

# Analysis of an itinerary-based airline fleet assignment model with passenger-choice driven recapture and pricing

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OR DAYS

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# Motivation

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- Flexibility in decision support tools,
- demand responsive transportation systems  
*... through ...*
- a better understanding of demand behavior,
- integration of explicit supply-demand interactions,
- endogenous demand variables that can be controlled by the optimization models.

# Itinerary choice model

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- Market segments,  $s$ , defined by the class and each OD pair
- Itinerary choice among the set of alternatives,  $I_s$ , for each segment  $s$
- For each itinerary  $i \in I_s$  the utility is defined by:

$$V_i = ASC_i + \beta_p \cdot \ln(p_i) + \beta_{time} \cdot time_i + \beta_{morning} \cdot morning_i$$

$$V_i = V_i(p_i, z_i, \beta)$$

- $ASC_i$  : alternative specific constant
  - $p$  is the **only policy variable** and included as log
  - $p$  and time are interacted with non-stop/stop
  - morning is 1 if the itinerary is a morning itinerary
- *No-revenue* represented by the subset  $I'_s \in I_s$  for segment  $s$ .

# Estimation

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- **Revealed preferences (RP) data:** Booking data from a major European airline
  - Lack of variability
  - Price inelastic demand
- RP data is combined with a **stated preferences (SP) data**
- Time, cost and morning parameters are **fixed** to be the same for the two datasets.
- A **scale** parameter is introduced for SP to capture the differences in variance.

# Market shares

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Market share for itinerary  $i$  in market segment  $s$ :

$$ms_i = \frac{\exp(V_i(p_i, z_i, \beta))}{\sum_{j \in I_s} \exp(V_j(p_j, z_j, \beta))}$$

- Spill and recapture information in airline fleet assignment
- Pricing decision

# Integrated airline scheduling, fleetling and pricing

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## Decision variables:

- $x_{k,f}$ : binary, assignment of aircraft  $k$  to flight  $f$
- $\pi_{k,f}^h$ : allocated seats for class  $h$  on flight  $f$  aircraft  $k$
- $p_i$ : price of itinerary  $i$
- $d_i$ : demand of itinerary  $i$
- $t_{i,j}$ : spilled passengers from itinerary  $i$  to  $j$

# Integrated model - Scheduling & fleeting

$$\text{Max} \sum_{h \in H} \sum_{s \in S^h} \sum_{i \in (I_s \setminus I'_s)} (d_i - \sum_{j \in I_s} t_{i,j} + \sum_{j \in (I_s \setminus I'_s)} t_{j,i} b_{j,i}) p_i - \sum_{\substack{k \in K \\ f \in F}} C_{k,f} x_{k,f} : \text{revenue - cost} \quad (1)$$

$$\text{s.t.} \sum_{k \in K} x_{k,f} = 1 : \text{mandatory flights} \quad \forall f \in F^M \quad (2)$$

$$\sum_{k \in K} x_{k,f} \leq 1 : \text{optional flights} \quad \forall f \in F^O \quad (3)$$

$$y_{k,a,t}^- + \sum_{f \in \text{In}(k,a,t)} x_{k,f} = y_{k,a,t}^+ + \sum_{f \in \text{Out}(k,a,t)} x_{k,f} : \text{flow conservation} \quad \forall [k, a, t] \in N \quad (4)$$

$$\sum_{a \in A} y_{k,a, \min E_a^-} + \sum_{f \in CT} x_{k,f} \leq R_k : \text{fleet availability} \quad \forall k \in K \quad (5)$$

$$y_{k,a, \min E_a^-} = y_{k,a, \max E_a^+} : \text{cyclic schedule} \quad \forall k \in K, a \in A \quad (6)$$

$$\sum_{h \in H} \pi_{k,f}^h = Q_k x_{k,f} : \text{seat capacity} \quad \forall f \in F, k \in K \quad (7)$$

$$x_{k,f} \in \{0, 1\} \quad \forall k \in K, f \in F \quad (8)$$

$$y_{k,a,t} \geq 0 \quad \forall [k, a, t] \in N \quad (9)$$

- Itinerary-based fleet assignment & Spill and recapture
- Lohatepanont and Barnhart 2004

