Fast Algorithms for Capacitated Continuous Pricing with Discrete Choice Demand Models

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Outline

- Introduction
- Methodology
- Experimental Results
- Conclusions

The Continuous Pricing Problem (CPP)

CPP

- Supplier offers J products for sale. Goal: determine optimal **price** for each product to maximize total **profit**.
- There always exists an **opt-out** option (competition, etc).
- Demand for each product is modeled using a **discrete choice model** (DCM).

DCM

- For every **costumer** n and **product** i a stochastic **utility** U_{in} is defined, which depends on **socio-economic** characteristics of the individual and **attributes** of the products (e.g. the price).

The Continuous Pricing Problem (CPP)

Utility

- **Utility** of alternative i for costumer n:

$$U_{in} = \sum_{k \neq p} \beta_k x_{ink} + \beta_p p_i + \varepsilon_{in}$$

- β_k : parameters (exogenous)
- x_{ink} : attributes (exogenous)
- p_i : price of alternative i
- ε_{in} : stochastic error term

The Continuous Pricing Problem (CPP)

Probability

- **Probability** that costumer n chooses alternative i:

$$P_n(i) \quad = \quad \mathbb{P}(U_{in} \geq U_{jn} \; \forall j \in J)$$

- **Logit** ($\varepsilon_{in} \sim$ i.i.d. Gumbel(0, 1)):

$$P_n(i) \quad = \quad \frac{e^{V_{in}}}{\sum_{j \in C_n} e^{V_{jn}}}$$

- **Mixed Logit** (Logit $+ \; \beta_k \sim F(\beta_k | \theta)$):

$$P_n(i) \quad = \quad \int \frac{e^{V_{in}(\beta_{kn})}}{\sum_{j \in C_n} e^{V_{jn}(\beta_{kn})}} f(\beta_k | \theta) d\beta_k$$

Literature

Integrating **Logit** into...

- Facility location [Mai and Lodi, 2017, Ljubić and Moreno, 2018]
- Revenue Management [Shen and Su, 2007, Korfmann, 2018]
- Railway Timetabling [Cordone and Redaelli, 2011, Robenek et al., 2018]

Integrating **Nested Logit** into...

- Toll setting [Wu et al., 2012]
- Pricing [Gallego and Wang, 2014]

Integrating **Mixed Logit** into...

- School location [Haase and Müller, 2013]
- Toll setting [Gilbert et al., 2014]
- Pricing [Marandi and Lurkin, 2020, van de Geer and den Boer, 2022]

Literature

Integrating general DCM into optimization problems

- Formulation as a mixed-integer-linear program (**MILP**) using **Monte-Carlo simulation** [Paneque et al., 2021]
- **Heuristic** based on **Lagrangian decomposition** and grouping of scenarios [Paneque et al., 2022]
- **Heuristic** based on **Benders decomposition** (w/out capacity constraints) [Haering et al., 2022]
- **Exact** method based on **spatial Branch-and-Bound** and **Benders decomposition** (w/out capacity constraints) [Haering et al., 2023]

So are we done?

- No. We want to go **faster**. In general. And include **capacity constraints**.

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Base layer: Monte Carlo Simulation

- **Simulate** R scenarios (draws), each with **deterministic** utilities U_{inr}:

$$\begin{aligned} U_{inr} &= \sum_{k \neq p} \beta_k x_{ink} + \beta_p p_i + \varepsilon_{inr} \qquad && \forall n \in \mathcal{N}, i \in C_n, r \in \mathcal{R} \\ &= c_{inr} + \beta_p p_i && \forall n \in \mathcal{N}, i \in C_n, r \in \mathcal{R} \end{aligned}$$

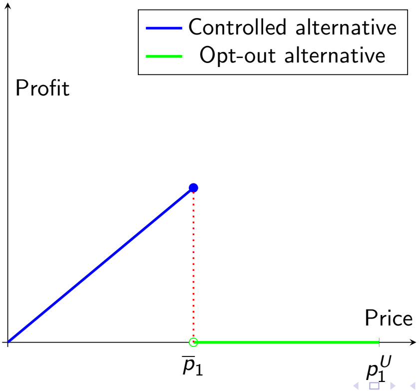
Breakpoints

- **Don't worry**. We're not doing *complicated* decomposition methods or actual *math*.
- Instead we will use the simple idea of decision making **breakpoints**.

