

Combinatorial choices for activity-based models

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Outline

Choice model as an optimization problem

Travel demand: activity based models

Model

Graph-based model

Illustrations and results

Predicting choice behavior



Decision rule

Homo economicus

Rational and narrowly self-interested economic actor who is optimizing her outcome

Behavioral assumptions

- ▶ The decision maker solves an optimization problem.
- ▶ The analyst needs to define
 - ▶ the decision variables,
 - ▶ the objective function,
 - ▶ the constraints.

Continuous case: classical microeconomics

Optimization problem

$$\max_q \tilde{U}(q; \theta)$$

subject to

$$p^T q \leq I, \quad q \geq 0.$$

Demand function

- ▶ Solution of the optimization problem.
- ▶ KKT optimality conditions:

$$q^* = f(I, p; \theta)$$

Discrete choices



How does it work for discrete choices?

Utility maximization

Optimization problem

$$\max_{q, w} \tilde{U}(q, w; \theta)$$

subject to

$$\begin{aligned} p^T q + c^T w &\leq I \\ \sum_j w_j &= 1 \\ w_j &\in \{0, 1\}, \forall j. \end{aligned}$$

where $c^T = (c_1, \dots, c_i, \dots, c_J)$ contains the cost of each alternative.

Derivation of the demand functions

- ▶ Mixed integer optimization problem
- ▶ No optimality condition
- ▶ Impossible to derive demand functions directly

Derivation of the demand functions

Step 1: condition on the choice of the discrete good

- ▶ Fix the discrete good(s), that is select a feasible w .
- ▶ Derive the conditional demand functions from KKT.

Step 2: enumerate all alternatives

- ▶ Enumerate all alternatives.
- ▶ Compute the conditional indirect utility function U_i .
- ▶ Select the alternative with the highest U_i .



Enumerate all alternatives ??????



Starbucks has 383 billion unique latte combinations. [Merritt, 2023]

Activity-based models

- ▶ Activity participation
- ▶ Activity type
- ▶ Activity location
- ▶ Activity timing
- ▶ Activity duration
- ▶ Activity scheduling
- ▶ Activity frequency
- ▶ Travel mode choice
- ▶ Route choice
- ▶ Departure time choice
- ▶ Trip chaining / Tour formation
- ▶ Vehicle usage
- ▶ Parking choice
- ▶ Joint activity participation
- ▶ Ride-sharing / Carpooling decision
- ▶ Household resource allocation
- ▶ Teleworking decision
- ▶ Trip cancellation or rescheduling
- ▶ Use of on-demand mobility services
- ▶ ... and many more

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Activities



Why do people travel?

- ▶ Most of the time, not for the sake of it.
- ▶ Activities.
- ▶ Spread in space and time.

Activity-based models: literature

Econometric models

- ▶ Discrete choice models.
- ▶ Curse of dimensionality.
- ▶ Decomposition: sequence of choices
 - ▶ Activity pattern
 - ▶ Primary tour: time of day
 - ▶ Primary tour: destination and mode
 - ▶ Secondary tour: time of day
 - ▶ Secondary tour: destination and mode
 - ▶ e.g.
[Bowman and Ben-Akiva, 2001]

Rule-based models

- ▶ If the selected activity is at location L ,
- ▶ and the travel time from current location C to L exceeds $T_{\max}$,
- ▶ then reject the activity–location combination,
- ▶ unless it is a high-utility or infrequent activity (e.g., doctor appointment).
- ▶ e.g. [Arentze et al., 2000]

Research question: can we combine the two?

	Econometric	Rule-based
Micro-economic theory	X	—
Parameter inference	X	—
Testing/validation	X	—
Joint decisions	—	X
Complex rules	—	X
Complex constraints	—	X

Combinatorial choices

Mathematical optimization

- ▶ Each individual is solving a combinatorial optimization problem.
- ▶ Decisions: see the long list before...
- ▶ Objective function: utility (to be maximized).
- ▶ Constraints: complex rules.

