Event-based modeling approach for the multi-agent daily scheduling problem

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Outline

- Introduction
- Multi-agent Daily Scheduling Problem
- Event-based modeling
- Interest of the model and computational results

Transport demand modeling

Activity-Based Models

- Participation in activities.
- How people plan activities → transport demand

Simultaneous Choice Modeling

- Choices are not made sequentially.

An optimization problem: the Daily Scheduling Problem

- Variables: Activity participation, Scheduling and Transport mode.
- Objective: Maximize agent's utility over the time period.

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Goal of this work

	[Pougala et al., 2022]	[Rezvany et al., 2023]	[Cortes Balcells et al., 2023]
Method	MILP	MILP	Dynamic programming
Scope	One agent	Several agents (≤ 5)	One agent
	No private vehicle	Private vehicle	Additional constraints

Our challenge: increase the number of dependent agents (social networks $\rightarrow$ multi-agent) + solve it **quickly** because **broader framework**.
Our contribution: a new formulation as a **minimum cost flow problem in a graph**.
Our hope: improve the **computation time**.

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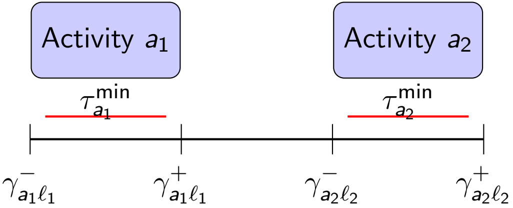
Multi-agent Daily Scheduling Problem: activities

- N: set of agents.
- A_n: **activities** that can be performed by agent n.



- L_a: **locations** for activity a.
- $c_{a\ell}$: cost of performing activity a at location ℓ.
- $[\gamma_{a\ell}^{-}, \gamma_{a\ell}^{+}]$: opening hours for activity a at location ℓ.
- $\tau_a^{\min}$ & $\tau_a^{\max}$: min & max duration for activity a.
- C_a: maximum capacity of activity a.
- N_a: collection of required agents for activity a.

Multi-agent Daily Scheduling Problem: special activities and trips

Special activities:

- G_k: **group of activities** that must be performed at least n_k times.
- dawn $\in A_n$: first activity at location home(n).
- dusk $\in A_n$: last activity at location home(n).

Trips:

- $M_n^{k\ell}$: available **transport modes** between locations k and ℓ for agent n.
- $\rho_{k\ell m}$ and $d_{k\ell m}$: cost and duration of trip (k, ℓ) with mode m.

Utility function

Collective decisions $\Rightarrow$ maximize the **utility of the group**

$$U = \sum_{n \in N} U_n = \sum_{n \in N} \left(\sum_{a \in A_n} U_a^n + \xi_{an} + \sum_{\ell, \ell' \in L} \sum_{m \in M} U_{\ell\ell' m}^n + \xi_{\ell\ell' mn} \right) \quad (1)$$

- U_a^n: reward + joint activity reward - deviation from the prefered schedule - cost
- $U_{\ell\ell' m}^n$: joint travel reward - travel cost - travel time
- ξ_{an} and $\xi_{\ell\ell' mn}$: random term with a known distribution