Breakpoints: Illustration

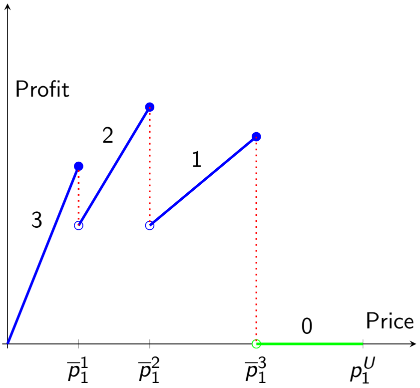
- **1** customer, **1** controlled price + opt-out
- **Breakpoint** $\overline{p}_1$:

$$U_0 = U_1 \implies U_0 = c_1 + \beta_p^1 \overline{p}_1 \implies \overline{p}_1 = \frac{U_0 - c_1}{\beta_p^1}.$$



Breakpoints: Illustration II

- **3** customers, **1** controlled price + opt-out
- Numbers: how many customers are **captured**



Breakpoint Exact Algorithm (BEA) [Haering et al., 2023]

Simplified:

For all possible orderings of prices $p_1 \leq p_2 \leq \cdots \leq p_J$ do:

- Introduce the **cheapest** alternative 1.
- Compute the **breakpoints** $\overline{p}_1^s$ (from $U_{0s} = U_{1s}$) for all simulated customers s.
- **Iterate** over all breakpoints $\overline{p}_1^s$, highest to lowest. For each $\overline{p}_1^s$, fix $p_1 = \overline{p}_1^s$ and do:
  - Introduce alternative **2**. Now compute the **breakpoints** $\overline{p}_2^s$ (from $\max(U_{0s}, U_{1s}) = U_{2s}$).
  - **Iterate** over all breakpoints $\overline{p}_2^s$, highest to lowest. For each $\overline{p}_2^s$, fix $p_2 = \overline{p}_2^s$ and do:
    - Introduce alternative **3**. ...
      - Once **all prices** are **fixed**, **compute profit**

Keep the price combination with **highest profit**!

BEA

- Very fast for one or two alternatives, then it **rapidly** breaks down.
- Complexity $O(J!(NR)^J log(NR))$ **exponential** in number of alternatives J.
- Our new contributions:
  1. Extend BEA to **include capacity constraints**
  2. Introduce Breakpoint **Heuristic** Algorithm (BHA) to solve high-dimensional problems much faster (and with high accuracy!)

Capacity constraints

- Need to compute breakpoints from not only $\max(U_{0s}, \ldots, U_{(i-1)s},) = U_{is}$) but from all $U_{js} = U_{is}$ separately, due to **people no longer always choosing highest utility** alternative.

Breakpoint Exact Algorithm with Capacities (BEAC)

For all possible orderings of prices $\{p_1, p_2, \ldots, p_J\}$ do:

- Introduce the **cheapest** alternative 1.
- Compute the **breakpoints** $\overline{p}_1^s$ (from $U_{0s} = U_{1s}$) for all simulated customers s.
- **Iterate** over all breakpoints $\overline{p}_1^s$. For each $\overline{p}_1^s$, fix $p_1 = \overline{p}_1^s$ and do:
  - Introduce alternative **2**. Now compute the breakpoints $\overline{p}_{02}^s, \overline{p}_{12}^s$ (from $U_{0s} = U_{2s}$ and $U_{1s} = U_{2s}$).
  - **Iterate** over all breakpoints $\overline{p}_{x2}^s$. For each, fix $p_2 = \overline{p}_{x2}^s$ and do:
    - Introduce alternative 3. ...
      - Once **all prices** are **fixed**, **compute capacitated profit**

Keep the price combination with **highest profit**!