# Integrated model - Revenue management - Pricing

$$\sum_{s \in S^h} \sum_{i \in (I_s \setminus I'_s)} \delta_{i,f} (d_i - \sum_{j \in I_s} t_{i,j} + \sum_{j \in (I_s \setminus I'_s)} t_{j,i} b_{j,i}) \leq \sum_{k \in K} \pi_{k,f}^h : \text{demand-capacity} \quad \forall h \in H, f \in F \quad (10)$$

$$\sum_{j \in I_s} t_{i,j} \leq d_i : \text{total spill} \quad \forall h \in H, s \in S^h, i \in (I_s \setminus I'_s) \quad (11)$$

$$\tilde{d}_i = D_s \frac{\exp(V_i(p_i, z_i, \beta))}{\sum_{j \in I_s} \exp(V_j(p_j, z_j, \beta))} : \text{logit demand} \quad \forall h \in H, s \in S^h, i \in I_s \quad (12)$$

$$b_{j,i} = \frac{\exp(V_j(p_j, z_j, \beta))}{\sum_{k \in I_s \setminus \{i\}} \exp(V_k(p_k, z_k, \beta))} : \text{recapture ratio} \quad \forall h \in H, s \in S^h, i \in (I_s \setminus I'_s), j \in I_s \quad (13)$$

$$d_i \leq \tilde{d}_i : \text{realized demand} \quad \forall h \in H, s \in S^h, i \in I_s \quad (14)$$

$$LB_i \leq p_i \leq UB_i : \text{bounds on price} \quad \forall h \in H, s \in S^h, i \in I_s \quad (15)$$

$$t_{i,j}, b_{i,j} \geq 0 \quad \forall h \in H, s \in S^h, i \in (I_s \setminus I'_s), j \in I_s \quad (16)$$

$$\pi_{k,f}^h \geq 0 \quad \forall h \in H, k \in K, f \in F \quad (17)$$

- Schön (2008): integration of pricing

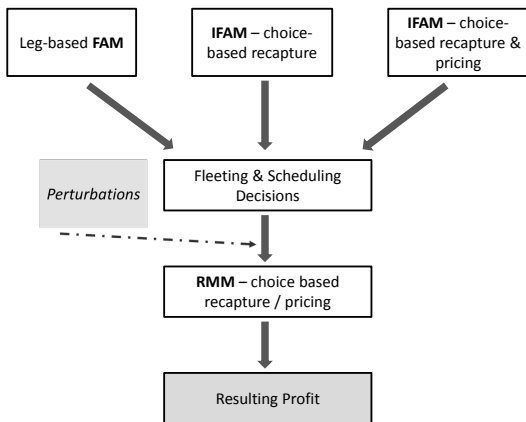


# Heuristic method

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- Mixed Integer Non-convex Problem
- We devised a heuristic procedure based on two subproblems:
  - $FAM^{LS}$ : price-inelastic schedule planning model  $\Rightarrow$  MILP
    - Prices fixed
    - Optimizes the schedule design and fleet assignment
  - $REV^{LS}$ : Revenue management with fixed capacity  $\Rightarrow$  NLP
    - Schedule design and fleet assignment fixed
    - Optimizes the revenue
- Local search based on spill information

