Airline Disruptions: Aircraft Recovery with Maintenance Constraints

Niklaus Eggenberg
Dr. Matteo Salani and Prof. Michel Bierlaire

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Index

- Airline Scheduling
- The Airplane Recovery Problem (ARP)
- The Column Generation (CG) approach
- Column Description
- The pricing algorithm with the Recovery Network
- Implementation and results
- Future work and conclusions
Airline Scheduling Approach

1. Route Choice
2. Fleet Assignment
3. Tail Assignment
4. Crew Pairing
5. Crew Roistering
6. Passenger Routing (catering)
Disrupted Schedule and Recovery

The Airplane Recovery Problem (ARP)

**Input**
- Planes’ States
- Initial Schedule
- Maintenances
- Cancelation Costs
- Delay Cost

**Output**
- $T$
- New schedule up to $T$
- Recovery cost
Multi-objective optimization:

Minimize both $T$ and recovery costs

Strategy: for fixed $T$ find optimal recovery plan

Give several recovery plans for different values of $T$ (decision aid)
Definitions:

**PLANES:**
- Initial State: position, initial time, initial resource consumption
- Final State: position, expected time, expected resource consumption
- Feasible Flight Set: coverable flights

**AIRPORTS:**
- Activity Slots: periods when take-off/landings are permitted
- Maintenance Slots: periods when given plane type can perform maintenance
Definitions (2):

**Flights:**
- Origin and Destination
- Scheduled Departure Time (SDT)
- Flight Duration
- Flight Cost
- Cancelation Cost
Solution to the ARP:

A recovery scheme for each plane:

- Initial State
- Flights and Maintenance
- Expected Final State
Column Generation Approach

Find out optimal solution by combining individual recovery schemes $r \in R'$ (master problem) on a subset $R' \subseteq R$ of all feasible recovery schemes.

Generate potentially improving recovery schemes $r \in R-R'$ dynamically for each plane (pricing problem).
Master Problem: MIP formulation

\[
\min z_{MP} = \sum_{r \in R} c_r x_r + \sum_{f \in F} c_f y_f + \sum_{s \in S} c_s z_s
\]

s. c.

\[
\sum_{r \in R} b_r^f x_r + y_f = 1 \quad \forall f \in F \quad (\lambda_f)
\]

\[
\sum_{r \in R} b_r^s x_r + z_s = 1 \quad \forall s \in S \quad (\eta_s)
\]

\[
\sum_{r \in R} b_r^p x_r \leq 1 \quad \forall p \in P \quad (\mu_p)
\]

\[
x_r \in \{0,1\} \quad \forall r \in R
\]

\[
y_f \in \{0,1\} \quad \forall f \in F
\]

\[
z_s \in \{0,1\} \quad \forall s \in S
\]
What is a column?

- **cost**
- **vector**

\[ c_r \]

\[ b_r = (b^f_r, b^s_r, b^p_r)^T \]

Where

- \( b^f_r = 1 \) if flight \( f \) is covered by column \( r \)
- \( b^s_r = 1 \) if final state \( s \) is covered by \( r \)
- \( b^p_r = 1 \) if column \( r \) is affected to plane \( p \)
The Pricing Problem

Find new columns minimizing the reduced cost $\tilde{c}_r^p$:

$$\min_{r \in R} \tilde{c}_r^p = c_r^p - \sum_{f \in F} b_r^f \lambda_f - \sum_{s \in S} b_r^s \eta_s - b_r^p \mu_p \quad \forall p \in P$$
Recovery Networks (Argüello et al. 97)

1. Generate a recovery network for each plane
2. Update arc costs according to dual variables
3. Solve Resource Constrained Elementary Shortest Path (RCESPP)
4. Add Columns to $R'$
5. Resolve restricted LP until optimality and branch
Generating Recovery Networks

- Create Source node $n_0$ (initial time, location, resource cons.)
- $S = \{n_0\}$
- While $S \neq \emptyset$:
  - Select $n \in S$, $S \leftarrow S - \{n\}$
  - For all feasible flights:
    - create flight and maintenance arcs
    - create destinations node $n_f$ and $n_m$
    - $S = S \cup \{n_f, n_m\}$
- Clean network
Updating arc costs

- flight arcs: \( c = c^f - \lambda_f \)
- maintenance arcs: \( c = c^f + c^M - \lambda_f \)
- termination arcs: \( c = -\eta_s \)

Solve RCESPP on networks returns column minimizing the reduced cost!

Righini & Salani (2006), which is an extension of Desrochers et al. (1988)
Some References

• **Argüello et al.** (1997): recovery without maintenance
  up to 27 planes, 162 flights, 30 airports

• **Desrosiers et al.** (1997): daily scheduling NOT recovery
  up to 91 planes, 383 flights, 33 airports; max delay of 30 minutes

• **Clarke** (1997): maintenances requirements but no decision on them
  up to 177 planes, 612 flights, 37 airports; only 0 or 30 min delay

• **Kohl et al.** (2004): Descartes project, good survey of state of the art
  no instance size mentioned for DAR

• **Barnhart and Bratu** (2006): passenger oriented recovery algorithm
  up to 302 planes, 1032 flights, 74 airports
Implementation issues

- Implemented in C++ with COIN-OR BCP framework
- Used interior point methods to solve the LP
- Used linear time and logarithmical resource discretisation
- 2 phase pricing:
  - generation (keep also non optimal columns, heuristic pricing)
  - proving optimality (optimal column only, exact pricing)
Linear Discretization