Computing the capacitated profit

- Having to compute the profit for each price combination is **computationally expensive, but:**
- It **adds flexibility** as to how we evaluate the profit
- We propose **three** different strategies:
  1. Based on an **exogenous priority queue**
  2. Supplier controls access and assigns people in such a way that **maximizes total profit**
  3. Same as 2. but now we want to minimize profit. Finding the price such that the worst-case profit is highest = **robust optimization**

Breakpoint Heuristic Algorithm (BHA)

1. Choose **starting point** for the heuristic, ex. $p^* = (\frac{p_i^L + p_i^U}{2})_{i \in C}$.
2. $o^* =$ `compute_objective_function`(p^*).
3. Set $j = 1$.
4. **Fix the bounds** to be $= p^*$ **except for alternative** j and solve this simplified problem using the BEA / BEAC. If the resulting $(\hat{p}, \hat{o})$ is better than (p^*, o^*), set $p^* = \hat{p}, o^* = \hat{o}$.
6. Set j = j + 1 and repeat from step 4. If $j = J$ then set $j = 1$.
7. **Terminate** after **no improvement** is found **over** J **iterations**.

... i.e., a **coordinate descent** (ascent).

BHA extended via dynamic line search (DLS)

Idea: escape local optima by adding small controlled perturbations in starting point.

- Base point = solution from BHA.
- **Change one coordinate** by δ and **start BHA from there**.
- If the BHA converges to a better solution than the best known, make it the **new base point**.
- Iterate through all coordinates.
- **Gradually increase the search distance** δ along each direction until a maximal step size is reached.

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Case Study

Parking space operator [Ibeas et al., 2014]

- **Alternatives**: Paid-Street-Parking (PSP), Paid-Underground-Parking (PUP) and Free-Street-Parking (FSP).
- Optimize prices for PSP and PUP, FSP is the **opt-out** alternative.
- **Socio-economic characteristics**: trip origin, vehicle age, driver income, residence area.
- **Product attributes**: access time to parking, access time to destination, and parking fee (price).
- Choice model is a **Mixed Logit**, $\beta_{\text{fee}}, \beta_{\text{time\_parking}} \sim \mathcal{N}(\mu, \sigma)$.

Results

Table 1: Test 1: MILP vs. BEAC in the capacitated case

			MILP		BEAC	
N	R	J	Time (s)	Profit	Time (s)	Profit
50	2	2	4.17	27.61	0.43	27.61
50	5	2	46.95	26.51	1.72	26.51
50	10	2	180.85	27.06	11.42	27.06
50	25	2	3119.66	27.08	169.08	27.08
50	50	2	$>$5 hours	$\geq$25.15	1272.68	26.85
50	100	2	$>$25 hours	$\geq$25.11	9928.57	26.85
50	250	2	$>$45 hours	$\geq$23.45	$>$45 hours	$\geq$25.00

Results

Table 2: Test 3: BHA and DLS vs. B&BD and BEA in the uncapacitated case

			B&BD		BEA		BHA		DLS	
N	R	J	Time (s)	Profit	Time (s)	Profit	Time (s)	Profit	Time (s)	Profit
20	100	4	12478	10.40	>24 hours	≥9.81	0.00	10.40	0.14	10.40
20	200	4	29213	10.40	>24 hours	≥10.40	0.01	10.40	0.41	10.40
20	300	4	>24 hours	≥10.38	>24 hours	≥10.13	0.02	10.24	0.64	10.24
20	400	4	>24 hours	≥9.81	>24 hours	≥9.42	0.05	10.26	0.78	10.26
20	500	4	>24 hours	≥10.01	>24 hours	≥9.67	0.13	10.24	1.37	10.24

