Human Behavior and Optimization

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Outline

Motivation

Human dimension in

- engineering
- business
- marketing
- planning
- policy making

Need for

- behavioral *theories*
- quantitative *methods*
- operational mathematical *models*

Motivation

Concept of demand

- marketing
- transportation
- energy
- finance

Concept of choice

- brand, product
- mode, destination
- type, usage
- buy/sell, product

In this lecture...

Focus

- Individual behavior (vs. aggregate behavior)
- Theory of behavior which is
  - descriptive (how people behave) and not normative (how they should behave)
  - general: not too specific
  - operational: can be used in practice for forecasting
- Type of behavior: choice

In this lecture...

Microeconomics
in a nutshell

In this lecture...

Microeconomics in a nutshell $\longrightarrow$ Logit and MEV models

In this lecture...

Microeconomics in a nutshell → Logit and MEV models → Profit maximization, facility location

In this lecture...

$$
\begin{array}{ccc}
\text{Microeconomics in a nutshell} & \longrightarrow & \text{Logit and MEV models} \\
\text{Profit maximization, facility location} & & \text{Activity-based models}
\end{array}
$$

In this lecture...

Optimization

Microeconomics in a nutshell

Logit and MEV models

Profit maximization, facility location

Activity-based models

In this lecture...

Optimization

Discrete choice

Microeconomics in a nutshell $\longrightarrow$ Logit and MEV models

Profit maximization, facility location

Activity-based models

In this lecture...

Optimization

Discrete choice

Tutorial

Microeconomics in a nutshell

Logit and MEV models

Profit maximization, facility location

Activity-based models

In this lecture...

	Optimization	**Discrete choice**
Tutorial	Microeconomics in a nutshell	Logit and MEV models
Research	Profit maximization, facility location	Activity-based models

Outline

Decision rule

Homo economicus

Rational and narrowly self-interested economic actor who is optimizing her outcome

Utility

$$U_n : \mathcal{C}_n \longrightarrow \mathbb{R} : a \rightsquigarrow U_n(a)$$

- captures the attractiveness of an alternative
- measure that the decision maker wants to optimize

Behavioral assumption

- the decision maker associates a utility with each alternative
- the decision maker is a perfect optimizer
- the alternative with the highest utility is chosen

Microeconomic consumer theory

Continuous choice set

- Consumption bundle

$$Q = \begin{pmatrix} q_1 \\ \vdots \\ q_L \end{pmatrix} ; p = \begin{pmatrix} p_1 \\ \vdots \\ p_L \end{pmatrix}$$

- Budget constraint

$$p^T Q = \sum_{\ell=1}^{L} p_\ell q_\ell \leq I.$$

- No attributes, just quantities

Preferences

Operators $\succ$, $\sim$, and $\succsim$

- $Q_a \succ Q_b$: Q_a is preferred to Q_b,
- $Q_a \sim Q_b$: indifference between Q_a and Q_b,
- $Q_a \succsim Q_b$: Q_a is at least as preferred as Q_b.

Rationality

- Completeness: for all bundles a and b,

$$Q_a \succ Q_b \text{ or } Q_a \prec Q_b \text{ or } Q_a \sim Q_b.$$

- Transitivity: for all bundles a, b and c,

$$\text{if } Q_a \succsim Q_b \text{ and } Q_b \succsim Q_c \text{ then } Q_a \succsim Q_c.$$

- "Continuity": if Q_a is preferred to Q_b and Q_c is arbitrarily "close" to Q_a, then Q_c is preferred to Q_b.

Utility

Utility function

- Parameterized function:

$$\widetilde{U} = \widetilde{U}(q_1, \ldots, q_L; \theta) = \widetilde{U}(Q; \theta)$$

- Consistent with the preference indicator:

