

# Activity-based models: an optimization perspective

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# Outline

- 1 Introduction
- 2 Model
- 3 Mixed integer optimization problem
- 4 Example
- 5 Parameter estimation



# Introduction



- Travel demand is derived from activity demand.
- Activity demand is influenced by socio-economic characteristics, social interactions, cultural norms, basic needs, etc. [Chapin, 1974]
- Activity demand is constrained in space and time [Hägerstrand, 1970].



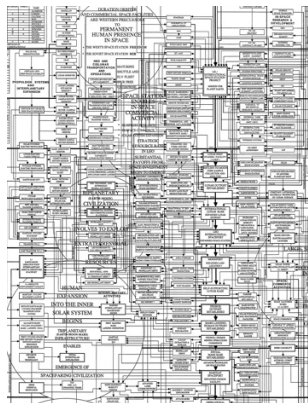
# Literature

## Econometric models

Handwritten mathematical derivations on a blackboard background:

- $S_1 = \frac{1}{n} \sum_{i=1}^n x_i$
- $VAR(S_1) = \frac{1}{n^2} \sum_{i=1}^n VAR(x_i) = \frac{1}{n^2} \sum_{i=1}^n \sigma^2 = \frac{\sigma^2}{n}$
- $VAR(S_2) = \frac{1}{n^2} \sum_{i=1}^n VAR(x_i) = \frac{\sigma^2}{n}$
- $COV(S_1, S_2) = \frac{1}{n^2} \sum_{i=1}^n COV(x_i, x_i) = \frac{1}{n^2} \sum_{i=1}^n \sigma^2 = \frac{\sigma^2}{n}$
- $VAR(S_1 + S_2) = VAR(S_1) + VAR(S_2) + 2COV(S_1, S_2) = \frac{\sigma^2}{n} + \frac{\sigma^2}{n} + 2 \cdot \frac{\sigma^2}{n} = \frac{4\sigma^2}{n}$
- $VAR(S_1 - S_2) = VAR(S_1) + VAR(S_2) - 2COV(S_1, S_2) = \frac{\sigma^2}{n} + \frac{\sigma^2}{n} - 2 \cdot \frac{\sigma^2}{n} = 0$

## Rule-based models



# State of the art: econometric approach

[Pinjari et al., 2011]

- ... individuals make their activity-travel decisions to maximize the utility derived from the choices they make.
- These model systems usually consist of a series of ... discrete choice models ... that are used to predict ... individuals' activity-travel decisions.
- these model systems employ econometric systems of equations ... to capture relationships between ... socio-demographics and ... attributes on the one hand and the observed activity-travel decision outcomes on the other.



# State of the art: econometric approach

[Pinjari et al., 2011]: main criticisms

- *individuals are not necessarily fully rational utility maximizers*
- *the approach does not explicitly model the underlying decision processes and behavioral mechanisms that lead to observed activity-travel decisions.*



# State of the art: econometric approach

[Bhat, 2005]

- Multiple Discrete Continuous Extreme Value
- Based on first principles.
- Decision-maker solves an optimization problem, with a time budget.
- Several alternatives may be chosen.
- Model derived from KKT conditions.



# Research question

## Relax the *series of discrete choice models* approach

- The interactions of all decisions is complex.
- Sequence of models is most of the time arbitrary.

## Integrated approach

Develop a model involving many decisions:

- activity participation,
- activity location,
- activity duration,
- activity scheduling,
- travel mode,
- travel route.



# Research objectives

- Integrated approach based on first principles.
- Theoretical framework: utility maximization.
- Individuals solve a scheduling problem.
- Important aspects: trade-offs on activity duration.



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# First principles



- Each individual  $n$  has a time-budget (a day).
- Each activity  $a$  considered by  $n$  is associated with a utility  $U_{an}$ .
- Individuals schedule their activities as to **maximize** the total utility, subject to their time-budget constraint.

# Further assumptions



## Individuals are **time sensitive**

- Have a desired *start time*, *duration* and/or *end time* for each activity
- Deviations from their desired times in the scheduling process decrease the utility function

# Time



- Time horizon: 24 hours.
- Discretization:  $T$  time intervals.
- Trade-off between model accuracy and computational time.