Results

Table 3: Test 4: BHA and DLS vs. MILP and BEAC in the capacitated case

			MILP		BEAC		BHA		DLS	
N	R	J	Time (s)	Profit	Time (s)	Profit	Time (s)	Profit	Time (s)	Profit
50	2	2	4.17	27.61	0.43	27.61	0.22	27.61	1.03	27.61
50	5	2	46.95	26.51	1.72	26.51	0.32	26.46	5.91	26.51
50	10	2	180.85	27.06	11.42	27.06	0.58	27.05	20.34	27.06
50	25	2	3119.66	27.08	169.08	27.08	3.40	27.05	129.66	27.08
50	50	2	$>$5 hours	$\geq$25.15	1272.68	26.85	8.31	26.53	559.04	26.85
50	100	2	$>$25 hours	$\geq$25.11	9928.57	26.85	51.77	26.72	2791.28	26.85
50	250	2	$>$45 hours	$\geq$23.45	$>$45 hours	$\geq$25.00	455.37	26.66	15867.67	26.71
50	10	4	$>$10 hours	$\geq$22.21	$>$10 hours	$\geq$25.41	7.08	26.78	527.34	26.83
50	50	4	$>$20 hours	$\geq$22.19	$>$20 hours	$\geq$27.00	166.21	27.00	7234.88	27.00
50	100	4	$>$45 hours	$\geq$20.50	$>$45 hours	$\geq$24.86	866.97	26.67	34050.57	26.67
50	200	4	$>$72 hours	$\geq$20.32	$>$72 hours	$\geq$24.79	2762.39	26.70	106286.13	26.70

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Conclusions

- **Exact method:** BEAC $\approx$20 times faster than MILP
- **Heuristic:** BHA $\approx$100-5000 times faster than MILP with capacities
- **Heuristic:** BHA several orders of magnitudes times faster than MILP and B&BD without capacities
- BHA **optimality gaps** $< 0.2\%$.
- DLS finds **global optimum**, but **too inefficient**. Alternatives should be explored.

Thank you for your attention!

Appendix

Table 4: Utility parameters reported in [Ibeas et al., 2014]

Parameter	Value
ASC_{FSP}	0.0
ASC_{PSP}	32.0
ASC_{PUP}	34.0
Fee (€)	$\sim \mathcal{N}(-32.328, 14.168)$
Fee PSP - low income (€)	-10.995
Fee PUP - low income (€)	-13.729
Fee PSP - resident (€)	-11.440
Fee PUP - resident (€)	-10.668
Access time to parking (min)	$\sim \mathcal{N}(-0.788, 1.06)$
Access time to destination (min)	-0.612
Age of vehicle (1/0)	4.037
Origin (1/0)	-5.762

MILP formulation [Paneque et al., 2021]

$$\max_{p,\omega,U,h} \frac{1}{R} \sum_{r\in\mathcal{R}} \sum_{n\in\mathcal{N}} \sum_{i\in C_n} p_i \omega_{inr} \qquad (o)$$

s.t.

$$\sum_{i\in C_n\cup\{0\}} \omega_{inr} = 1 \qquad \forall n\in\mathcal{N}, r\in\mathcal{R} \quad (\mu_{nr})$$

$$h_{nr} = c_{0nr}\omega_{0nr} + \sum_{i\in C_n} U_{inr}\omega_{inr} \qquad \forall n\in\mathcal{N}, r\in\mathcal{R} \quad (\zeta_{nr})$$

$$h_{nr} \geq c_{0nr} \qquad \forall n\in\mathcal{N}, r\in\mathcal{R} \quad (\alpha_{0nr})$$

$$h_{nr} \geq U_{inr} \qquad \forall i\in C_n, n\in\mathcal{N}, r\in\mathcal{R} \quad (\alpha_{inr})$$

$$U_{inr} = c_{inr} + \beta_p^{in} p_i \qquad \forall i\in C_n, n\in\mathcal{N}, r\in\mathcal{R} \quad (\kappa_{inr})$$

$$\omega \in \{0,1\}^{(J+1)NR}$$

$$p \in [p_1^L, p_1^U] \times \ldots \times [p_J^L, p_J^U]$$

$$U, h \in \mathbb{R}^{JNR}, \mathbb{R}^{NR}$$

Breakpoint Exact Algorithm (BEA) [Haering et al., 2023]

Algorithm 1: Breakpoint Exact Algorithm (BEA) to solve the CPP

Result: optimal solution p^* and profit o^* for CPP.