# Sensitivity Analysis

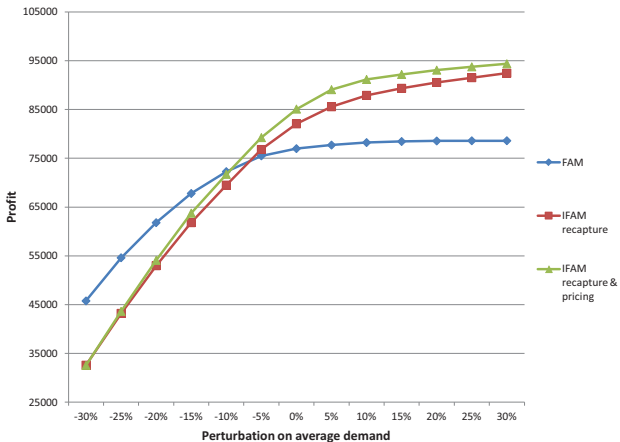


# Sensitivity to demand fluctuations

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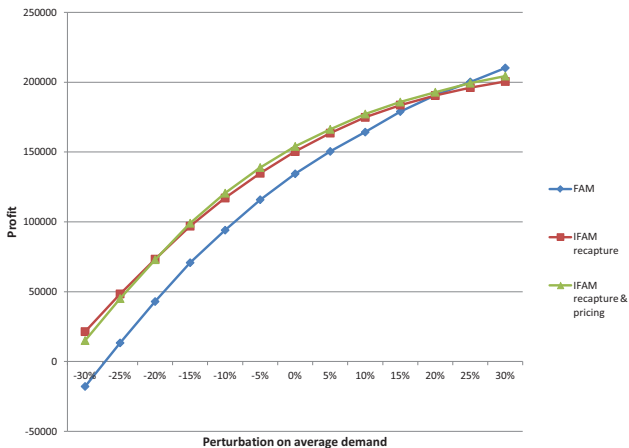
- Total market segment demand is assumed to be known
- Fluctuations in reality
  
- Average demand is perturbed in a range  $[-30\%, +30\%]$
- For each average demand 500 simulations with Poisson

# Sensitivity to demand fluctuations



23 flights 4 aircraft types

# Sensitivity to demand fluctuations



77 flights 4 aircraft types - heuristic solution

# Transformation of the logit model

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$$ms_i = \frac{\exp(V_i)}{\sum_{j \in I_s} \exp(V_j)}, \quad V_i = \beta \ln(p_i) + c_i$$

A new variable  $v_s = \frac{1}{\sum_{j \in I_s} \exp(V_j)}$

$$ms_i = v_s \exp(\beta \ln(p_i) + c_i)$$

$$\sum_{i \in I_s} ms_i = 1$$

$$v_s \geq 0$$

# Transformation of the logit model

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## A logarithmic transformation:

$$ms_i = v_s \exp(\beta \ln(p_i) + c_i)$$

$$ms'_i = v'_s + \beta p'_i + c_i$$

$$ms'_i \Rightarrow \ln(ms_i), v'_s \Rightarrow \ln(v_s), p'_i \Rightarrow \ln(p_i).$$

# Transformation of the logit model

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## A logarithmic transformation:

$$ms_i = v_s \exp(\beta \ln(p_i) + c_i)$$

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$$ms'_i \Rightarrow \ln(ms_i), v'_s \Rightarrow \ln(v_s), p'_i \Rightarrow \ln(p_i).$$

## Revenue function:

$$\sum_{s \in S} \sum_{i \in (I_s \setminus I'_s)} D_s ms_i p_i = \exp(\ln(D_s) + ms'_i + p'_i)$$

$\exp()$  can be removed



# Reformulated revenue problem

Given the supply decisions...