$$\widetilde{U}(Q_a; \theta) \geq \widetilde{U}(Q_b; \theta)$$

  is equivalent to

$$Q_a \succsim Q_b.$$

- Unique up to an order-preserving transformation

Optimization

Optimization problem

$$\max_{Q} \widetilde{U}(Q;\theta)$$

subject to

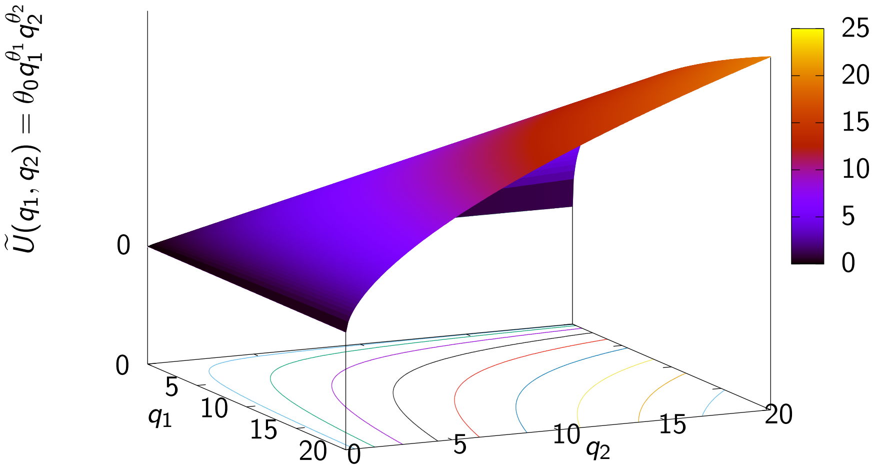
$$p^T Q \leq I,\ \ Q \geq 0.$$

Demand function

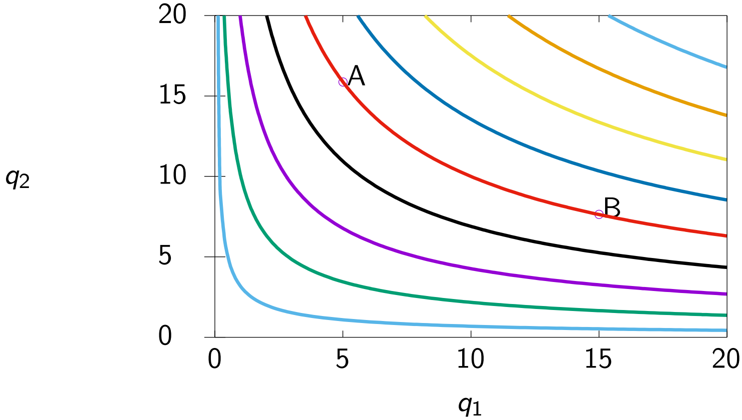
- Solution of the optimization problem
- Quantity as a function of prices and budget

$$Q^* = f(I, p; \theta)$$

Example: Cobb-Douglas



Example



Example

Optimization problem

$$\max_{q_1,q_2} \widetilde{U}(q_1, q_2; \theta_0, \theta_1, \theta_2) = \theta_0 q_1^{\theta_1} q_2^{\theta_2}$$

subject to

$$p_1 q_1 + p_2 q_2 = I.$$

Lagrangian of the problem:

$$L(q_1, q_2, \lambda) = \theta_0 q_1^{\theta_1} q_2^{\theta_2} + \lambda(I - p_1 q_1 - p_2 q_2).$$

Necessary optimality condition

$$\nabla L(q_1, q_2, \lambda) = 0$$

Example

Necessary optimality conditions

$$
\begin{array}{rcrcll}
\theta_0\theta_1 q_1^{\theta_1-1} q_2^{\theta_2} & - & \lambda p_1 & = & 0 & (\times q_1) \\
\theta_0\theta_2 q_1^{\theta_1} q_2^{\theta_2-1} & - & \lambda p_2 & = & 0 & (\times q_2) \\
p_1 q_1 + p_2 q_2 & - & I & = & 0. &
\end{array}
$$

We have

$$
\begin{array}{rcrcl}
\theta_0\theta_1 q_1^{\theta_1} q_2^{\theta_2} & - & \lambda p_1 q_1 & = & 0 \\
\theta_0\theta_2 q_1^{\theta_1} q_2^{\theta_2} & - & \lambda p_2 q_2 & = & 0.
\end{array}
$$

Adding the two and using the third condition, we obtain

$$\lambda I = \theta_0 q_1^{\theta_1} q_2^{\theta_2} (\theta_1 + \theta_2)$$

or, equivalently,

$$\theta_0 q_1^{\theta_1} q_2^{\theta_2} = \frac{\lambda I}{(\theta_1 + \theta_2)}$$

Solution

From the previous derivation

$$\theta_0 q_1^{\theta_1} q_2^{\theta_2} = \frac{\lambda I}{(\theta_1 + \theta_2)}$$