$p_j^* \leftarrow 0 \quad \forall j \in \{1, \ldots, J\}$

$o^* \leftarrow 0$

for s **in** S **do**

- $p_{s_j} \leftarrow 0 \quad \forall j \in \{1, \ldots, J\}$
- $h_{nr}^{s_1} \leftarrow c_{0nr} \quad \forall (n, r) \in \mathcal{N} \times \mathcal{R}$
- $\eta_{nr} \leftarrow 0 \quad \forall (n, r) \in \mathcal{N} \times \mathcal{R}$
- $(\hat{p}, \hat{o}) \leftarrow \texttt{enumerate}(s, p, h^{s_1}, \eta, 1)$
- **if** $\hat{o} > o^*$ **then**
  - $p^* \leftarrow \hat{p}$;
  - $o^* \leftarrow \hat{o}$;
- **end**

end

return (p^*, o^*)

Recursive enumeration function within BEA

Algorithm 2: Recursive enumeration function within BEA

Function enumerate(s, p, h^{s_j}, η, j):

$p_{s_j} \leftarrow p^U_{s_j}$

$\bar{p}^{nr}_{s_j} \leftarrow \frac{h^{s_j}_{nr} - c_{s_j nr}}{\beta^{s_j n}_p} \quad \forall (n, r) \in \mathcal{N} \times \mathcal{R}$

$\mathcal{N}_1 \leftarrow \{(n, r) | \bar{p}^{nr}_{s_j} \geq p^U_{s_j}\}$

$\mathcal{N}_2 \leftarrow \{(n, r) | \max\{p^L_{s_j}, p_{s_{j-1}}\} < \bar{p}^{nr}_{s_j} < p^U_{s_j}\}$

$h^{s_{j+1}}_{nr} \leftarrow h^{s_j}_{nr} \quad \forall (n, r) \in \mathcal{N} \times \mathcal{R} \setminus \mathcal{N}_1$

$h^{s_{j+1}}_{nr} \leftarrow c_{s_j nr} + \beta^{s_j n}_p p^U_{s_j} \quad \forall (n, r) \in \mathcal{N}_1$

$\eta_{nr} \leftarrow p^U_{s_j} \quad \forall (n, r) \in \mathcal{N}_1$

$o^* \leftarrow \sum_{n \in \mathcal{N}} \sum_{r \in \mathcal{R}} \eta_{nr}$

$p^* \leftarrow p$

Sort the elements of $\mathcal{N}_2$ so that $\bar{p}^{n_1 r_1}_{s_j} \geq \bar{p}^{n_2 r_2}_{s_j} \geq \cdots \geq \bar{p}^{n_{|\mathcal{N}_2|} r_{|\mathcal{N}_2|}}_{s_j}$

if $j \leq J - 1$ **then**

  for $i \in \{1, \ldots, |\mathcal{N}_2|\}$ **do**

    $p_{s_j} \leftarrow \bar{p}^{n_i r_i}_{s_j}$

    $h^{s_{j+1}}_{nr} \leftarrow c_{s_j nr} + \beta^{s_j n}_p p_{s_j} \quad \forall (n, r) \in \{(n_1, r_1), \ldots, (n_i, r_i)\} \cup \mathcal{N}_1$

    $\eta_{nr} \leftarrow p_{s_j} \quad \forall (n, r) \in \{(n_1, r_1), \ldots, (n_i, r_i)\} \cup \mathcal{N}_1$

