An integrated fleet assignment model with supply-demand interactions

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Motivation

- Demand responsive transportation systems
  - Better representation of demand ⇒ Appropriate demand models
  - Flexibility in supply ⇒ New concept: Clip-Air
  - Integration of supply-demand interactions in transportation models

Itinerary choice model ▸ DCA

- Market segments, s, defined by the class and each OD pair
- Itinerary choice among the set of alternatives, I_s, for each segment s
- For each itinerary $i \in I_s$ the utility is defined by:

$$V_i = \mathrm{ASC}_i + \beta_p \cdot \ln(p_i) + \beta_{time} \cdot \mathrm{time}_i + \beta_{morning} \cdot \mathrm{morning}_i$$

$$V_i = V_i(p_i, z_i, \beta)$$

  - ASC_i : alternative specific constant
  - p is a policy variable and included as log
  - p and time are interacted with non-stop/stop
  - $\mathrm{morning}$ is 1 if the itinerary is a morning itinerary
- *No-revenue* represented by the subset $I'_s \in I_s$ for segment s.

Itinerary choice model

- Demand for class h for each itinerary i in market segment s:

$$\tilde{d}_i = D_s \frac{\exp(V_i(p_i, z_i, \beta))}{\sum_{j \in I_s} \exp(V_j(p_j, z_j, \beta))}$$

  - D_s is the total expected demand for market segment s.

- **Spill and recapture effects**: Capacity shortage $\Rightarrow$ passengers may be recaptured by other itineraries (instead of their desired itineraries)
- Recapture ratio is given by:

$$b_{i,j} = \frac{\exp(V_j(p_j, z_j, \beta))}{\sum_{k \in I_s \setminus \{i\}} \exp(V_k(p_k, z_k, \beta))}$$

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Estimation

- **Revealed preferences (RP) data:** Booking data from a major European airline
  - Lack of variability
  - Price inelastic demand
- RP data is combined with a **stated preferences (SP) data**
- Time, cost and morning parameters are **fixed** to be the same for the two datasets.
- A **scale** parameter is introduced for SP to capture the differences in variance.

Estimation results

	β_{fare}		β_{time}		
	non-stop	one-stop	non-stop	one-stop	$\beta_{morning}$
economy	-2.23	-2.17	-0.102	-0.0762	0.0283
business	-1.97	-1.97	-0.104	-0.0821	0.079

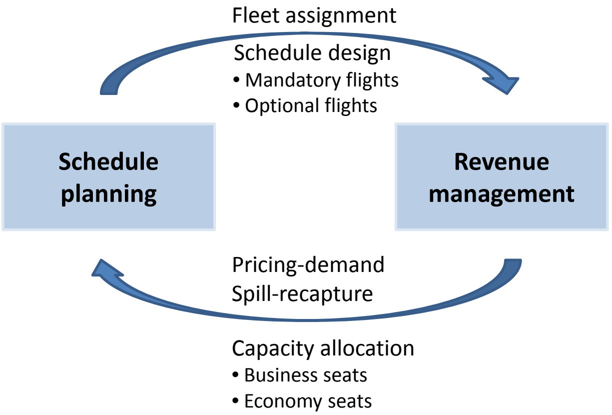
- **Price elasticity** of demand:

$$E^{P_i}_{price_i} = \frac{\partial P_i}{\partial price_i} \cdot \frac{price_i}{P_i}$$

An example

- for a non-stop itinerary
  - price elasticity for economy is -2.03 and -1.86 for business
- for a one-stop itinerary
  - price elasticity for economy is -2.14 and -1.95 for business