First condition

$$\theta_0 \theta_1 q_1^{\theta_1} q_2^{\theta_2} = \lambda p_1 q_1.$$

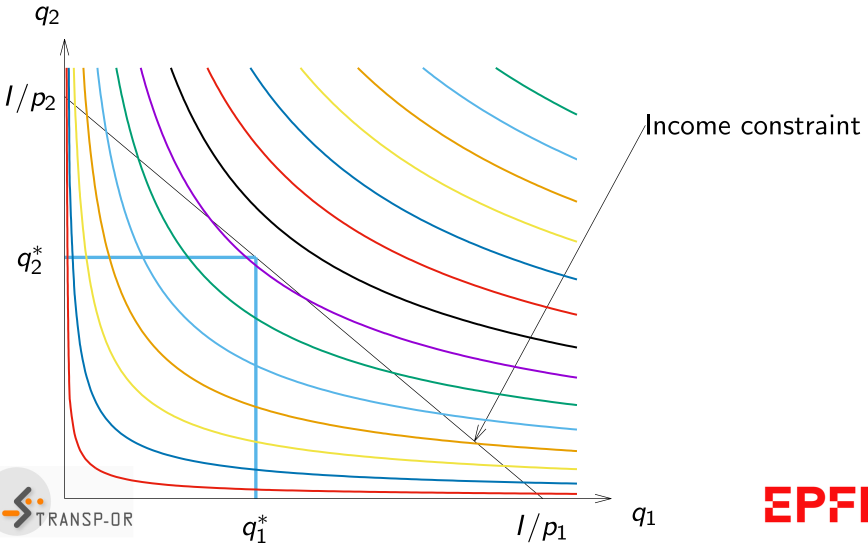
Solve for q_1

$$q_1^* = \frac{I\theta_1}{p_1(\theta_1 + \theta_2)}$$

Similarly, we obtain

$$q_2^* = \frac{I\theta_2}{p_2(\theta_1 + \theta_2)}$$

Optimization problem



Demand functions

Product 1

$$q_1^* = \frac{I}{p_1}\frac{\theta_1}{\theta_1+\theta_2}$$

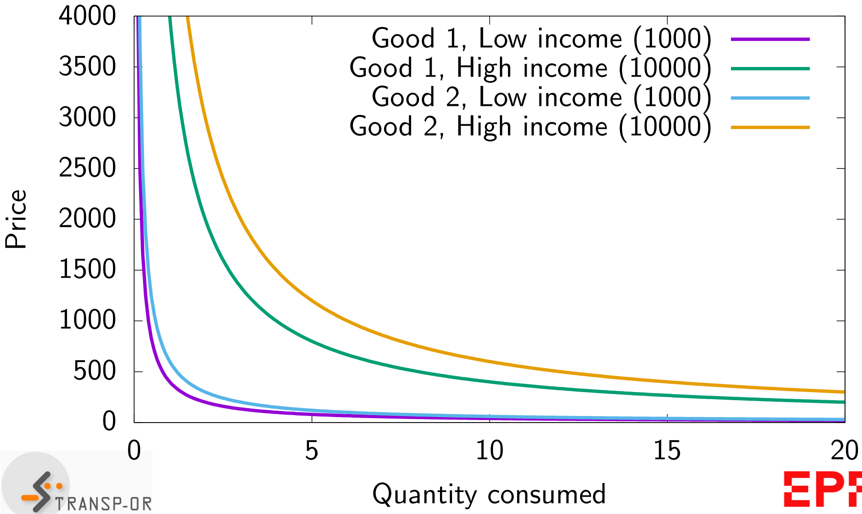
Product 2

$$q_2^* = \frac{I}{p_2}\frac{\theta_2}{\theta_1+\theta_2}$$

Comments

- Demand decreases with price
- Demand increases with budget
- Demand independent of θ_0, which does not affect the ranking
- Property of Cobb Douglas: the demand for a good is only dependent on its own price and independent of the price of any other good.

Demand curve (inverse of demand function)



Indirect utility

Substitute the demand function into the utility

$$U(I, p; \theta) = \theta_0 \left(\frac{I}{p_1} \frac{\theta_1}{\theta_1 + \theta_2} \right)^{\theta_1} \left(\frac{I}{p_2} \frac{\theta_2}{\theta_1 + \theta_2} \right)^{\theta_2}$$

Indirect utility

Maximum utility that is achievable for a given set of prices and income

In discrete choice...

- only the indirect utility is used
- therefore, it is simply referred to as "utility"

Microeconomic theory of discrete goods

Expanding the microeconomic framework

- Continuous goods
- and discrete goods

The consumer

- selects the quantities of continuous goods: $Q = (q_1, \ldots, q_L)$
- chooses an alternative in a discrete choice set $i = 1, \ldots, j, \ldots, J$
- discrete decision vector: $(y_1, \ldots, y_J)$, $y_j \in \{0, 1\}$, $\sum_j y_j = 1$.

Note

- In theory, one alternative of the discrete choice combines all possible choices made by an individual.
- In practice, the choice set will be more restricted for tractability

Utility maximization

Utility

$$\widetilde{U}(Q, y, \tilde{z}^T y; \theta)$$

- Q: quantities of the continuous good
- y: discrete choice
- $\tilde{z}^T = (\tilde{z}_1, \ldots, \tilde{z}_i, \ldots, \tilde{z}_J) \in \mathbb{R}^{K \times J}$: K attributes of the J alternatives
- $\tilde{z}^T y \in \mathbb{R}^K$: attributes of the chosen alternative
- θ: vector of parameters

Utility maximization

Optimization problem

$$\max_{Q,y} \widetilde{U}(Q, y, \tilde{z}^T y; \theta)$$

subject to

$$\begin{array}{l} p^T Q + c^T y \leq I \\ \sum_j y_j = 1 \\ y_j \in \{0, 1\}, \forall j. \end{array}$$

where $c^T = (c_1, \ldots, c_i, \ldots, c_J)$ contains the cost of each alternative.