$$\max \sum_{s \in S} \sum_{i \in (I_s \setminus I'_s)} (\ln(D_s) + ms'_i + p'_i) - M(ms_i - \exp(ms'_i))^2 \quad (18)$$

$$\sum_{i \in I_s} ms_i = 1 \quad \forall s \in S \quad (19)$$

$$ms'_i = v'_s + \beta p'_i + c_i \quad \forall s \in S, i \in I_s \quad (20)$$

$$\sum_{s \in S} D_s \sum_{i \in (I_s \setminus I'_s)} \delta_{i,f} ms_i \leq \sum_{k \in K} Q_k X_{k,f} \quad \forall f \in F \quad (21)$$

$$p'_i \leq \ln(UB_i) \quad \forall s \in S, i \in (I_s \setminus I'_s) \quad (22)$$

$$ms'_i \in \mathfrak{R} \quad \forall s \in S, i \in I_s \quad (23)$$

$$ms_i \geq 0 \quad \forall s \in S, i \in I_s \quad (24)$$

$$v'_s \in \mathfrak{R} \quad \forall s \in S \quad (25)$$

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$$v'_s \in \mathfrak{R} \quad \forall s \in S \quad (25)$$

## Added value of the reformulation

- Even for a small instance with 11 flights, 2 aircraft types
- If the elasticity is very high, non-convexity misleads

	Original - BONMIN Profit: <b>-12,516.2</b> Op. costs: 140,207		Reformulated - MOSEK Profit: <b>6,816.7</b> Op. costs: 140,207	
	ms	price	ms	price
1	0.80	140	0.80	140
2	0.80	140	0.80	140
3	0.13	250	0.13	231
4	0.29	200	0.29	200
5	0.30	200	0.27	200
6	0	-	0.12	120
7	0	-	0.13	180
8	0.29	237	0.29	198
9	0	-	0.13	200
10	0	-	0.12	225
11	0.30	228	0.22	208

# Conclusions

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- The integrated model has promising results
- ... which motivates the effort in devising solution methodologies
- Logarithmic transformation provides a concave formulation of the revenue problem
- ... is expected to facilitate efficient solution methodologies
- More analysis should be done on the added-value of the reformulation

## On-going work

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- The framework is flexible for the integration of advanced demand models
- $$\sum_{i \in I_s} (\ln(D_s) + ms'_i + p'_i) \Rightarrow \sum_{i \in I_s} \sum_{n \in N} p'_i + \text{Prob}'_{i,n}$$
- Generalized framework for the integration of endogenous demand models

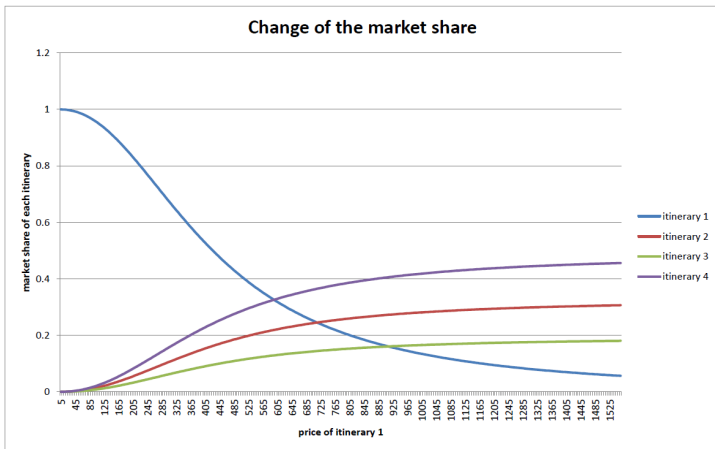
## On-going work

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- $$\sum_{i \in I_s} (\ln(D_s) + ms'_i + p'_i) \Rightarrow \sum_{i \in I_s} \sum_{n \in N} p'_i + \text{Prob}'_{i,n}$$
- Generalized framework for the integration of endogenous demand models

Thank you for your attention!

# Logit behavior



# Itinerary choice model

- Market share and demand for itinerary  $i$  in market segment  $s$ :

$$ms_i = \frac{\exp(V_i(p_i, z_i, \beta))}{\sum_{j \in I_s} \exp(V_j(p_j, z_j, \beta))} \Rightarrow d_i = D_s ms_i$$

- $D_s$  is the total expected demand for market segment  $s$ .