Integrated schedule planning and revenue management



Integrated model - Schedule planning

$$Max \sum_{h\in H}\sum_{s\in S^h}\sum_{i\in(I_s\setminus I_s')}(d_i - \sum_{j\in I_s} t_{i,j} + \sum_{j\in(I_s\setminus I_s')} t_{j,i}b_{j,i})p_i - \sum_{\substack{k\in K\\ f\in F}} C_{k,f}x_{k,f}: \text{ revenue - cost} \quad (1)$$

$$s.t. \sum_{k\in K} x_{k,f} = 1: \text{ mandatory flights} \qquad \forall f \in F^M \quad (2)$$

$$\sum_{k\in K} x_{k,f} \leq 1: \text{ optional flights} \qquad \forall f \in F^O \quad (3)$$

$$y_{k,a,t^-} + \sum_{f\in In(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f\in Out(k,a,t)} x_{k,f}: \text{ flow conservation} \qquad \forall [k,a,t] \in N \quad (4)$$

$$\sum_{a\in A} y_{k,a,minE_a^-} + \sum_{f\in CT} x_{k,f} \leq R_k: \text{ fleet availability} \qquad \forall k \in K \quad (5)$$

$$y_{k,a,minE_a^-} = y_{k,a,maxE_a^+}: \text{ cyclic schedule} \qquad \forall k \in K, a \in A \quad (6)$$

$$\sum_{h\in H} \pi^h_{k,f} = Q_k x_{k,f}: \text{ seat capacity} \qquad \forall f \in F, k \in K \quad (7)$$

$$x_{k,f} \in \{0,1\} \qquad \forall k \in K, f \in F \quad (8)$$

$$y_{k,a,t} \geq 0 \qquad \forall [k,a,t] \in N \quad (9)$$

Integrated model - Revenue management

$$\sum_{s\in S^h}\sum_{i\in(I_s\setminus I_s^{'})}\delta_{i,f}d_i-\sum_{j\in I_s}\delta_{i,f}t_{i,j}+\sum_{\substack{j\in(I_s\setminus I_s^{'})\\ i\neq j}}\delta_{i,f}t_{j,i}b_{j,i}\leq\sum_{k\in K}\pi_{k,f}:\ \textit{capacity}\qquad\forall h\in H,f\in F\qquad(10)$$

$$\sum_{\substack{j\in I_s\\ i\neq j}}t_{i,j}\leq d_i:\ \textit{total spill}\qquad\forall h\in H,s\in S^h,i\in(I_s\setminus I_s^{'})\qquad(11)$$

$$\tilde{d}_i=D_s\frac{\exp(V_i(p_i,z_i,\beta))}{\sum_{j\in I_s}\exp(V_j(p_j,z_j,\beta))}:\ \textit{logit demand}\qquad\forall h\in H,s\in S^h,i\in I_s\qquad(12)$$

$$b_{i,j}=\frac{\exp(V_j(p_j,z_j,\beta))}{\sum_{k\in I_s\setminus\{i\}}\exp(V_k(p_k,z_k,\beta))}:\ \textit{recapture ratio}\qquad\forall h\in H,s\in S^h,i\in(I_s\setminus I_s^{'}),j\in I_s\qquad(13)$$

$$d_i\leq\tilde{d}_i:\ \textit{realized demand}\qquad\forall h\in H,s\in S^h,i\in I_s\qquad(14)$$

$$0\leq p_i\leq UB_i:\ \textit{upper bound on price}\qquad\forall h\in H,s\in S^h,i\in I_s\qquad(15)$$

$$t_{i,j}\geq 0\qquad\forall h\in H,s\in S^h,i\in(I_s\setminus I_s^{'}),j\in I_s\qquad(16)$$

$$b_{i,j}\geq 0\qquad\forall h\in H,s\in S^h,i\in(I_s\setminus I_s^{'}),j\in I_s\qquad(17)$$

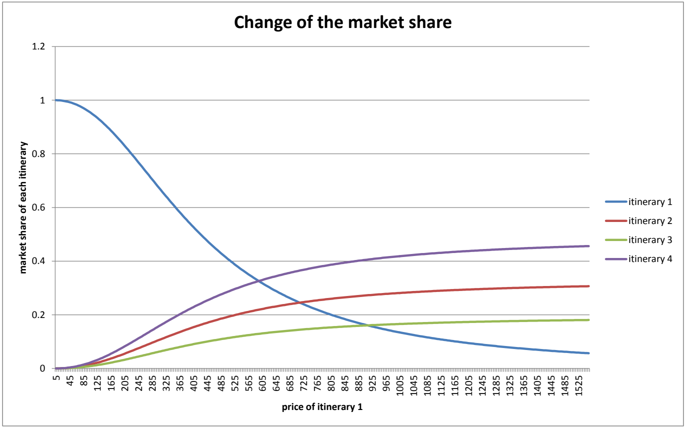
$$\pi_{k,f}^h\geq 0\qquad\forall h\in H,k\in K,f\in F\qquad(18)$$