[Pougala et al., 2023]

Challenges

- ▶ Stochasticity: random utility → rely on simulation.
- ▶ Large number of variables and constraints → decomposition methods.
- ▶ Interacting individuals (households, social groups) → this talk.
- ▶ Time horizon → future work.

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Social groups

We consider a social group N of agents that cooperate and desire to maximize their aggregated utility.



- ▶ Coordination, joint activities.
- ▶ Group decision making
- ▶ Service to the group, maintenance.
- ▶ Resource constraints.
- ▶ Escorting.

Objective function: utility of the group

- ▶ Function of the utility of each member. But which function?
- ▶ Lack of consensus in the literature.
- ▶ Additive: the (weighted) sum of the utility of each member.
- ▶ Autocratic: the utility of the “strongest” member.
- ▶ Egalitarian: the utility of the “weakest” member.
- ▶ Important for our framework: must be easy to linearize.



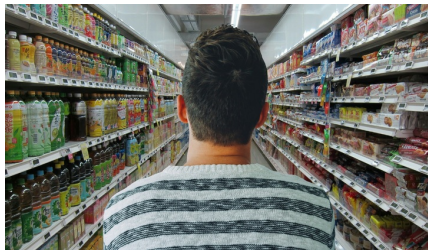
Constraints



Coordinated activities

- ▶ a is an activity that must be performed by all members of the group.
- ▶ Dining out.
- ▶ Family gathering.
- ▶ Sport events.

Constraints



Distributed activities

- ▶ a is an activity that must be performed for the group.
- ▶ Maintenance.
- ▶ Grocery shopping.
- ▶ Meal preparation.
- ▶ Accounting of the sport club.

Constraints

Resource constraints

- ▶ One car per household.
- ▶ One meeting room in a shared office space.
- ▶ Modeling approach: treat the resource as an individual.
- ▶ “The car is a member of the family”.
- ▶ It is associated with “activities” and a schedule.
- ▶ We can then introduce “coordinated activities” constraints.



Constraints



Escorting a child to school

- ▶ Specific instance of a resource constraint.
- ▶ The person escorting becomes a resource.
- ▶ As individuals and resources are modeled in the same way, coordinated activities constraints can be applied.

Space



Discrete and finite set L of locations.

For each (ℓ, ℓ') :

- ▶ $M_n^{\ell\ell'}$: available modes for agent n .
- ▶ $\rho_{\ell\ell'm}$: travel cost of the trip with mode m .
- ▶ $d_{\ell\ell'm}$: travel time of the trip with mode m .

Assumption: travel time and cost are exogenous.

Activities: notations

Set A_n of potential activities for each agent n .

For each activity a :

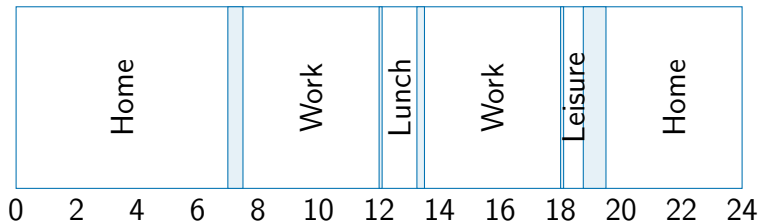
- ▶ L_a : set of possible **locations** for a
- ▶ $c_{a\ell}$: cost of a at location ℓ
- ▶ $[\gamma_{a\ell}^-, \gamma_{a\ell}^+]$: opening hours for a at location ℓ
- ▶ $\tau_a^{\min}$ & $\tau_a^{\max}$: min & max duration of a .
- ▶ C_a : maximum capacity for a .
- ▶ N_a : set of required agents for a .