Solving the problem

- Mixed integer optimization problem
- No optimality condition
- Impossible to derive demand functions directly

Solving the problem

Step 1: condition on the choice of the discrete good

- Fix the discrete good, that is select a feasible y.
- The problem becomes a continuous problem in Q.
- Conditional demand functions can be derived:

$$q_{\ell|y} = f(I - c^T y, p, \tilde{z}^T y; \theta),$$

or, equivalently, for each alternative i,

$$q_{\ell|i} = f(I - c_i, p, \tilde{z}_i; \theta).$$

- $I - c_i$ is the income left for the continuous goods, if alternative i is chosen.
- If $I - c_i < 0$, alternative i is declared unavailable and removed from the choice set.

Solving the problem

Conditional indirect utility functions

Substitute the demand functions into the utility:

$$U_i = U(I - c_i, p, \tilde{z}_i; \theta) \text{ for all } i \in \mathcal{C}.$$

Step 2: Choice of the discrete good

$$\max_y \; U(I - c^T y, p, \tilde{z}^T y; \theta)$$

- Enumerate all alternatives.
- Compute the conditional indirect utility function U_i.
- Select the alternative with the highest U_i.
- Note: no income constraint anymore.

Simple example: mode choice

Attributes

	Attributes	
Alternatives	Travel time (t)	Travel cost (c)
Car (1)	t_1	c_1
Bus (2)	t_2	c_2

Utility

$$\widetilde{U} = \widetilde{U}(y_1, y_2),$$

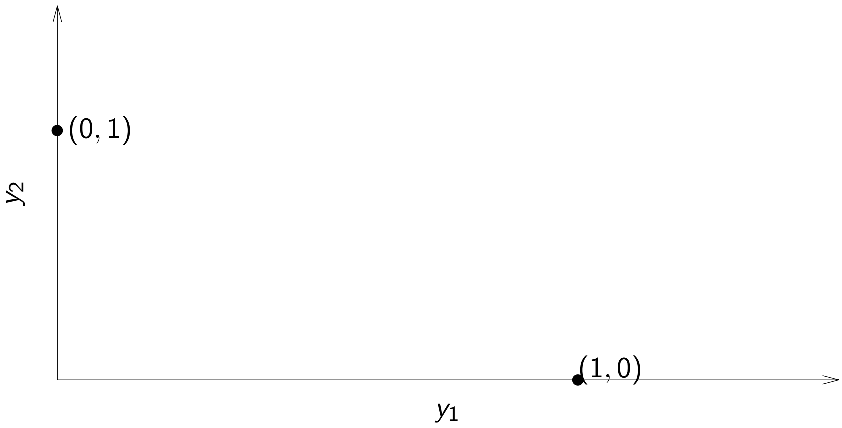
where we impose the restrictions that, for $i = 1, 2$,

$$y_i = \begin{cases} 1 & \text{if travel alternative i is chosen,} \\ 0 & \text{otherwise;} \end{cases}$$

and that only one alternative is chosen: $y_1 + y_2 = 1$.

Simple example: mode choice

Choice set



Simple example: mode choice

Utility functions

$$\begin{aligned} U_1 &= -\beta_t t_1 - \beta_c c_1, \\ U_2 &= -\beta_t t_2 - \beta_c c_2, \end{aligned}$$

where $\beta_t > 0$ and $\beta_c > 0$ are parameters.

Equivalent specification

$$\begin{aligned} U_1 &= -(\beta_t/\beta_c) t_1 - c_1 &= -\beta t_1 - c_1 \\ U_2 &= -(\beta_t/\beta_c) t_2 - c_2 &= -\beta t_2 - c_2 \end{aligned}$$

where $\beta > 0$ is a parameter.

Choice

- Alternative 1 is chosen if $U_1 \geq U_2$.
- Ties are ignored.

Simple example: mode choice

Choice

Alternative 1 is chosen if

$$-\beta t_1 - c_1 \geq -\beta t_2 - c_2$$

or

$$-\beta(t_1 - t_2) \geq c_1 - c_2$$

Alternative 2 is chosen if

$$-\beta t_1 - c_1 \leq -\beta t_2 - c_2$$

or

$$-\beta(t_1 - t_2) \leq c_1 - c_2$$

Dominated alternative

- If $c_2 > c_1$ and $t_2 > t_1$, $U_1 > U_2$ for any $\beta > 0$
- If $c_1 > c_2$ and $t_1 > t_2$, $U_2 > U_1$ for any $\beta > 0$

Simple example: mode choice

Trade-off

- Assume $c_2 > c_1$ and $t_1 > t_2$.
- Is the traveler willing to pay the extra cost $c_2 - c_1$ to save the extra time $t_1 - t_2$?
- Alternative 2 is chosen if