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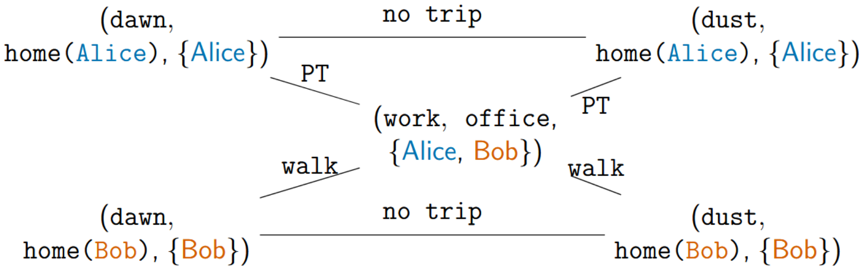
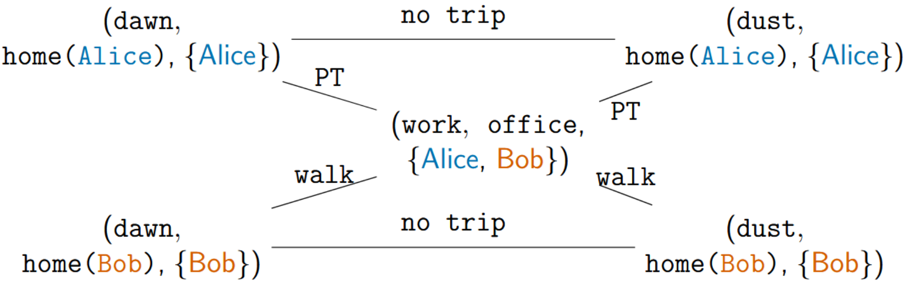
Event-based modeling approach [Gaul et al., 2022]

- Formulation as a shortest path problem in a graph $G = (V, E)$ with additional constraints.



- Vertices V: triplet $v = (\text{activity } a_v, \text{ location } \ell_v, \text{ subgroup of agents } S_v)$
  $\rightarrow$ also encoding C_a and N_a
- Arcs E: transition of agents between activities
  $\rightarrow$ labeled with the transport mode

Event-based modeling approach



- **One dawn(n)-dusk(n) path in G $\Leftrightarrow$ One sequence of activities/trips for an agent n**
- Problem reformulation: find one path per agent under time-consistency, combinatorial and budget constraints.

Variables

- Graph variables
  - $z_e^n \in \{0, 1\}$ — equals 1 if agent n travels along arc e
  - $w_v \in \{0, 1\}$ — equals 1 if vertex v is part of the path for all agents in S_v
- Time variables for each vertex v
  - $x_v \in \mathbb{R}_+$ — starting time of activity a_v
  - $\tau_v \in \mathbb{R}_+$ — duration of activity a_v

These apply to all agents in S_v at location ℓ_v.

Constraints

1. **flow** constraints
   - path definition
2. **combinatorial** constraints
   - eligibility to pass through a vertex
   - group consistency
   - location uniqueness
   - group of activities
3. **time-consistency** constraints
   - schedule consistency
   - full time period covered
   - opening hours
   - duration bounds
4. a **budget** constraint

Constraints

- **flow** constraints: dawn(n)-dusk(n) path definition

$$\sum_{e \in \delta^+(v)} z_e^n = \sum_{e \in \delta^-(v)} z_e^n \qquad \forall v \in V \quad \forall n \in N$$

$$\sum_{e \in \delta^+(\text{dawn}(n))} z_e^n = 1 \qquad \forall n \in N$$

$$\sum_{e \in \delta^+(\text{dusk}(n))} z_e^n = 1 \qquad \forall n \in N$$

Constraints

- **combinatorial** constraints $u = (a_u, l_u, N_u) \longrightarrow v = (a_v, l_v, N_v)$
  - Group consistency

$$w_v = \sum_{e \in \delta^+(v)} z_e^n \qquad \forall v \in V \quad \forall n \in S_v$$

  - Eligibility

$$z_e^n = 0 \qquad \forall e = (u, v) \in E \quad \forall n \notin N_u \cap N_v$$

  - Group of activities

$$\sum_{v \in V:\ a_v \in G_k} w_v \geq n_k \qquad \forall k \in K$$

  - Location uniqueness

$$w_v + w_{v'} \leq 1 \qquad \forall v, v' \in V \text{ s.t. } a_v = a_{v'}, S_v = S_{v'}, \ell_v \neq \ell_{v'}$$