Activities: further assumptions

- ▶ **Start and end at home:** The first activity (dawn) and the last activity (dusk) always take place at the agent's home.
- ▶ **Group of activities:**
 - ▶ Some activity groups (e.g., shopping) must be performed at least a specified number of times over the planning horizon.
 - ▶ Examples: shopping, domestic tasks, sport, etc.

Scheduling



Utility function

Collective decisions $\Rightarrow$ maximize the **utility of the group**

$$U = \sum_{n \in N} U_n = \sum_{n \in N} \left(\sum_{a \in A_n} w_{an} (U_a^n + \xi_{an}) + \sum_{\ell, \ell' \in L} \sum_{m \in M} w_{\ell\ell'mn} (U_{\ell\ell'm}^n + \xi_{\ell\ell'mn}) \right)$$

- ▶ U_a^n : reward + joint activity reward - deviation from the preferred schedule - cost
- ▶ $U_{\ell\ell'm}^n$: joint travel utility (travel cost, travel time, etc.), usually negative.
- ▶ ξ_{an} and $\xi_{\ell\ell'mn}$: random term with a known distribution
- ▶ w_{an} and $w_{\ell\ell'mn}$ decision variables

Utility function



Error terms

- ▶ Rely on simulation.
- ▶ Draw ξ_{anr} and $\xi_{\ell\ell'mn}$, $r = 1, \dots, R$.
- ▶ Optimization problem for each r .
- ▶ Utility: U_{anr} .

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Model

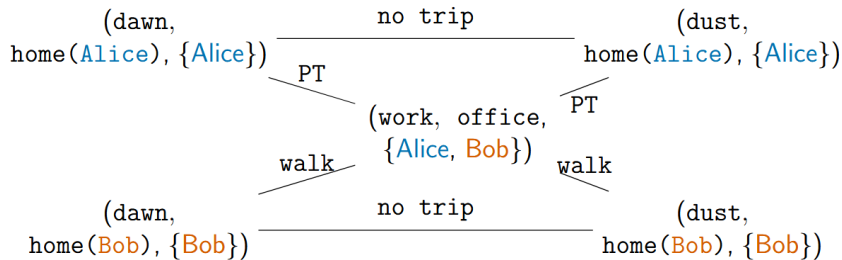
Graph-based model

Illustrations and results

Graph-based modeling approach

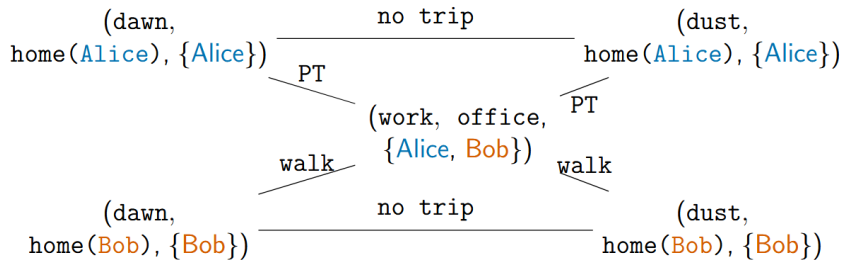
- ▶ Formulated as an **optimization problem on a graph** $G = (V, E)$
- ▶ Connects to well-studied problems from the **vehicle routing literature**:
 - ▶ Team Orienteering Problem with Synchronization
 - ▶ Longest Path Problem with Time Constraints
 - ▶ Vehicle Routing Problem with Synchronization
- ▶ **Synchronization constraints** can be directly encoded in the graph structure

