8	217
3	50,149	859	10	300	-	-	10	300
4	43,990	882	10	331	4.45%	-17.80%	8	207
5	69,901	931	22	274	1.43%	14.18%	24	324
6	82,311	1,145	16	333	-	-	16	333
7	84,186	1,131	14	329	3.47%	-3.80%	14	329
8	904,054	1,448	10	1,148	0.30%	-	10	1,312
9	135,656	1,814	32	498	-	-	32	498
10	115,983	2,236	26	691	-	-	26	691
11	93,920	2,270	26	747	0.30%	-0.97%	26	747
12	854,902	1,270	10	1,016	0.43%	5.83%	10	1,090
13	27,076	448	10	207	-	-	10	207
14	52,369	599	10	267	1.45%	16.69%	12	267
15	51,160	793	8	402	-	-	8	402
16	37,100	1,067	12	377	2.89%	-2.72%	12	377
17	137,428	1,517	34	391	0.83%	4.94%	34	476
18	93,347	1,144	20	387	3.36%	1.40%	20	457
19	83,251	1,104	12	536	-	-	12	536

Data instances are derived from ROADEF 2009 dataset.

Heuristic method

Available solvers are able to converge on instances with up to 4 airports and about 35 flights.
We devised a heuristic procedure based on two simplified versions of the model:

- FAM^{LS}: price-inelastic schedule planning model $\Rightarrow$ MILP
  - Explores new fleet assignment solutions based on a local search
  - Price sampling
  - Variable neighborhood search (VNS)
- REV^{LS}: Revenue management with fixed capacity $\Rightarrow$ NLP
  - Optimizes the revenue for the explored fleet assignment solution

Heuristic method

Require: x^0, y^0, d^0, p^0, t^0, b^0, π^0, time_{max}, n_{min}, n_{max}, notImpr, tabuListSize

$g := 0$, $\mathrm{time} := 0$, $n_{\mathrm{fixed}} := n_{\mathrm{min}}$, $\mathrm{notImpr} := 0$, $z^* := -\mathrm{INF}$, $\mathrm{tabuList} := \emptyset$

repeat

- p^g := Price sampling(t^{g-1}, p^{g-1}, d^{g-1})
- $\{d^g, b^g\}$:= Logit model(p^g)
- L := Fixing(x^{g-1}, t^{g-1}, n_{fixed})
- $\{x^g, y^g, \pi^g, t^g\}$:= solve $z_{\mathrm{FAM}^{\mathrm{LS}}(p^g, d^g, b^g, L)}$
- **if** ($\bar{x}^g \notin \mathrm{tabuList}$) **then**
  - $\mathrm{tabuList} := \mathrm{tabuList} \bigcup x^g$
  - $\{p^g, d^g, b^g, \pi^g, t^g\}$:= solve $z_{\mathrm{REV}^{\mathrm{LS}}(x^g, y^g)}$
  - **if** ($z_{\mathrm{REV}^{\mathrm{LS}} \geq z^*}$) **then**
    - Update z^*
    - Intensification: $n_{\mathrm{fixed}} := n_{\mathrm{fixed}} + 1$ when $n_{\mathrm{fixed}} < n_{\mathrm{max}}$
    - $\mathrm{notImpr} := 0$
  - **else if** ($\mathrm{notImpr} == 3$) **then**
    - Diversification: $n_{\mathrm{fixed}} := n_{\mathrm{fixed}} - 1$ when $n_{\mathrm{fixed}} > n_{\mathrm{min}}$
    - $\mathrm{notImpr} := \mathrm{notImpr} - 1$
  - **end if**
- **end if**
- $g := g + 1$

until $\mathrm{time} \geq \mathrm{time}_{max}$

Local search

- Neighborhood solutions are visited based on the spill rather than a fully random search
- Price sampling:
    - A random price is drawn for each itinerary
    - If the spilled passengers are higher than the average $\Rightarrow$ decrease the price
    - Otherwise $\Rightarrow$ increase the price
- Fixing FAM solutions - VNS:
    - The itineraries are sorted according to their spilled number of passengers
    - Low spill value $\Rightarrow$ associated flights have a higher probability to be fixed to their current aircraft
    - If the solution is improved more assignments are fixed and vice versa.