Logarithmic Discretization
Real Instances

• Got real schedules from Thomas Cook Airlines (APM’s main customer)

• Solved original schedules up to 250 flights (algorithm validation)

• Generated disruption scenarios
  - delayed planes (initial states)
  - grounded planes (initial states)
  - airport closures (activity slots)
  - forced maintenances (initial resource consumption)
## Solved Instances (2): Problem Sizes

<table>
<thead>
<tr>
<th>Instance</th>
<th>2D_5AC</th>
<th>2D_5AC_1del</th>
<th>2D_10AC</th>
<th>2D_10AC_1del</th>
<th>2D_10AC_2del</th>
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<tbody>
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<td>5</td>
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<td>21175(*)</td>
<td>750(*)</td>
<td>21545(*)</td>
<td>21745(*)</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<td>run time [s]</td>
<td>&lt; 0.1</td>
<td>&lt; 0.1</td>
<td>0.7</td>
<td>0.7</td>
<td>1.0</td>
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<table>
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<th>4D_10AC</th>
<th>5D_5AC</th>
<th>5D_10AC</th>
<th>7D_16AC</th>
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<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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Solved Instances (3): Behavior against disruptions

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<tr>
<th>Instance</th>
<th>Den2del1</th>
<th>Den2grd</th>
<th>Den4del</th>
<th>Den4grd</th>
<th>Den2del2grd</th>
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<th>Den6grd</th>
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<td>6</td>
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<td># grounded planes</td>
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<td>2</td>
<td>0</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>6</td>
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<tr>
<td># affected flights</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>8</td>
<td>5</td>
<td>5</td>
<td>16</td>
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<tr>
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<td>2</td>
<td>0</td>
<td>8</td>
<td>4</td>
<td>0</td>
<td>16</td>
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<tr>
<td># delayed flights</td>
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<td>4</td>
<td>7</td>
<td>2</td>
<td>7</td>
<td>13</td>
<td>2</td>
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<td>920</td>
<td>230</td>
<td>380</td>
<td>490</td>
<td>640</td>
<td>380</td>
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<td>85</td>
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<td>200</td>
<td>100</td>
<td>200</td>
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<td>cost</td>
<td>36100(*)</td>
<td>83200(*)</td>
<td>38300(*)</td>
<td>163800(*)</td>
<td>84900(*)</td>
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<td>251800(*)</td>
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<table>
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<th>Den_1x300</th>
<th>Den_Storm1</th>
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<td>3</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td># grounded planes</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td># affected flights</td>
<td>9</td>
<td>11</td>
<td>7</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td># cancelled flights</td>
<td>6</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td># delayed flights</td>
<td>12</td>
<td>11</td>
<td>11</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>total delay</td>
<td>950</td>
<td>675</td>
<td>2560</td>
<td>350</td>
<td>1550</td>
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<tr>
<td>max delayed flight</td>
<td>200</td>
<td>90</td>
<td>385</td>
<td>140</td>
<td>340</td>
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<tr>
<td>cost</td>
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<td>42750(*)</td>
<td>125600(*)</td>
<td>39500(*)</td>
<td>51500(*)</td>
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<tr>
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<td>35</td>
<td>1</td>
<td>3</td>
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<tr>
<td>run time</td>
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<td>0.3</td>
<td>0.8</td>
<td>0.5</td>
<td>0.5</td>
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</table>
Solved Instances (3): Added value of maintenances

Average results of 10 randomly generated instances

<table>
<thead>
<tr>
<th>Instance</th>
<th>No maintenance</th>
<th>Dummy maintenance</th>
<th>Maintenance optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td># cancelled fits</td>
<td>63.3</td>
<td>5.4</td>
<td>4.8</td>
</tr>
<tr>
<td># delayed fits</td>
<td>4.3</td>
<td>3.1</td>
<td>1.1</td>
</tr>
<tr>
<td># uncovered final states</td>
<td>2.2</td>
<td>0.5</td>
<td>0.3</td>
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<td>total delay [min]</td>
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<td>103.3</td>
<td>36.6</td>
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<td>max delay [min]</td>
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<td>31.6</td>
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<td>36581.5</td>
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<td>optimality gap [%]</td>
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<td>0.28</td>
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<tr>
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<td>12</td>
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<td>run time [s]</td>
<td>20.3</td>
<td>57.9</td>
<td>41.8</td>
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</table>

Considering maintenances is crucial!!!
Example of instance

<table>
<thead>
<tr>
<th>Instance</th>
<th>No maintenance</th>
<th>Dummy maintenance</th>
<th>Maintenance optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td># cancelled fits</td>
<td>57</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td># delayed fits</td>
<td>9</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>total delay [min]</td>
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<td>79</td>
</tr>
<tr>
<td>max delay [min]</td>
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<td>34</td>
<td>50</td>
</tr>
<tr>
<td>cost</td>
<td>339195</td>
<td>13310(*)</td>
<td>5760(*)</td>
</tr>
<tr>
<td>tree size</td>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>run time [s]</td>
<td>8.8</td>
<td>30.5</td>
<td>47.0</td>
</tr>
</tbody>
</table>
Future Work

• Benchmark solutions against practitioners

• Allow repositioning flights and early departures

• Extend Pricing Solver for acceleration

• Include in APM solutions
Conclusions

- Developed a flexible and fast algorithm
- Solutions are very promising
- Maintenance planning is an added value
THANKS for your attention!

Any Questions?