Synchronized graph



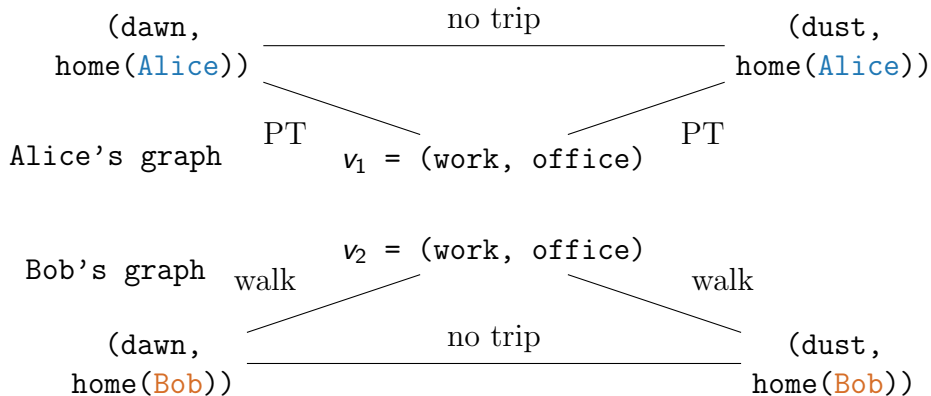
- ▶ Vertices V : triplet $v = (\text{activity } a_v, \text{location } \ell_v, \text{subgroup of agents } S_v)$
→ also encoding C_a and N_a
- ▶ Arcs E : transition of agents between activities
→ labeled with the transport mode

Synchronized graph



- ▶ **One $\text{dawn}(n)$ - $\text{dusk}(n)$ path in $G \Leftrightarrow$ One sequence of activities/trips for an agent n**
- ▶ Problem reformulation: find one path per agent under time-consistency, combinatorial and budget constraints.

Unsynchronized graph



- ▶ Synchronization constraints hard-coded in the MILP.
- ▶ Example: if Alice's path goes through v_1 , Bob's path should go through v_2 and they should arrive and stay the same time.

Variables

- ▶ Graph variables
 - ▶ $z_e^n \in \{0, 1\}$ — equals 1 if agent n travels along arc e
 - ▶ $w_v \in \{0, 1\}$ — equals 1 if vertex v is part of the path for all agents in S_v
- ▶ Time variables for each vertex v
 - ▶ $x_v \in \mathbb{R}_+$ — starting time of activity a_v
 - ▶ $\tau_v \in \mathbb{R}_+$ — duration of activity a_v

These apply to all agents in S_v at location ℓ_v .

Constraints

1. **flow** constraints

- ▶ path definition

2. **combinatorial** constraints

- ▶ eligibility to pass through a vertex
- ▶ group consistency
- ▶ location uniqueness
- ▶ group of activities

3. **time-consistency** constraints

- ▶ schedule consistency
- ▶ full time period covered
- ▶ opening hours
- ▶ duration bounds

5. a **budget** constraint

Using the unsynchronized graph:

6. **synchronization** constraints.

- ▶ same starting time for all the agents synchronizing for the activity
- ▶ same duration for all the agents synchronizing for the activity

Constraints

- **flow conservation** constraints: dawn(n)-dusk(n) path definition

$$\sum_{e \in \delta^+(v)} z_e^n = \sum_{e \in \delta^-(v)} z_e^n \quad \forall v \in V \quad \forall n \in N$$

$$\sum_{e \in \delta^+(\text{dawn}(n))} z_e^n = 1 \quad \forall n \in N$$

$$\sum_{e \in \delta^-(\text{dusk}(n))} z_e^n = 1 \quad \forall n \in N$$

Constraints

- ▶ **combinatorial** constraints

- ▶ Group consistency

$$w_v = \sum_{e \in \delta^+(v)} z_e^n \quad \forall v \in V \quad \forall n \in S_v$$

- ▶ Eligibility

$$z_e^n = 0 \quad \forall e = (u, v) \in E \quad \forall n \notin N_u \cap N_v$$

- ▶ Group of activities

$$\sum_{v \in V: a_v \in G_k} w_v \geq n_k \quad \forall k \in K$$

- ▶ Location uniqueness

$$w_v + w_{v'} \leq 1 \quad \forall v, v' \in V \text{ s.t. } a_v = a_{v'}, S_v = S_{v'}, l_v \neq l_{v'}$$