Constraints

- **time-consistency** constraints

$$
\begin{aligned}
& x_v \geq x_u + \tau_u + d_{uv} - T(1 - z_e^n) && \forall e = (u, v) \in E \quad \forall n \in N \\
& x_v \leq x_u + \tau_u + d_{uv} + T(1 - z_e^n) && \forall e = (u, v) \in E \quad \forall n \in N \\
& \gamma^-_{a_v,\ell_v} w_v \leq x_v \leq \gamma^+_{a_v,\ell_v} + T(1 - w_v) && \forall v \in V \\
& \tau^{\min}_{a_v} w_v \leq \tau_v \leq T(1 - w_v) && \forall v \in V \\
& \sum_{v \in V:\ n \in S_v} \tau_v + \sum_{e \in E} d_e z_e^n = T && \forall n \in N
\end{aligned}
$$

Additional constraints

- a **budget** constraint

$$\sum_{v \in V:\ n \in S_v} c_{a_v \ell_v} w_v + \sum_{e \in E} \rho_e z_e^n \leq B \qquad \forall n \in N$$

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Illustration

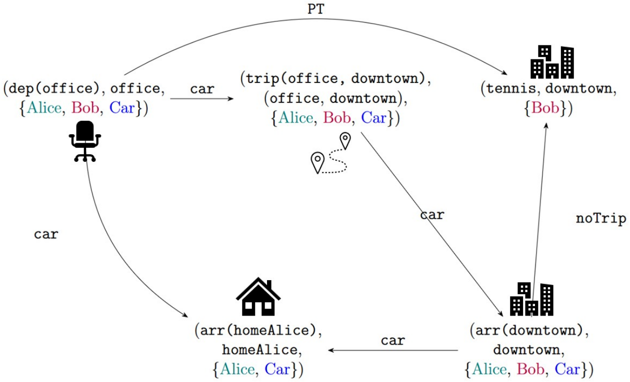
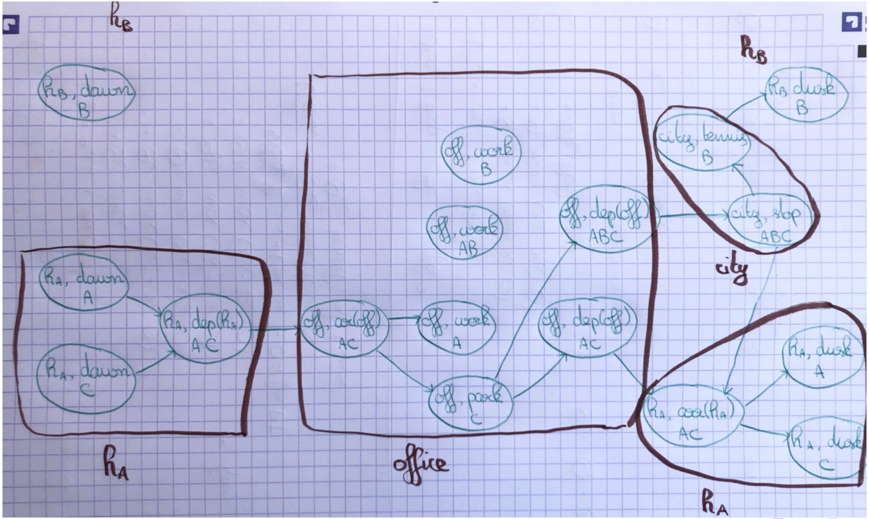


Figure 1: Example of ride sharing modeling

Example

- Alice (A) and Bob (B): two colleagues
- Alice has a car.
- Bob has another activity: tennis.
- **Trick:** model the car as an agent

Full graph



Hypotheses

- Alice and Bob derive a **social reward** by working together.
- Alice prefers to work in the afternoon.
- Bob can only play tennis **between 16 and 19**.
- The trip from the office to the tennis takes **much more time** with **public transport** than with the **car**.

Different scenarios

- If Alice and Bob work together, without the car, Bob can't go to tennis.

- If he arrives at work early, he can go to tennis, but he doesn't work with Alice.