Integrated model

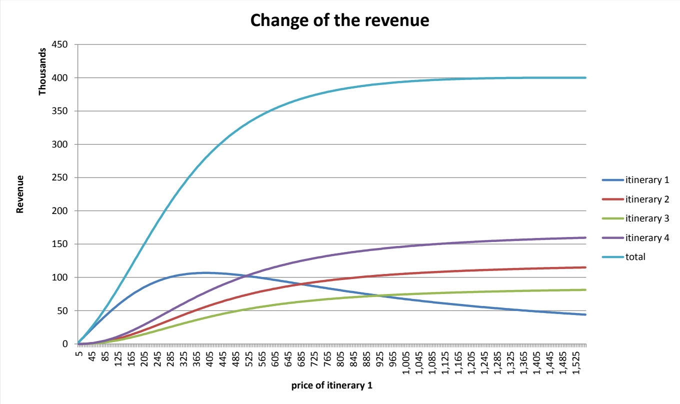
- We consider reference models to evaluate the integrated model
  - **Price-inleastic schedule planning**: M. Lohatepanont and C. Barnhart (2004)
  - **Sequential approach**: Revenue management considers fixed supply capacity

- The resulting model is a mixed integer nonlinear problem
- Nonlinearity is due to the explicit supply-demand interactions
- The model is implemented in AMPL and BONMIN solver is used
- BONMIN does not guarantee optimality

Illustration



Illustration



Sequential versus integrated

	Sequential approach				Integrated model - % Change			
	Profit	Pax.	Flights	Seats	Profit	Pax.	Flights	Seats
1	15,091	284	8	124	-	-	8	124
2	35,372	400	8	150	5.55%	33.50%	8	217
3	50,149	859	10	300	-	-	10	300
4	43,990	882	10	331	4.45%	-17.80%	8	207
5	69,901	931	22	274	1.43%	14.18%	24	324
6	82,311	1,145	16	333	-	-	16	333
7	84,186	1,131	14	329	3.47%	-3.80%	14	329
8	904,054	1,448	10	1,148	0.30%	-	10	1,312
9	135,656	1,814	32	498	-	-	32	498
10	115,983	2,236	26	691	-	-	26	691
11	93,920	2,270	26	747	0.30%	-0.97%	26	747
12	854,902	1,270	10	1,016	0.43%	5.83%	10	1,090
13	27,076	448	10	207	-	-	10	207
14	52,369	599	10	267	1.45%	16.69%	12	267
15	51,160	793	8	402	-	-	8	402
16	37,100	1,067	12	377	2.89%	-2.72%	12	377
17	137,428	1,517	34	391	0.83%	4.94%	34	476
18	93,347	1,144	20	387	3.36%	1.40%	20	457
19	83,251	1,104	12	536	-	-	12	536

Heuristic method

- We are limited in terms of the computational time
- A heuristic based on two simplified versions of the model:
  - FAM^{LS}: price-inelastic schedule planning model ⇒ MILP
    - Explores new fleet assignment solutions based on a local search
    - Price sampling
    - Variable neighborhood search
  - REV^{LS}: Revenue management with fixed capacity ⇒ NLP
    - Optimizes the revenue for the explored fleet assignment solution

Heuristic method

Require: $\bar{x}_0$, $\bar{y}_0$, $\bar{d}_0$, $\bar{p}_0$, $\bar{t}_0$, $\bar{b}_0$, $\bar{\pi}_0$, z^*, z_{opt}, k_{max}, ε, n_{min}, n_{max}

  $k := 0$, $n_{fixed} := n_{min}$

  repeat

    $\bar{p}_k$:= Price sampling

    $\{\bar{d}_k, \bar{b}_k\}$:= Demand model($\bar{p}_k$)