Constraints

► time-consistency constraints

$$x_v \geq x_u + \tau_u + d_{uv} - T(1 - z_e^n) \quad \forall e = (u, v) \in E \quad \forall n \in N$$

$$x_v \leq x_u + \tau_u + d_{uv} + T(1 - z_e^n) \quad \forall e = (u, v) \in E \quad \forall n \in N$$

$$\gamma_{a_v, \ell_v}^- w_v \leq x_v \leq \gamma_{a_v, \ell_v}^+ + T(1 - w_v) \quad \forall v \in V$$

$$\tau_{a_v}^{\min} w_v \leq \tau_v \leq T(1 - w_v) \quad \forall v \in V$$

$$\sum_{v \in V: n \in S_v} \tau_v + \sum_{e \in E} d_e z_e^n = T \quad \forall n \in N$$

Additional constraints

- ▶ a **budget** constraint

$$\sum_{v \in V: n \in S_v} c_{a_v} \ell_v w_v + \sum_{e \in E} \rho_e z_e^n \leq B \quad \forall n \in N$$

Using the unsynchronized graph:

- ▶ **synchronization** constraints

Methods to solve the graph-based approach

Two directions

- ▶ **MILP formulation** — commercial (Gurobi, CPLEX) or open-source (GLPK, HiGHS) solvers
- ▶ **Graph optimization** — decomposition, shortest path, dynamic programming

MILP: two graph formulations tested

Synchronization encoded either **in the graph** or **in the constraints**

Tested on real-life instances (UK NTS, individuals with 11–45 activities):

- ▶ **Sync in graph** faster for **single** synchronization
- ▶ **Sync in constraints** faster for **subgroup** synchronization

Limit of this approach: scalability (up to 4 agents and 20 activities).

MILP results: which sync. encoding wins?

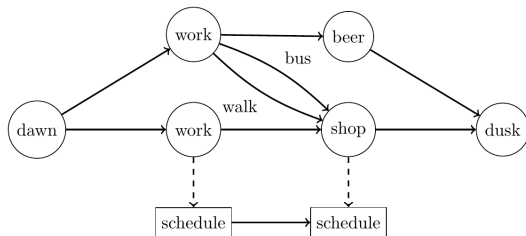
Single synchronization		
	B&R	V. et al.
H1	0.11	0.09
H3	2.27	2.16
H4	0.06	0.08
H5	0.67	0.65
H6	0.23	0.18
H7	1.69	1.08
H12	0.12	0.18

Subgroup synchronization		
	B&R	V. et al.
H1	0.25	0.19
H3	1.81	2.65
H4	0.11	0.09
H5	0.41	0.31
H6	0.18	0.28
H7	5.65	284.85
H12	0.43	1.10

- ▶ On timed-out instances, **B&R** reaches the better gap on 5/5 subgroup cases vs. 2/4 single-sync
- ▶ On **H7** (subgroup), B&R is $\sim 50\times$ faster (5.65s vs 284.85s)

Ongoing work: Pulse algorithm

- ▶ Applied to the **Single Agent** problem
- ▶ Extend a path from dawn to dusk
- ▶ Prune it once an upper bound shows it can't beat the best path found so far



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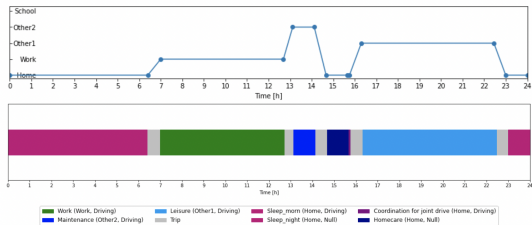
Model