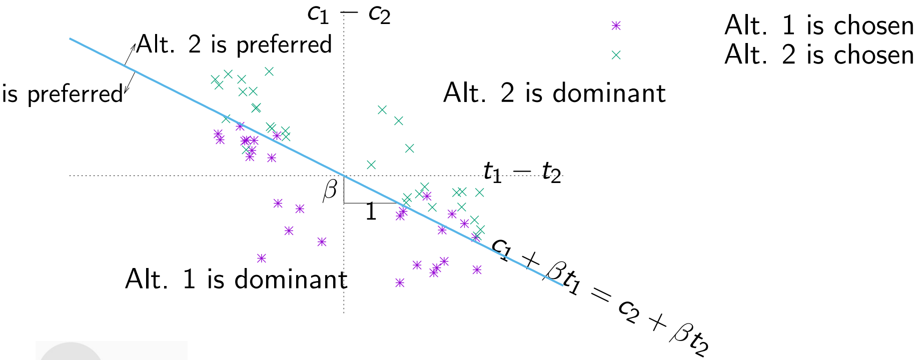
$$-\beta(t_1 - t_2) \leq c_1 - c_2$$

  or

$$\beta \geq \frac{c_2 - c_1}{t_1 - t_2}$$

- β is called the *willingness to pay* or *value of time*

Simple example: mode choice



Behavioral validity of the utility maximization?

Assumptions

Decision-makers

- are able to process information
- have perfect discrimination power
- have transitive preferences
- are perfect maximizer
- are always consistent

Relax the assumptions

Use a probabilistic approach: what is the probability that alternative i is chosen?

Random utility model

Probability model

$$P(i|\mathcal{C}_n) = \Pr(U_{in} \geq U_{jn}, \forall j \in \mathcal{C}_n),$$

Random utility

$$U_{in} = V_{in} + \varepsilon_{in} = \beta^T X_{in} + \varepsilon_{in}.$$

Similarity with linear regression

$$Y = \beta^T X + \varepsilon$$

Here, U is not observed. Only the choice is observed.

Derivation

Joint distributions of ε_n

- Assume that $\varepsilon_n = (\varepsilon_{1n}, \ldots, \varepsilon_{J_n n})$ is a multivariate random variable
- with CDF

$$F_{\varepsilon_n}(\varepsilon_1, \ldots, \varepsilon_{J_n})$$

- and pdf

$$f_{\varepsilon_n}(\varepsilon_1, \ldots, \varepsilon_{J_n}) = \frac{\partial^{J_n} F}{\partial \varepsilon_1 \cdots \partial \varepsilon_{J_n}}(\varepsilon_1, \ldots, \varepsilon_{J_n}).$$

The random utility model: $P_n(i|\mathcal{C}_n) =$

$$\int_{\varepsilon=-\infty}^{+\infty} \frac{\partial F_{\varepsilon_{1n},\varepsilon_{2n},\ldots,\varepsilon_{J_n}}}{\partial \varepsilon_i}(\ldots, V_{in} - V_{(i-1)n} + \varepsilon, \varepsilon, V_{in} - V_{(i+1)n} + \varepsilon, \ldots) d\varepsilon$$

Random utility model

- The general formulation is complex.
- We can derive specific models based on simple assumptions.

Outline

Road map

Optimization → **Discrete choice**

Microeconomics in a nutshell → Logit and MEV models

Error term

Random utility

$$U_{in} = V_{in} + \varepsilon_{in}.$$

Assumptions about the distribution

- Probit: central limit theorem: the sum of many i.i.d. random variables approximately follows a normal distribution.
- Logit: Gumbel theorem: the maximum of many i.i.d. random variables approximately follows an Extreme Value distribution: EV(η, μ).

The Extreme Value distribution EV(η, μ)

Probability density function (pdf)

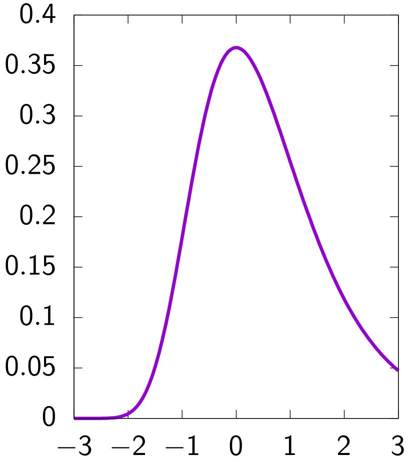
$$f(t) = \mu e^{-\mu(t-\eta)} e^{-e^{-\mu(t-\eta)}}$$

Cumulative distribution function (CDF)

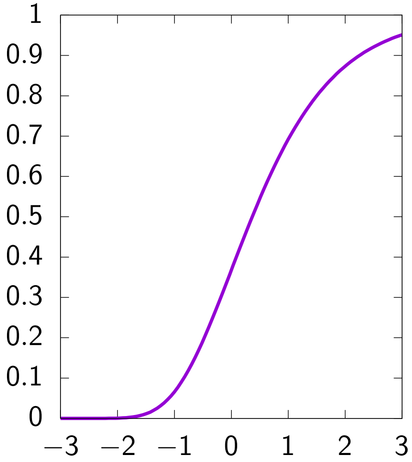
$$\begin{aligned} P(c \geq \varepsilon) = F(c) &= \int_{-\infty}^{c} f(t) dt \\ &= e^{-e^{-\mu(c-\eta)}} \end{aligned}$$