    $\{\bar{x}_k, \bar{y}_k, \bar{\pi}_k, \bar{t}_k\}$:= solve $z_{\mathrm{FAM}^{\mathrm{LS}}(\bar{d}_k, \bar{b}_k, n_{fixed})}$

    $\{\bar{p}_k, \bar{d}_k, \bar{b}_k, \bar{\pi}_k, \bar{t}_k\}$:= solve $z_{\mathrm{REV}^{\mathrm{LS}}(\bar{x}_k, \bar{y}_k)}$

    if improvement($z_{\mathrm{REV}^{\mathrm{LS}}}$) **then**

      Update z^*

      Intensification: $n_{fixed} := n_{fixed} + 1$ when $n_{fixed} < n_{max}$

    else

      Diversification: $n_{fixed} := n_{fixed} - 1$ when $n_{fixed} > n_{min}$

    end if

    $k := k + 1$

  until $||z_{opt} - z^*||^2 \leq \varepsilon$ **or** $k \geq k_{max}$

Performance of the heuristic

The omitted instances are the ones where the sequential approach has the same solution as the integrated model.

		Sequential approach		Best solution reported by Bonmin		Heuristic results Average over 5 replications			
	flights	profit	% dev.	profit	time(sec)	profit	%dev.	time(sec)	time red.
2	11	35,372	5.26%	37,335	27	37,335	0.00%	13	53.33%
4	12	43,990	4.45%	46,037	2,686	46,037	0.00%	3	99.90%
5	26	69,901	1.41%	70,904	2,479	70,679	**0.32%**	6	99.75%
7	19	84,186	3.47%	87,212	42,628	87,212	0.00%	60	99.86%
8	12	904,054	0.30%	906,791	12,964	906,791	0.00%	2	99.98%
11	32	93,920	0.30%	94,203	1,724	94,203	0.00%	10	99.42%
12	11	854,902	0.42%	858,544	7,343	858,545	0.00%	1	99.99%
13	39	137,428	0.83%	138,575	37,177	138,575	0.00%	173	99.54%
14	23	93,347	3.25%	96,486	17,142	96,486	0.00%	89	99.48%
16	19	37,100	2.89%	38,205	240	38,205	0.00%	1	99.50%
18	14	52,369	1.43%	53,128	141	53,128	0.00%	1	99.53%
20	33	146,464	0.00%	146,467	31,945	147,506	-0.71%	380	98.81%
21	77	208,561	-7.18%	194,598	42,360	210,395	-8.12%	791	98.13%
22	61	226,615	0.33%	227,364	22,174	227,284	0.04%	1,283	94.21%
23	48	163,114	-6.06%	153,789	4,387	163,393	-6.24%	126	97.12%
					max 43200			*max 3600*	

Conclusions and future work

- Integrated schedule planning and revenue management
  - More efficient schedule planning with the information on supply-demand interactions
- Heuristic
  - Inclusion of larger instances to test the limits of the heuristic
- Further solution methods for the resulting mixed integer nonlinear problem
  - Convex approximation of the nonlinearity
  - Decomposition methods $\Rightarrow$ FAM and REV models

Thank you for your attention!

Discrete choice analysis

- Finite and discrete set of alternatives
  - Choice of transportation mode: car, bus, etc.
  - Choice of brand: Leonidas, Lindt, Suchard, Toblerone, etc.
  - Choice of flight: GVA-NCE 10:00, GVA-NCE 06:30, etc.
- Individual n associates a utility to alternative i
- Represented by a random function

$$U_{in} = V_{in} + \varepsilon_{in} = \sum_k \beta_k x_{ink} + \varepsilon_{in}$$

Discrete choice analysis

Choice Model

- Individual n chooses alternative i if $U_{in} \geq U_{jn}$, for all j.
- Utility is random, so we have a probabilistic model

$$P_n(i|C_n) = Pr(U_{in} \geq U_{jn}) = Pr(V_{in} + \varepsilon_{in} \geq V_{jn} + \varepsilon_{jn})$$

- Concrete models require
  - specification of V_{in}
  - assumptions about ε_{in}
  - estimation of the parameters from data