Graph-based model

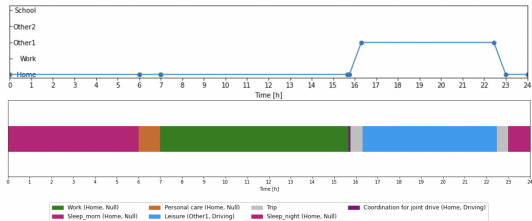
Illustrations and results

Car as a resource

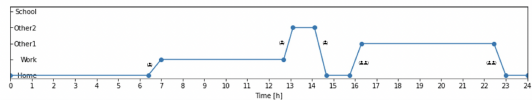
Sara



David



Car



To Tennis or Not to Tennis

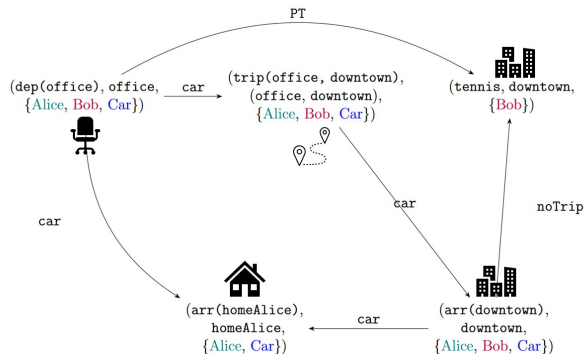
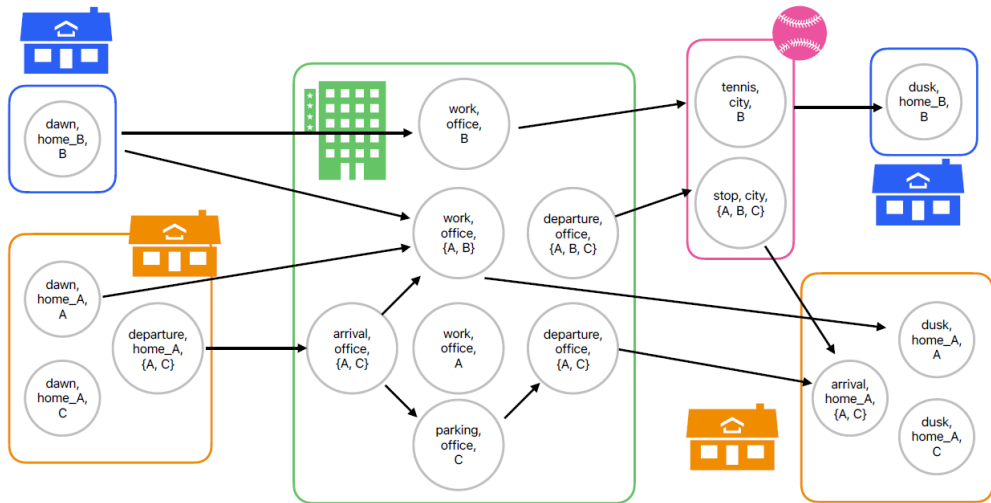


Figure: Example of ride sharing modeling

Example

- ▶ Alice (A) and Bob (B): two colleagues
- ▶ Alice has a car.
- ▶ Bob has another activity: tennis.

Full graph



Hypotheses

- ▶ Alice and Bob derive a social reward by working together.
- ▶ Alice prefers to work in the afternoon.
- ▶ Bob can only play tennis between 4pm and 7pm.
- ▶ The trip from the office to the tennis takes much more time with public transport than with the car.



Different scenarios

- ▶ If Alice and Bob work together, without the car, Bob can't go to tennis.



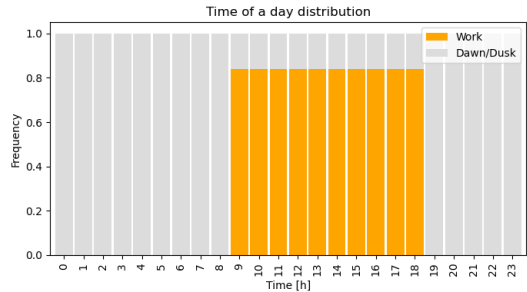
- ▶ If he arrives at work early, he can go to tennis, but he doesn't work with Alice.