# Space



- Discrete and finite set  $S$  of locations, indexed by  $s$ .
- For each individual, each activity is associated with a list of potential locations.

# Travel

- For each pair OD, list of possible modes.
- For each mode, list of possible routes.
- For each  $(O, D, m, r)$ ,  $\rho(O, D, m, r)$  is the travel time.
- Exogenously given.



# Activities

## Definition: Activity

An activity is associated with a location and a trip.





# Activities

## Location, mode and route choices

- Lunch at location  $A$ , followed by trip by bus on route  $A$ .
- Lunch at location  $A$ , followed by trip by bus on route  $B$ .
- Lunch at location  $A$ , followed by trip by car on route  $A$ .
- Lunch at location  $B$ , followed by trip by car on route  $B$ .

## Constraint

Only one of the “duplicates” can be chosen.



# Activities



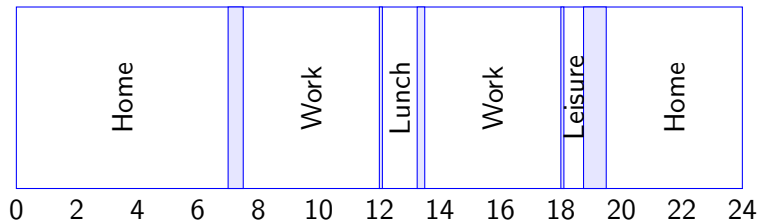
## Given

- Set  $A$  of activities.
- Location  $s_a$ .
- Feasible time interval:  $[\gamma_a^-, \gamma_a^+]$  (e.g. opening hours).

## Decisions

- Participation:  $w_a \in \{0, 1\}$ .
- Starting time  $x_a$ ,  $0 \leq x_a \leq T$ .
- Schedule:  $z_{ab} \in \{0, 1\}$ .
- Duration:  $\tau_a \geq 0$ .

# Scheduling



# Categories



- [Castiglione et al., 2014]: mandatory, maintenance, discretionary.
- Flexible, somewhat flexible, not flexible.

## Category

Activities that share the same preference profile.



# Preferences

## Preferences

- desired starting time  $x_a^*$ ,
- desired duration  $\tau_a^*$ .

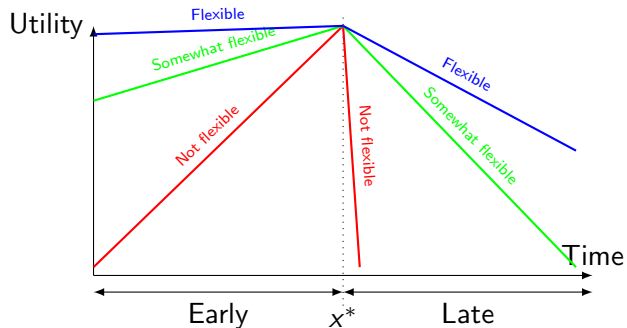
## Penalties

- Starting early [Small, 1982]:  
 $\theta_e \max(x_a^* - x_a, 0)$ .
- Starting late [Small, 1982]:  
 $\theta_\ell \max(x_a - x_a^*, 0)$ .
- Shorter activity:  $\theta_{ds} \max(\tau_a^* - \tau_a, 0)$ .
- Longer activity:  $\theta_{dl} \max(\tau_a - \tau_a^*, 0)$ .



# Preferences

Parameters depend on the category type



# Disutility of travel



Each activity is followed by a trip

- Travel time from  $a$  to  $a^+$ :  $t_a$ .
- Depends on the next activity.

$$t_a = \sum_b z_{ab} \rho(s_a, s_b, m_a, r_a).$$

- Over variables can be included (cost, etc.)
- Note: If  $s_a = s_b$ ,  $\rho(s_a, s_a, m_a, r_a) = 0$
- Exception: last activity of the day (home).

# Utility function

An individual  $n$  derives the following utility from performing activity  $a$ , with a schedule flexibility  $k$ :

$$\begin{aligned}U_{an} = & \theta_e \max(x_a^* - x_a, 0) \\ & + \theta_\ell \max(x_a - x_a^*, 0) \\ & + \theta_{ds} \max(\tau_a^* - \tau_a, 0) \\ & + \theta_{dl} \max(\tau_a - \tau_a^*, 0) \\ & + c_{an} + \varepsilon_{an},\end{aligned}$$

where  $\varepsilon_{an}$  are error components.





