The Single-Agent Routing and Scheduling Problem with Schedule Deviation Penalties

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Outline

Single Agent Routing and Scheduling Problem

One agent. One time horizon T. Activities A.
Choose **what** to do, **when** to do it,
for how long, and **how to travel**.

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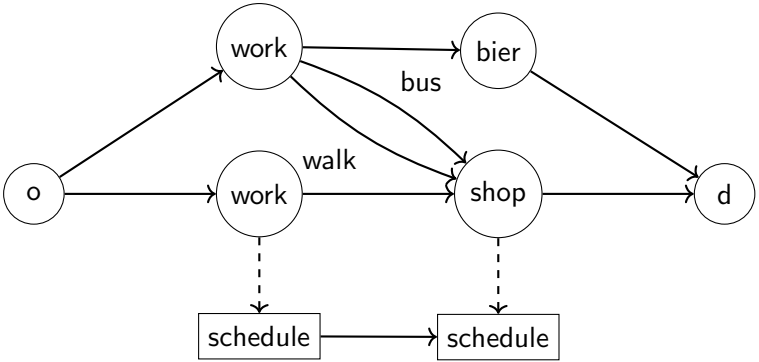


Temporal constraints

Penalties for the deviation from the **preferred schedule**

Looking for an **o-d path** and a **feasible schedule** (start time and duration)

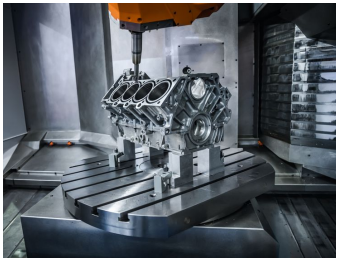
Why study this problem?

Activity-based models

How people plan their activities → transport demand [Pougala et al., 2022]

General structure

The SARSP contains several classical routing and scheduling problems as special cases or subproblems.



Other applications

Machine scheduling [Shabtay and Steiner, 2007], home health care routing [Di Mascolo et al., 2021].

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Problem definition

Input: one agent + a directed multigraph (o, d $\in V$) + scheduling parameters + utility parameters.
Problem: "Find an $o - d$ path AND a feasible schedule maximizing the utility of the agent."

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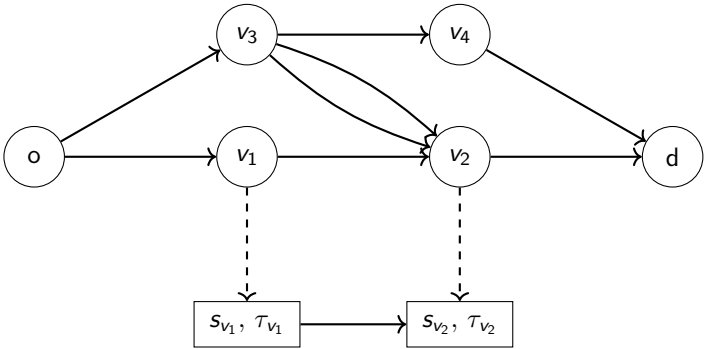

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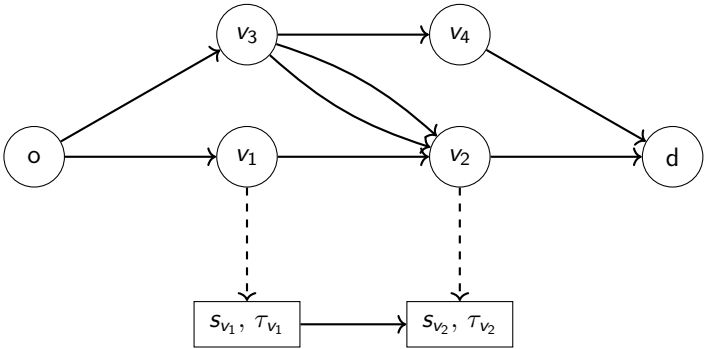


Variables:

- path variables (binary),
- scheduling variables (continuous): x_v (start), τ_v (duration) for all $v \in V$.