The Extreme Value distribution

pdf EV(0,1)



CDF EV(0,1)



The Extreme Value distribution

Properties

If

$$\varepsilon \sim \text{EV}(\eta, \mu)$$

then

$$\text{E}[\varepsilon] = \eta + \frac{\gamma}{\mu} \quad \text{and} \quad \text{Var}[\varepsilon] = \frac{\pi^2}{6\mu^2}$$

where γ is Euler's constant.

Euler's constant

$$\gamma = \lim_{k \to \infty} \sum_{i=1}^{k} \frac{1}{i} - \ln k = -\int_0^{\infty} e^{-x} \ln x dx \approx 0.5772$$

Logit model

The random utility model: $P_n(i|\mathcal{C}_n) =$

$$\int_{\varepsilon=-\infty}^{+\infty} \frac{\partial F_{\varepsilon_{1n},\varepsilon_{2n},\ldots,\varepsilon_{J_n}}}{\partial \varepsilon_i}(\ldots, V_{in} - V_{(i-1)n} + \varepsilon, \varepsilon, V_{in} - V_{(i+1)n} + \varepsilon, \ldots) d\varepsilon$$

CDF

Assumption: i.i.d. EV distributions:

$$F_{\varepsilon_n}(\varepsilon_1, \ldots, \varepsilon_{J_n}) = \prod_{i=1}^{J_n} e^{-e^{-\mu\varepsilon_i}}.$$

Logit model

$$P_n(i|\mathcal{C}_n) = \frac{y_{in} e^{V_{in}}}{\sum_{j=1}^{J} y_{jn} e^{V_{jn}}}.$$

Logit model

$$U_{in} = V_{in} + \varepsilon_{in}.$$

Why "logit"?

If U_{in} and U_{jn} are EV distributed, $U_{in} - U_{jn}$ follows a logistic distribution.

Availability of alternatives

$$y_{in} = \begin{cases} 1 & \text{if } i \in \mathcal{C}_n, \\ 0 & \text{otherwise.} \end{cases}$$

$y_{in=1}$ if alternative i is available to individual n.

Logit model

Expected maximum utility

If ε_{in}, $i = 1, \ldots, J_n$ are i.i.d. $EV(0, \mu)$, then

$$\mathrm{E}[\max_i U_{in}] = \frac{1}{\mu} \ln \sum_{i=1}^{J} y_{jn} e^{\mu V_{in}}.$$

Example

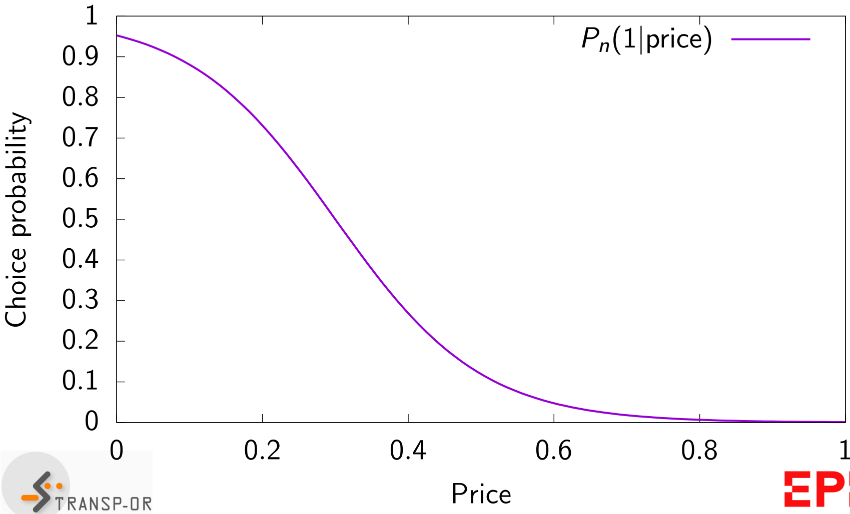
Two alternatives

$$V_{0n} = 0$$
$$V_{1n} = -10 * \text{price} + 3$$

Choice probability

$$P_n(1|\text{price}) = \frac{e^{-10*\text{price}+3}}{e^0 + e^{-10*\text{price}+3}} = \frac{e^{-10*\text{price}+3}}{1 + e^{-10*\text{price}+3}}$$

Example



$P_n(1|\text{price})$
Choice probability
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
Price

Beyond logit

- Other distributional assumptions can be used.
- Logit is not always consistent with observed behavior.
- Trade-off between model complexity and behavioral realism.
- Example: Multivariate Extreme Value models

MEV models

Definition

$$\varepsilon_n = (\varepsilon_{1n}, \ldots, \varepsilon_{Jn})$$

follows a multivariate extreme value distribution if it has the CDF

$$F_{\varepsilon_n}(\varepsilon_{1n}, \ldots, \varepsilon_{Jn}) = e^{-G(e^{-\varepsilon_{1n}}, \ldots, e^{-\varepsilon_{Jn}})},$$

where $G : \mathbb{R}_+^{J_n} \to \mathbb{R}_+$ is a positive function with positive arguments, that must verify some properties.