# Utility function

## Utility of a schedule

$$U_{sn} = \sum_a w_a U_{an} + \theta_t \sum_a \sum_b z_{ab} \rho(a, b, m_a, r_a)$$

## Error components

$$\sum_a w_a \varepsilon_{an}$$

where  $\varepsilon_{an}$  normally distributed.



# Utility function



## Error terms

- Rely on simulation.
- Draw  $\varepsilon_{anr}$ ,  $r = 1, \dots, R$ .
- Optimization problem for each  $r$ .
- Utility:  $U_{anr}$ .

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# Decision variables for individual $n$ and draw $r$

For each (potential) activity  $a$ :

- Activity participation:  $w_{anr} \in \{0, 1\}$ .
- Starting time:  $x_{anr} \in \{0, \dots, T\}$ .
- Duration:  $\tau_{anr} \in \{0, \dots, T\}$ .
- Scheduling:  $z_{abnr} \in \{0, 1\}$ : 1 if activity  $b$  immediately follows  $a$ .



# Objective function

## Additive utility

$$\max \sum_{a \in A} w_{anr} U_{anr} + \theta_t \sum_{a \in A} \sum_{b \in A} z_{abnr} \rho(a, b, m_a, r_a).$$



# Constraints

## Time budget

$$\sum_{a \in A} w_{anr} \tau_{anr} + \sum_{a \in A} \sum_{b \in A} z_{abnr} \rho(a, b, m_a, r_a) = T, \forall n, r.$$

## Time windows

$$0 \leq \gamma_a^- \leq x_{anr} \leq x_{anr} + \tau_{anr} \leq \gamma_a^+ \leq T, \forall a, n, r.$$



# Constraints

## Precedence constraints

$$z_{abnr} + z_{banr} \leq 1, \forall a, b, n, r.$$

## Single successor/predecessor

$$\sum_{b \in A \setminus \{a\}} z_{abnr} = w_{anr}, \forall a, n, r,$$

$$\sum_{b \in A \setminus \{a\}} z_{banr} = w_{anr}, \forall a, n, r.$$



# Constraints

## Consistent timing

$$(z_{abnr} - 1)T \leq x_{anr} + \tau_{anr} + t_{anr} - x_b \leq (1 - z_{abnr})T, \forall a, b, n, r.$$

where

$$t_{anr} = \sum_{b \in A} z_{abnr} \rho(s_a, s_b).$$

## Mutually exclusive duplicates

$$\sum_{a \in B_k} w_{anr} = 1, \forall k, n, r.$$



# Optimization problem

## Simulation-based optimization

- For each realization of the error terms, we have an optimal schedule.
- It includes all the choice dimensions (activity participation, location, duration, scheduling, and mode and route).
- We can generate an empirical distribution of chosen schedules.

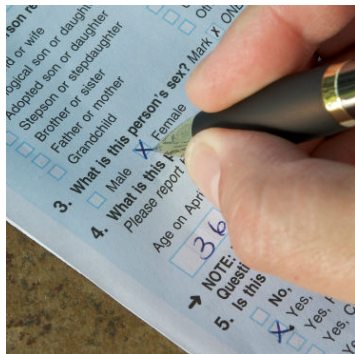


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# Real data



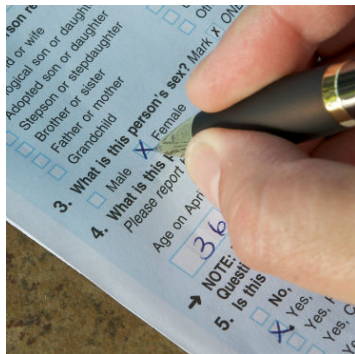
## Dataset

- 2015 Swiss Mobility and Transport Microcensus.
- Daily trip diaries for 57'000 individuals.
- Records of activities and visited location.

## Challenges: classical RP issues

- No information about unchosen alternatives.
- Latent preferences.