- Spill and recapture effects:** Capacity shortage  $\Rightarrow$  passengers may be recaptured by other itineraries (instead of their desired itineraries)
- Recapture ratio** is given by:

$$b_{i,j} = \frac{\exp(V_j(p_j, z_j, \beta))}{\sum_{k \in I_s \setminus \{i\}} \exp(V_k(p_k, z_k, \beta))}$$



# Itinerary choice model

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- **Value of time (VOT):**

$$\begin{aligned}VOT_i &= \frac{\partial V_i / \partial time_i}{\partial V_i / \partial cost_i} \\ &= \frac{\beta_{time} \cdot cost_i}{\beta_{cost}}\end{aligned}$$

For the same OD pair...

- VOT for economy, non-stop: 8 €/hour
- VOT for economy, one-stop: 19.8, 11, 9.2 €/hour
- VOT for business, non-stop: 21.7 €/hour

# Data

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no	airports	flights	flights per route	demand per flight	fleet composition
1	3	10	1.67	51.90	2 50-37
2	3	11	2.75	83.10	2 117-50
3	3	12	2.00	113.80	2 164-100
4	3	12	2.00	113.80	6 164-146-128-124-107-100
5	3	26	4.33	56.10	3 100-50-37
6	3	19	3.17	96.70	3 164-117-72
7	3	19	3.17	96.70	5 124-107-100-85-72
8	3	12	3.00	193.40	3 293-195-164
9	3	33	8.25	71.90	3 117-70-37
10	3	32	5.33	100.50	3 164-117-85
11	3	32	5.33	100.50	5 128-124-107-100-85
12	2	11	5.50	173.70	3 293-164-127
13	4	39	4.88	64.50	4 117-85-50-37
14	4	23	3.83	86.10	4 117-85-70-50
15	4	19	3.17	101.40	4 134-117-100-85
16	4	19	3.17	101.40	5 128-124-107-100-85
17	4	15	1.88	58.10	5 117-85-70-50-37
18	4	14	2.33	87.60	5 134-117-85-70-50
19	4	13	2.60	100.10	5 164-134-117-100-85
20	3	33	8.25	71.90	4 85-70-50-35
21	3	46	7.67	86.85	5 128-124-107-100-85
22	7	48	2.29	101.94	4 124-107-100-85
23	3	61	15.25	69.15	4 117-85-50-37
24	8	77	2.08	67.84	4 117-85-50-37
25	8	97	3.46	90.84	5 164-117-100-85-50

Data instances are derived from ROADEF 2009 dataset.

# Computational results

	<b>BONMIN Integrated model</b>		<b>Sequential approach (SA)</b>			<b>Local search heuristic Average over 5 replications</b>			
	Profit	Time (sec)	Profit	% deviation from BONMIN	Time (sec)	Profit	%deviation from BONMIN	%impr. over SA	Time (sec)
1	15,091	2	15,091	0.00%	1	15,091	0.00%	0.00%	1
2	37,335	22	35,372	-5.26%	1	37,335	0.00%	5.55%	13
3	50,149	62	50,149	0.00%	1	50,149	0.00%	0.00%	1
4	46,037	2,807	43,990	-4.45%	1	46,037	0.00%	4.65%	3
5	70,904	1,580	69,901	-1.41%	1	70,679	-0.32%	1.11%	6
6	82,311	1,351	82,311	0.00%	1	82,311	0.00%	0.00%	1
7	87,212	32,400	84,186	-3.47%	1	87,212	0.00%	3.59%	60
8	779,819	8,137	779,819	0.00%	1	779,819	0.00%	0.00%	1
9	135,656	666	135,656	0.00%	2	135,656	0.00%	0.00%	2
10	107,927	482	107,927	0.00%	1	107,927	0.00%	0.00%	1
11	85,820	31,705	85,535	-0.33%	2	85,820	0.00%	0.33%	88
12	858,544	5,598	854,902	-0.42%	1	858,544	0.00%	0.43%	1
13	112,881	32,713	109,906	-2.64%	1	112,881	0.00%	2.71%	151
14	85,808	10,643	82,440	-3.93%	1	85,808	0.00%	4.09%	9
15	49,448	33	49,448	0.00%	1	49,448	0.00%	0.00%	1
16	38,205	240	37,100	-2.89%	1	38,205	0.00%	2.98%	1
17	27,076	35	27,076	0.00%	1	27,076	0.00%	0.00%	1
18	45,070	78	44,339	-1.62%	1	45,070	0.00%	1.65%	1
19	26,486	13	26,486	0.00%	1	26,486	0.00%	0.00%	1
20	146,773	30 846	146,464	-0.21%	1	147,506	0.50%	0.71%	406
21	194,987	4,963	210,134	7.77%	10	214,251	9.88%	1.96%	1,499
22	152,126	68,864	158,978	4.50%	2	159,258	4.69%	0.18%	39
23	227,643	40,862	226,615	-0.45%	12	227,284	-0.16%	0.30%	1,283
24	153,384	59,708	154,301	0.60%	4	158,099	3.07%	2.46%	2,314
25	313,943	82,780	331,920	5.73%	13	332,744	5.99%	0.25%	1,451