MEV models

Choice model

$$P_n(i) = \frac{e^{V_{in} + \ln G_i\left(e^{V}\right)}}{\sum_j e^{V_{jn} + \ln G_j\left(e^{V}\right)}}.$$

Expected maximum utility

$$\mathrm{E}[\max_{j \in \mathcal{C}_n} U_{jn}] = \frac{1}{\mu}(\log G(e^{V_{1n}}, \ldots, e^{V_{J_n n}}) + \gamma),$$

where γ is Euler's constant

Road map

Optimization

Microeconomics
in a nutshell

Discrete choice

Logit and
MEV models

Profit maximization,
facility location

Outline

A simple example

Data

- $\mathcal{C}$: set of movies
- Population of N individuals
- Utility function:
  $U_{in} = \beta_{in} p_{in} + f(z_{in}) + \varepsilon_{in}$

Decision variables

- What movies to propose? y_{in}
- What price? p_{in}

Profit maximization

Data

- Two alternatives: my theater (m) and the competition (c)
- We assume an homogeneous population of N individuals

$$U_{cn} = 0 + \varepsilon_{cn}$$
$$U_{mn} = \beta_n p_m + c_{mn} + \varepsilon_{mn}$$

- $\beta_n < 0$
- Logit model: ε_{mn} i.i.d. EV

Heterogeneous population

Two groups in the population

$$U_{mn} = \beta_n p_m + c_{mn} + \varepsilon_{mn}$$

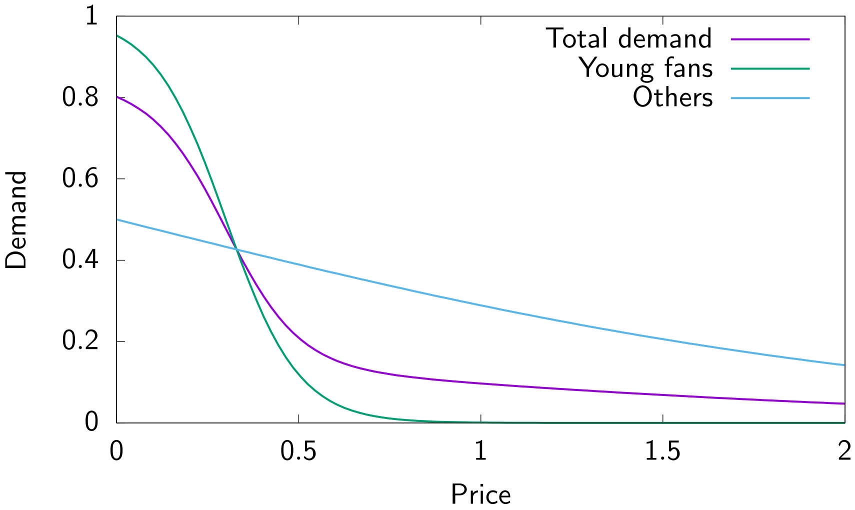
$n = 1$: Young fans: 2/3

$\beta_1 = -10$, $c_{m1} = 3$

$n = 2$: Others: 1/3

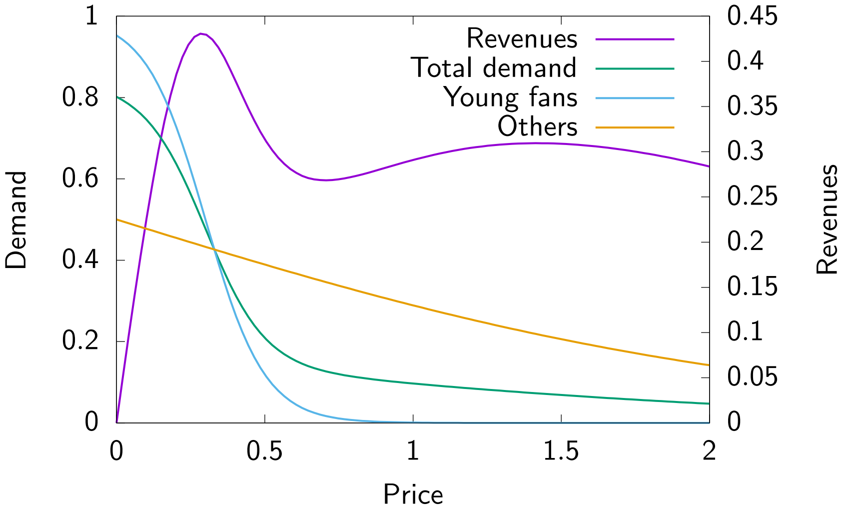
$\beta_1 = -0.9$, $c_{1m} = 0$

Demand



1
0.8
0.6
0.4
0.2
0
Demand
0
0.5
1
1.5
2
Price
Total demand
Young fans
Others

Demand and revenues



Optimization

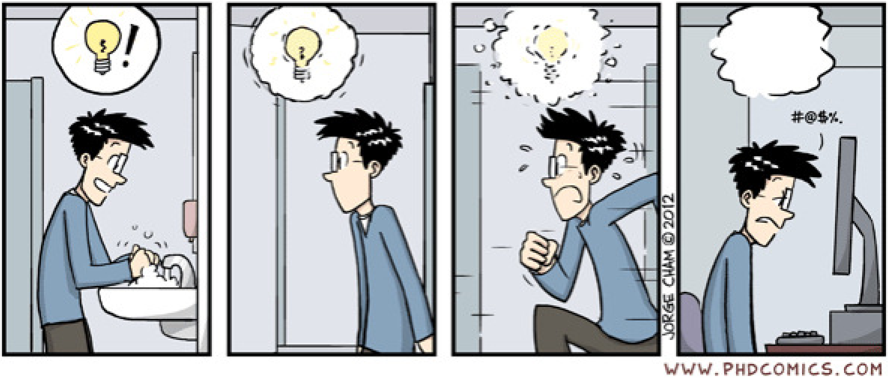
Profit maximization

- Non linear
- Non convex

Facility location

- Discrete

The main idea



The main idea

Linearization

- Hopeless to linearize the logit formula (we tried...)
- Anyway, we want to go beyond logit.

The main idea

Linearization

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- Anyway, we want to go beyond logit.

First principles

Each customer solves an optimization problem

The main idea

Linearization

- Hopeless to linearize the logit formula (we tried...)
- Anyway, we want to go beyond logit.

First principles