# Real data

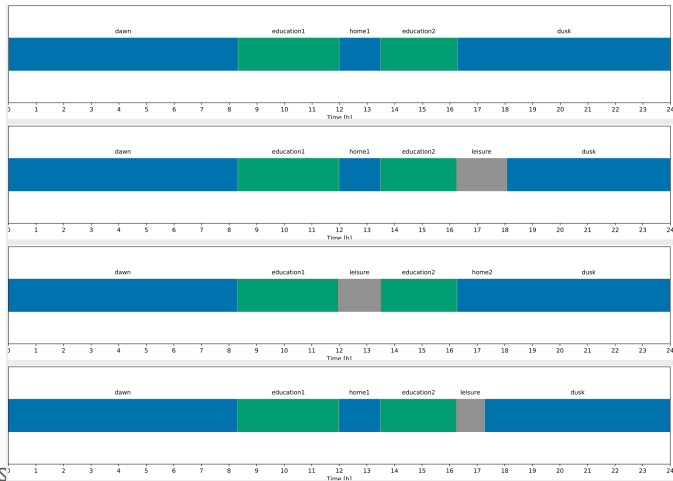


## Assumptions

- Desired start times and durations are the recorded ones.
- Feasible time windows: average start and end times from out of sample distribution.
- Only the recorded locations are considered.
- Uniform flexibility profile across population.

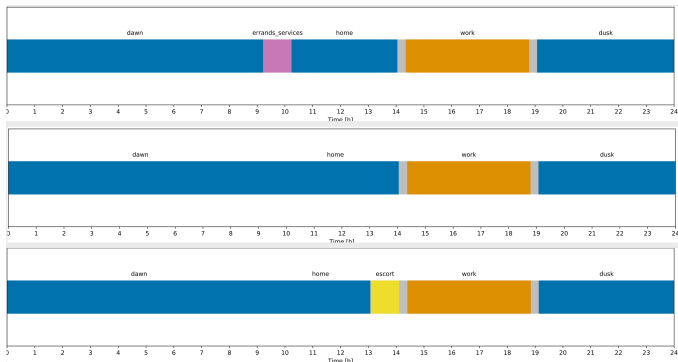
# Individual 1 (weekday)

Optimal schedules generated for random draws of  $\varepsilon_{an}$



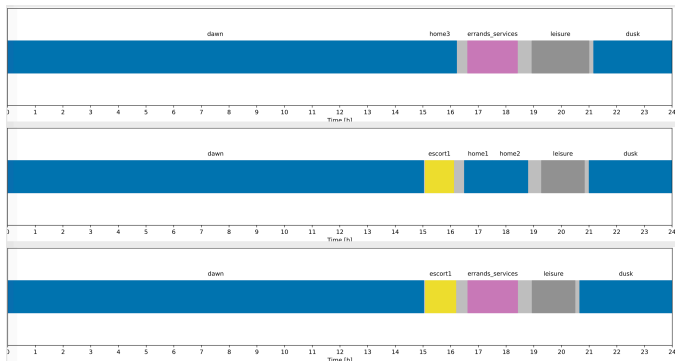
# Individual 2 (weekday)

Optimal schedules generated for random draws of  $\varepsilon_{a_n}$



# Individual 3 (weekday)

Optimal schedules generated for random draws of  $\varepsilon_{a_n}$



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# Parameter estimation

## Choice set generation

- Full set of schedules  $C_n$  is combinatorial, approximated with a sample of alternatives  $\tilde{C}_n$
- Sampling protocol using Metropolis-Hastings algorithm [Flötteröd and Bierlaire, 2013]

## Choice model estimation

- Include an EV error term to obtain a mixture of logit.
- Probability of choosing a schedule  $y$  for individual  $n$  is conditional on the parameters  $\beta_n$ , the variables  $x_n$  and the sampled choice set  $\tilde{C}_n$  [Guevara and Ben-Akiva, 2013]
- Maximum likelihood estimators of the parameters:

$$\max_{\hat{\beta}} L(y|\hat{\beta}, X) = \prod_n P(y|x_n, \hat{\beta}_n, \tilde{C}_n)$$

# Conclusions

## Achievements so far




- Formulation of the model.
- Applied on real data.
- The results make sense.
- We are able to draw from a distribution of activity schedules.

## Ongoing work





- Parameter estimation.



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