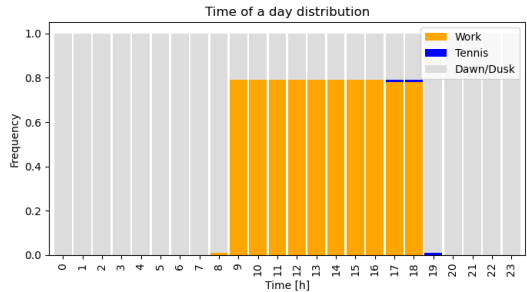
- ▶ If Alice and Bob work together and Alice comes by car, B can go to tennis by car with Alice.



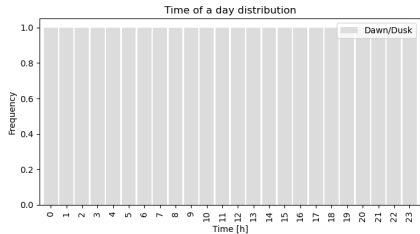
Simulation: From isolated individuals...



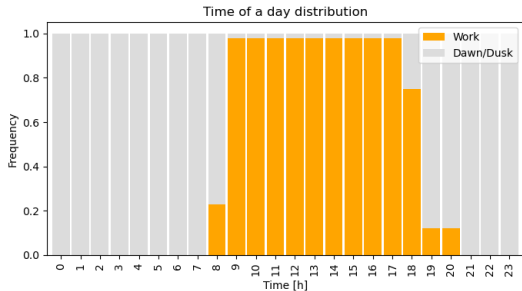
Alice



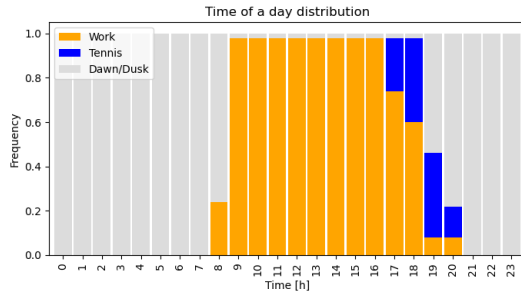
Bob



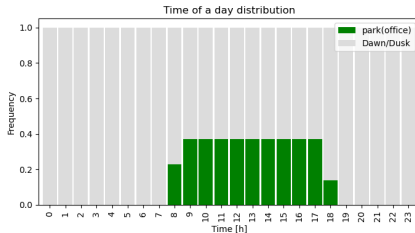
Simulation: ...to social groups



Alice



Bob



Conclusions

It works!

- ▶ Handles complex activity and schedule choices.
- ▶ Integrates behavioral and operational constraints.
- ▶ Enables realistic, data-driven simulations.

What's next?

- ▶ **Scalability** remains a major challenge (time, activities)
- ▶ **Simulation cost** is high — need for efficient algorithms
- ▶ **Efficient single-agent algorithm** — to be embedded in a **decomposition framework** (as in VRP)
- ▶ **Inference** could benefit from Bayesian approaches

Summary

- ▶ **Goal:** develop operational combinatorial choice models, such as activity-based models.
- ▶ **Approach:** integrate econometric modeling with rule-based logic.
- ▶ **Methodology:** leverage operations research, mathematical optimization and simulation.
- ▶ Simulation of activity schedule: [Pougala et al., 2022a].
- ▶ Application with the Swiss Railways: [Manser et al., 2021].
- ▶ Estimation of the parameters: [Pougala et al., 2022b].
- ▶ Household interactions: [Rezvary et al., 2023], [Rezvary et al., 2024].
- ▶ Main advantage of the framework: flexibility.

Combinatorial choices

Main philosophy

Leverage the power of modern combinatorial optimization to model complex choice behavior.

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