Scheduling constraints:

- minimum and maximum duration,
- opening hours,
- time consistency: a task starts after the end of the previous one and the travel time.

Utility function

Accumulated utility u_e for each arc e:

- Utility for each activated vertex (activity participation, activity cost)
- Utility for each crossed arc (travel time, travel cost)

Temporal utility for each vertex v:

- $\tilde{s}_v$ and $\tilde{\tau}_v$, exogenous preferred starting time and duration,
- $f_v(s) = \alpha_f(s - \tilde{s}_v)^+ + \beta_f(\tilde{s}_v - s)^+$, with $\alpha_f, \beta_f < 0$ given, (deviation from $\tilde{s}_v$)
- $g_v(\tau) = \alpha_h(\tau - \tilde{\tau}_v)^+ + \beta_h(\tilde{\tau}_v - \tau)^+$, with $\alpha_g, \beta_g < 0$ given.

$$\max \quad \underbrace{\sum_{e \in A} u_e z_e}_{\text{accumulated utility}} \quad \underbrace{- \sum_{i \in V} f_i(s_i) y_i - \sum_{i \in V} g_i(\tau_i) y_i}_{\text{temporal utility}}$$

MILP formulation

Decision variables

$$z_e \in \{0,1\} \quad \text{arc selection}, \qquad y_i \in \{0,1\} \quad \text{activity selection},$$
$$s_i \geq 0 \quad \text{starting time}, \qquad \tau_i \geq 0 \quad \text{duration}.$$

$$\max \quad \sum_{e \in A} u_e z_e - \sum_{i \in V} f_i(s_i) y_i - \sum_{i \in V} g_i(\tau_i) y_i \tag{1a}$$

s.t. o-d path definition, (1b)

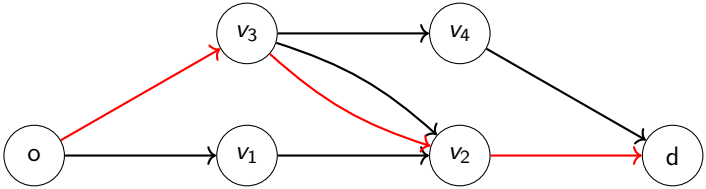
$$s_v \geq s_u + \tau_u + d_e z_e - b_u(1 - z_e), \qquad \forall e = (u,v) \in A, \tag{1c}$$
$$s_v \leq s_u + \tau_u + d_e z_e + (b_v - \underline{\tau}_u)(1 - z_e), \qquad \forall e = (u,v) \in A, \tag{1d}$$

opening and closing time, $\forall i \in V$, (1e)

bounds on activity durations τ_i, $\forall i \in V$, (1f)

$$z_e \in \{0,1\},\ y_i \in \{0,1\},\ s_i \geq 0,\ \tau_i \geq 0, \qquad \forall e \in A,\ \forall i \in V. \tag{1g}$$

Decomposition in two subproblems



Orienteering subproblem
Longest path under resource constraints.
Introduced by [Golden et al., 1987].
Survey [Shen et al., 2025].

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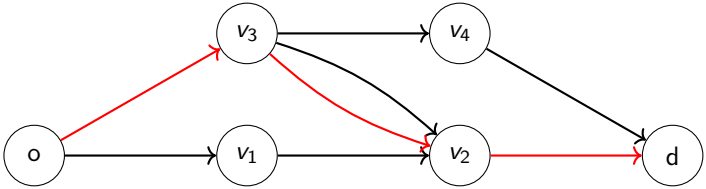

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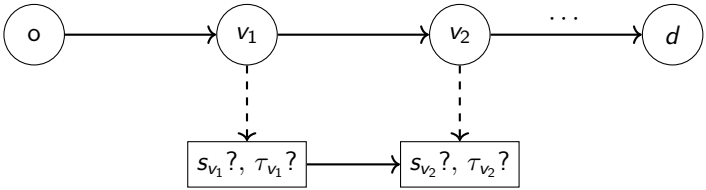