    $(\hat{p}, \hat{o}) \leftarrow$ enumerate$(s, p, h^{s_{j+1}}, \eta, j + 1)$

    if $\hat{o} > o^*$ **then**

      $o^* \leftarrow \hat{o}$

      $p^* \leftarrow \hat{p}$

    end

  end

end

else

  $\tilde{o} \leftarrow o^* - \sum_{(n,r) \in \mathcal{N}_1} \eta_{nr}$

  for $i \in \{1, \ldots, |\mathcal{N}_2|\}$ **do**

    $\tilde{o} \leftarrow \tilde{o} - \eta_{n_i r_i}$

    $p_{s_j} \leftarrow \bar{p}^{n_i r_i}_{s_j}$

    $o \leftarrow \tilde{o} + (|\mathcal{N}_1| + i) p_{s_j}$

    if $o > o^*$ **then**

      $o^* \leftarrow o$

      $p^* \leftarrow p$

    end

  end

  return (p^*, o^*)

end

end

Capacity constraints

$$\omega_{inr} \leq y_{inr} \qquad \forall i \in C_n, \in \mathcal{N}, r \in \mathcal{R}$$

$$\sum_{m=1}^{n} \omega_{imr} \leq (c_i - 1)y_{inr} + (n-1)(1 - y_{inr}) \qquad \forall i \in C_n, n > c_i \in \mathcal{N}, r \in \mathcal{R}$$

$$\sum_{m=1}^{n} \omega_{imr} \geq c_i(1 - y_{inr}) \qquad \forall i \in C_n, n > 1 \in \mathcal{N}, r \in \mathcal{R}$$

Compute Objective Value with Priority Queue

Function `compute_objective_value_with_priority_queue`($p, c, prio\_queue$):

```
    ς ← (0)_{i∈C}
    for idx ∈ prio_queue do
        u ← [U^i_{idx} for i ∈ C]
        a ← sort(u, descending)
        φ ← false
        j ← 1
        while j ≤ C − 1 and !φ do
            if ς_{a_j} ≤ c_{a_j} − 1 then
                ς_{a_j} += 1
                φ ← true
            end
            else
                j += 1
            end
        end
    end
    o ← Σ_{i∈C} ς_i · p_i
    return o
end
```

Compute Objective Value with Capacities (profit max/min)

Function `compute_objective_value_with_capacities`($p, c; max$):
- $s \leftarrow \text{sortperm}(p)$
- $\varsigma \leftarrow (0)_{i \in C}$
- $A \leftarrow \{\}$
- **for** $idx \in \mathcal{N} \times \mathcal{R}$ **do**
  - $u \leftarrow [U^i_{idx} \text{ for } i \in C]$
  - $a \leftarrow \text{sort}(u, \text{descending})$
  - $A \leftarrow A \cup \{a\}$
- **if** max **then**
  - $A \leftarrow \text{sort}(A, \text{ascending})$
- **else**
  - $A \leftarrow \text{sort}(A, \text{descending})$
- **while** $|A| \geq 1$ **do**
  - $\pi \leftarrow A_{11}$
  - $A \leftarrow A \setminus \{A_1\}$
  - **if** $\pi \geq 1$ **then**
    - $\varsigma_{s_{next\_pref}} \mathrel{+}= 1$
    - **if** $\varsigma_{s_{next\_pref}} = c_{s_{next\_pref}}$ **then**
      - Remove all entries π from A
      - **if** max **then**
        - $A \leftarrow \text{sort}(A, \text{ascending})$
      - **else**
        - $A \leftarrow \text{sort}(A, \text{descending})$
- $o \leftarrow \sum_{i \in C} \varsigma_i \cdot p_i$
- **return** o

Results

Table 5: Test 2: Priority queue vs. Max profit vs. Robust Optimization

			BEAC		BEAC-M		BEAC-R	
N	R	J	Time (s)	Profit	Time (s)	Profit	Time (s)	Profit
50	2	2	0.43	27.61	0.44	28.81	0.45	27.61
50	5	2	1.72	26.51	1.78	28.44	1.82	26.46
50	10	2	11.42	27.06	12.88	28.3	12.98	27.01
50	25	2	169.08	27.08	197.23	28.58	189.28	27.06
50	50	2	1272.68	26.85	1513.44	28.61	1523.89	26.85
50	100	2	9928.57	26.85	12093.8	28.57	12494.13	26.85
50	250	2	$>$45 hours	$\geq$25.00	$>$45 hours	$\geq$26.63	$>$45 hours	$\geq$24.34

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