Performance of the heuristic

The omitted instances are the ones where the sequential approach has the same solution as the integrated model.

		SA	Integrated model					
			Best solution by BONMIN		Heuristic - Avg. over 5 runs			
	Flights	Profit	Profit	Time (sec) *max 43,200*	%dev. from BONMIN	%imp. over SA	Time (sec) *max 3,600*	%time red.
2	11	35,372	37,335	27	0.00%	5.55%	13	53.33%
4	12	43,990	46,037	2,686	0.00%	4.66%	3	99.90%
5	26	69,901	70,904	2,479	**0.32%**	1.11%	6	99.75%
7	19	84,186	87,212	42,628	0.00%	3.59%	60	99.86%
8	12	904,054	906,791	12,964	0.00%	0.30%	2	99.98%
11	32	93,920	94,203	1,724	0.00%	0.30%	10	99.42%
12	11	854,902	858,544	7,343	0.00%	0.43%	1	99.99%
13	39	137,428	138,575	37,177	0.00%	0.83%	173	99.54%
14	23	93,347	96,486	17,142	0.00%	3.36%	89	99.48%
16	19	37,100	38,205	240	0.00%	2.98%	1	99.50%
18	14	52,369	53,128	141	0.00%	1.45%	1	99.53%
20	33	146,464	146,467	31,945	-0.71%	0.71%	380	98.81%
21	46	217,169	207,434	4,848	-5.64%	0.91%	1,395	71.22%
22	48	163,114	153,789	4,387	-6.24%	0.17%	126	97.12%
23	61	226,615	227,364	22,174	0.04%	0.30%	1,283	94.21%
24	77	208,561	194,598	42,360	-8.12%	0.88%	791	98.13%
25	97	469,136	463,731	31,535	-1.46%	0.29%	1,117	96.46%

Improvement due to the local search

	SA	Random neighborhood		Neighborhood based on spill		% Improvement	
	Profit	Profit	Time(sec)	Profit	Time(sec)	Quality of the solution	Reduction in time
2	35,372	37,335	116	37,335	13	-	89.10%
4	43,990	44,302	27	46,037	3	3.92%	-
5	69,901	*No imp. over SA*		70,679	6	1.11%	-
7	84,186	85,335	1,649	87,212	60	2.20%	-
8	904,054	906,791	209	906,791	2	-	99.04%
11	93,920	*No imp. over SA*		94,203	10	0.30%	-
12	854,902	*No imp. over SA*		858,545	1	0.43%	-
13	137,428	*No imp. over SA*		138,575	173	0.83%	-
14	93,347	96,365	943	96,486	89	0.13%	-
16	37,100	38,205	6	38,205	1	-	80.65%
18	52,369	53,128	334	53,128	1	-	99.80%
20	146,464	*No imp. over SA*		147,506	380	0.71%	-
21	217,169	*No imp. over SA*		219,136	1,395	0.91%	-
22	163,114	*No imp. over SA*		163,393	126	0.17%	-
23	226,615	*No imp. over SA*		227,284	1,283	0.30%	-
24	208,561	*No imp. over SA*		210,395	791	0.88%	-
25	469,136	*No imp. over SA*		470,494	1,117	0.29%	-

Conclusions and future work

- Integrated schedule planning and revenue management
  - More efficient schedule planning with the information on supply-demand interactions
- Heuristic
  - Obtaining upper bound in order to more appropriately quantify the performance of the heuristic
- Further solution methods for the resulting mixed integer nonlinear problem
  - Decomposition methods $\Rightarrow$ FAM and REV models

Thank you for your attention!

Clip-Air