# Improvement due to the local search

	Sequential approach (SA)	Random neighborhood		Neighborhood based on spill		% Improvement	
	Profit	Profit	Time(sec)	Profit	Time(sec)	Quality of the solution	Reduction in time
2	35,372	37,335	116	37,335	13	-	89.10%
4	43,990	44,302	27	46,037	3	3.92%	88.88%
5	69,901	<i>No imp. over SA</i>		70,679	6	1.11%	-
7	84,186	85,335	1,649	87,212	60	2.20%	96.36%
8	904,054	906,791	209	906,791	2	-	99.04%
11	93,920	<i>No imp. over SA</i>		94,203	10	0.30%	-
12	854,902	<i>No imp. over SA</i>		858,545	1	0.43%	-
13	137,428	<i>No imp. over SA</i>		138,575	173	0.83%	-
14	93,347	96,365	943	96,486	89	0.13%	90.56%
16	37,100	38,205	6	38,205	1	-	80.65%
18	52,369	53,128	334	53,128	1	-	99.80%
20	146,464	<i>No imp. over SA</i>		147,506	380	0.71%	-
21	217,169	<i>No imp. over SA</i>		219,136	1,395	0.91%	-
22	163,114	<i>No imp. over SA</i>		163,393	126	0.17%	-
23	226,615	<i>No imp. over SA</i>		227,284	1,283	0.30%	-
24	208,561	<i>No imp. over SA</i>		210,395	791	0.88%	-
25	469,136	<i>No imp. over SA</i>		470,494	1,117	0.29%	-

## A small example

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- 2 airports CDG-MRS
- 4 flights - all are mandatory
- 2 aircraft types: 37-50 seats

We start with an initial FAM solution:

	AC1	AC2
F1	X	
F2	X	
F3	X	
F4	X	

# A small example - GBD iterations

Iteration 1		
	Sub	Master
	12522.8	16923.4
	<b>LB</b>	<b>UB</b>
	12522.8	16923.4
	AC1	AC2
F1		X
F2		X
F3		X
F4		X

⇒

Iteration 2		
	Sub	Master
	10734.4	14822.8
	<b>LB</b>	<b>UB</b>
	12522.8	14822.8
	AC1	AC2
F1		X
F2		X
F3	X	
F4	X	

Iteration 3		
	Sub	Master
	12696.8	14822.8
	<b>LB</b>	<b>UB</b>
	12696.8	14822.8
	AC1	AC2
F1	X	
F2		X
F3		X
F4	X	

⇒

Iteration 4		
	Sub	Master
	12474.4	12696.8
	<b>LB</b>	<b>UB</b>
	<b>12696.8</b>	<b>12696.8</b>
	AC1	AC2
F1		X
F2		X
F3	X	
F4	X	