Timing subproblem
From the **single machine scheduling problem with earliness and tardiness costs**. [Lakshminarayan et al., 1978]
Timing subproblem
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[Sourd, 2005]

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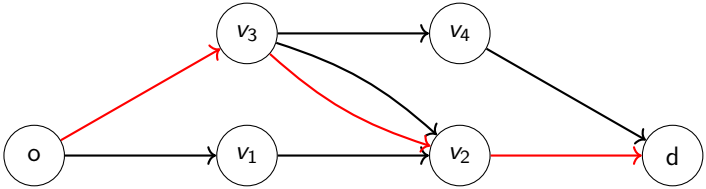

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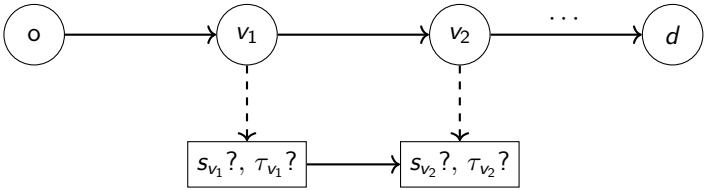

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Contributions: 2 complexity results AND a labeling algorithm to solve SARSP solving the timing subproblem (LP) at each iteration.

Outline

Complexity results

SARSP

NP-hard

Proof by reduction from the knapsack problem.

Timing subproblem of SARSP

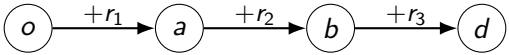
$\mathcal{O}(n \log n)$

Proof inspired by [Tseng and Luo, 1996]: serial compositions of sums and inf-convolutions of simple piecewise-linear functions.

where n is the number of breakpoints in the temporal utility function.

Outline

Labeling algorithm for Resource Constrained Shortest Path Problem



Along the path: $R = r_1 + r_2 + r_3$

Easy forward label extension and dominance rules, when resources are:

- **monotone** along the path,
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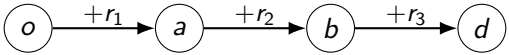

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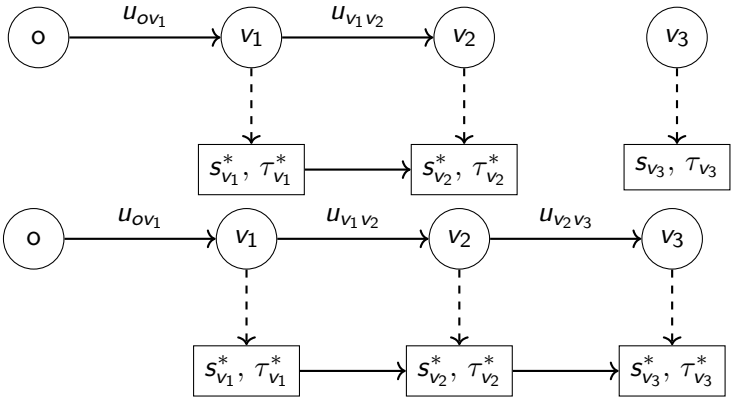
In our case:

$$\max \quad \underbrace{\sum_{e \in A} u_e z_e}_{\text{accumulated utility}} \quad \underbrace{- \sum_{i \in V} f_i(s_i) y_i - \sum_{i \in V} g_i(\tau_i) y_i}_{\text{temporal utility}}$$

- accumulated utility: accumulated (additive) but not monotone

Why the temporal utility is not additive?



- temporal utility: monotone but not accumulated

Solution: **augment the label state**, carrying upper bound and lower bound on the utility of the whole path.