- If Alice and Bob work together and Alice comes by car, B can go to tennis by car with Alice.



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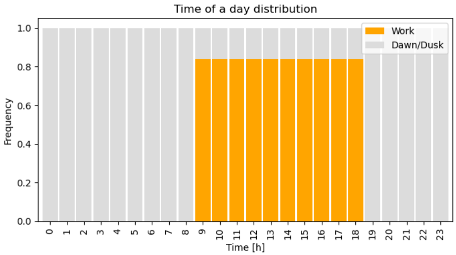
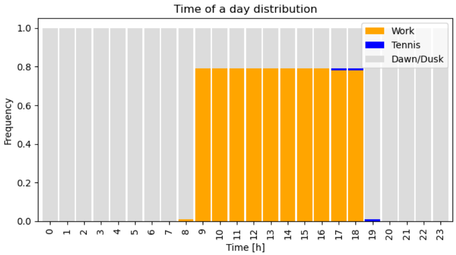
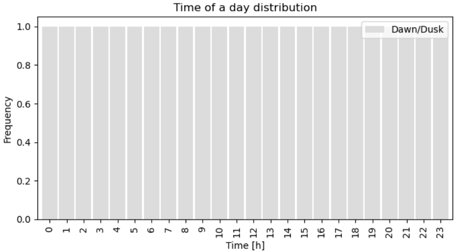
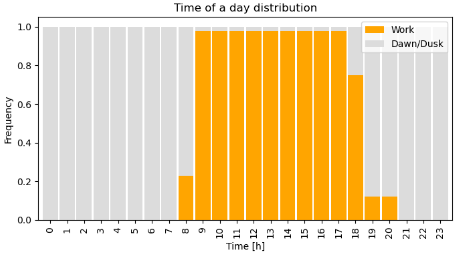
Simulation: From isolated individuals. . .

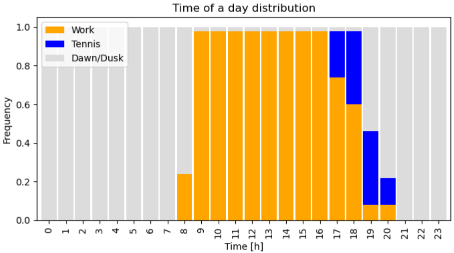


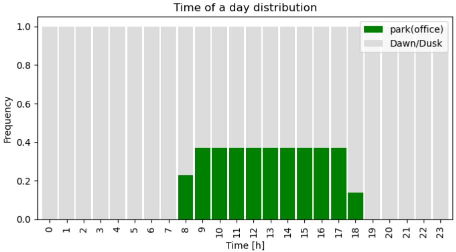




Simulation: . . . to social groups







Computational results

- One real instance (David and Sara from UK Time use survey (TUS) used by [Rezvany et al., 2023]).
- Generated instances with an increasing number of activities A.

N	A	L	M	Total time	Solving time	Gap	Root gap	Constr.	Var.
2	5	3	2	3.10	0.44	0%	457%	2,229	7,851
5	5	5	4	0.70	0.33	0%	312%	4,316	16,433
5	6	5	4	1.68	1.33	0%	428%	5,145	19,587
5	7	5	4	602.23	600.31	16%	553%	9,519	36,039
5	8	5	4	601.04	600.50	22%	389%	10,370	39,271
5	9	5	4	601.07	600.68	23%	321%	11,033	41,144

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Problem: weak relaxation, because of Big-M!

Hope: valid inequalities, nammely classical Miller-Tucker-Zemlin constraints (TWVRP).

Further steps

Explore the **flexibility** of this model on real data:

- Test on more data of the UKTUS

Speed up the **computation**:

- Add valid inequalities (Miller-Tucker-Zemlin constraints for TWVRP)
- Clean the graph to reduce the size of the problem
- Eventually, use column generation (convenient to derive a path-formulation).

Questions?

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