Labels definition

A label $\mathcal{L}$ is a tuple of:

- v,
- $p = e_1, ...e_{k-1}$, $k-1$ arcs from o to v,
- u^a: accumulated utility of p,
- $t^{\min}$: earliest feasible arrival time,
- $t^{\max}$: latest feasible arrival time,

(Accumulated resources)

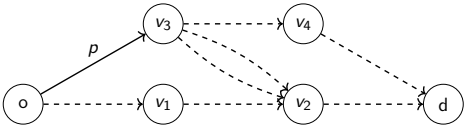
- u^t: **sofar optimal temporal utility of the subpath**,
- $\mathrm{LB}(\mathcal{L})$: **lower bound** on an optimal $o-d$ path starting with p,
- $\mathrm{UB}(\mathcal{L})$: **upper bound** on an optimal $o-d$ path starting with p.

(Computed resources)

Bounds: given a label $\mathcal{L}$ with a subpath p

Lower bound on the objective of **any extension of** p

- Construct a **feasible solution** up to d
- **Solve the LP** $\rightarrow$ temporal utility
- Return: accumulative utility + temporal utility



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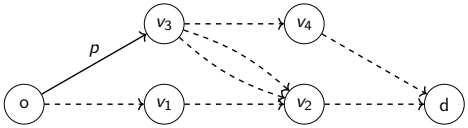

Upper bound on the objective of any extension of p

Consider an extension $p + q$ to d,

$$
\begin{aligned}
u_{p+q} &= u^a_p + u^a_q + u^t_{p+q} && (u^t_{p+q} \leq u^t_p + u^t_q) \\
&\leq u^a_p + u^a_q + u^t_p + u^t_q && (u^t_q \leq 0) \\
&\leq u^a_p + u^a_q + u^t_p && u^a_q \leq \mathrm{UB}^{\mathrm{knap}} \quad \forall q \\
&\leq u^a_p + \mathrm{UB}^{\mathrm{knap}} + u^t_p.
\end{aligned}
$$

$\mathrm{UB}^{\mathrm{knap}}$: knapsack relaxation with weight = time and gain = utility.

Label update rule

Let Q be the current set of labels and $z^\star$ the objective of the best solution found so far, i.e.,

$$z^\star = \max_{\mathcal{L} \in Q} \mathrm{LB}(\mathcal{L}).$$

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Given a new label $\mathcal{L}_{\text{new}}$:

1. (Pruning) If $\mathrm{UB}(\mathcal{L}_{\text{new}}) \leq z^\star$, discard $\mathcal{L}_{\text{new}}$.
2. If $\mathrm{LB}(\mathcal{L}_{\text{new}}) \leq z^\star$, add $\mathcal{L}_{\text{new}}$ to Q.
3. If $\mathrm{LB}(\mathcal{L}_{\text{new}}) > z^\star$:
   - Update $z^\star \leftarrow \mathrm{LB}(\mathcal{L}_{\text{new}})$,
   - (Dominance) Remove all labels $\mathcal{L}_j$ such that $\mathrm{UB}(\mathcal{L}_j) < z^\star$,
   - Add $\mathcal{L}_{\text{new}}$ to Q.

Theorem

At any iteration, for all $\mathcal{L}$ and $\mathcal{L}'$ in Q:

$$\mathrm{UB}(\mathcal{L}) > z^\star \geq \mathrm{LB}(\mathcal{L}').$$

Outline

Data and computational settings

Instances

- Based on the classical Solomon instances [Solomon, 1987]
- Two sets: first 20 and first 50 customers

Random utility parameters based on 3 activity categories [Pougala et al., 2023]

Activity	Prob.	γ	early/late	short/long
work	40%	8	-2.4	-9.6
shopping	30%	5	-0.61	-2.4
leisure	30%	2	-0.61	-2.4

Computational settings

- Planning horizon: 24 hours
- MILP and LPs in the labeling algorithm solved with GUROBI v12.0.3
- Time limit: 600 seconds (first set) 3000 seconds (second set)

Instance	$\|A\|$	Labels	Dom.	Prun.	Lab (s)	MILP (s)	MILP / Lab
from_c107	20	5515	7	463	8.26	1.43	0.17
from_c108	20	15320	20	2706	22.01	2.60	0.12
from_c207	20	7086	676	3445	7.96	7.23	0.91
from_c208	20	8044	81	4509	9.31	1.08	0.12
from_r102	20	2537	12	1126	2.92	4.09	1.40
from_r103	20	8025	9	3947	9.21	52.98	5.76
from_r104	20	43376	725	24296	52.02	600.45	11.54
from_r105	20	478	1	111	0.55	0.34	0.62
from_r202	20	4797	13	2859	4.75	3.37	0.71
from_r203	20	15997	219	8451	16.35	69.22	4.23
from_r204	20	93982	98	52288	94.16	600.27	6.37
from_r205	20	2098	7	620	2.04	2.37	1.16
from_rc101	20	716	0	100	0.84	0.61	0.72
from_rc102	20	17344	4	6290	21.72	16.22	0.75
from_rc201	20	614	4	106	0.62	0.23	0.37
from_rc202	20	31401	1227	14135	33.39	15.94	0.48

Table 1: Results on the small instances (20 vertices).

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Instance	$\|A\|$	Labels	Dom.	Prun.	Lab (s)	MILP (s)	MILP / Lab
from_c107	50	58027	601	24987	103.50	5.30	0.05
from_c108	50	229866	1558	106903	454.21	37.63	0.08
from_c207	50	1803633	62180	1056102	3003.28	3000.76	1.00
from_c208	50	836801	8125	439774	1268.86	468.15	0.37
from_r102	50	78614	1836	43311	110.31	2957.33	26.81
from_r103	50	1255485	38214	804083	2203.03	3000.21	1.36
from_r104	50	1525185	8719	976177	3003.85	3000.23	0.99
from_r105	50	21732	435	8277	37.70	4.18	0.11
from_r202	50	308490	27555	209913	400.74	3000.34	7.49
from_r203	50	2155119	88236	1533945	3000.91	3000.18	0.99
from_r204	50	1932961	18053	1263880	3001.13	3000.43	0.99
from_r205	50	207175	13575	118414	319.19	15.51	0.05
from_rc101	50	4195	69	1081	6.84	0.78	0.11
from_rc102	50	63981	1373	31664	114.78	93.95	0.82
from_rc201	50	2985	16	894	4.47	0.72	0.16
from_rc202	50	88268	603	52073	142.81	71.13	0.50

Table 2: Results on the medium instances (50 vertices).

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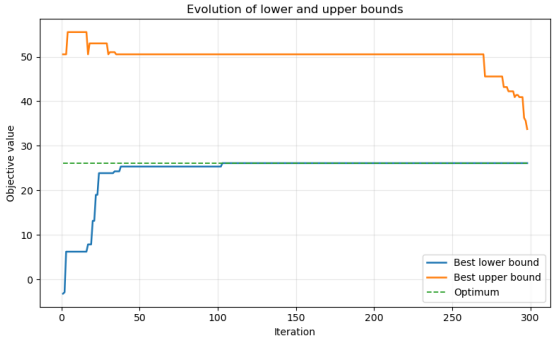
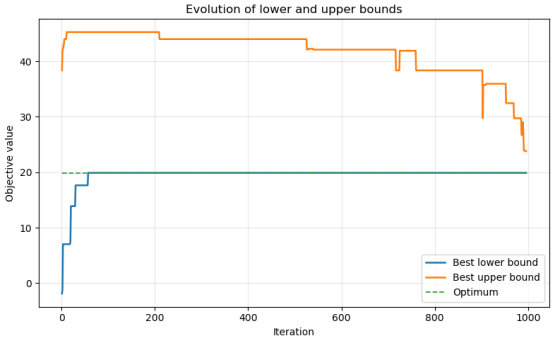
Lower and upper bound analysis





Outline

Further questions

Upper bound

- Improve the current upper bound
- Current version: greedy knapsack relaxation
- Limitation: selected arcs do not necessarily form a path

Experiments on realistic instances

- ABM instances
- Home care scheduling instances
- Comparison with existing home care scheduling methods

Future

- Use it in decomposition methods: Multi-agent scheduling problem
- ABM with housholds or social networks

Thanks for your attention :)

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