Each customer solves an optimization problem

Solution

Use the utility and not the probability

A linear formulation

Utility function

$$U_{in} = V_{in} + \varepsilon_{in} = \sum_k \beta_k x_{ink} + f(z_{in}) + \varepsilon_{in}.$$

Simulation

- Assume a distribution for ε_{in}
- E.g. logit: i.i.d. extreme value
- Draw R realizations ξ_{inr}, $r = 1, \ldots, R$
- The choice problem becomes deterministic

Scenarios

Draws

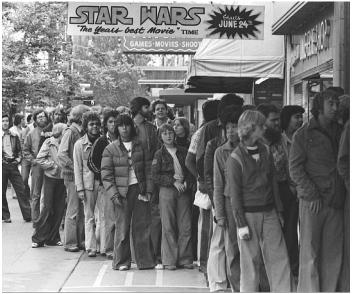
- Draw R realizations ξ_{inr}, $r = 1, \ldots, R$
- We obtain R scenarios

$$U_{inr} = \sum_k \beta_k x_{ink} + f(z_{in}) + \xi_{inr}.$$

- For each scenario r, we can identify the largest utility.
- It corresponds to the chosen alternative.

Capacities

- Demand may exceed supply
- Each alternative i can be chosen by maximum c_i individuals.
- An exogenous priority list is available.
- Can be randomly generated, or according to some rules.
- The numbering of individuals is consistent with their priority.



Choice set

Variables

$y_i \in \{0,1\}$	operator decision
$y^d_{in} \in \{0,1\}$	customer decision (data)
$y_{in} \in \{0,1\}$	product of decisions
$y_{inr} \in \{0,1\}$	capacity restrictions

Constraints

$$y_{in} = y^d_{in} y_i \quad \forall i, n$$
$$y_{inr} \leq y_{in} \quad \forall i, n, r$$

Utility

Variables

U_{inr} utility

$$z_{inr} = \begin{cases} U_{inr} & \text{if } y_{inr} = 1 \\ \ell_{nr} & \text{if } y_{inr} = 0 \end{cases}$$ discounted utility

(ℓ_{nr} smallest lower bound)

Constraint: utility

$$U_{inr} = \overbrace{\beta_{in} p_{in} + q_d(x_d)}^{V_{in}} + \xi_{inr} \; \forall i, n, r$$

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EPFL

Utility (ctd)

Constraints: discounted utility

$$
\begin{aligned}
\ell_{nr} &\leq z_{inr} & \forall i, n, r \\
z_{inr} &\leq \ell_{nr} + M_{inr} y_{inr} & \forall i, n, r \\
U_{inr} - M_{inr}(1 - y_{inr}) &\leq z_{inr} & \forall i, n, r \\
z_{inr} &\leq U_{inr} & \forall i, n, r
\end{aligned}
$$

Choice

Variables

$$U_{nr} = \max_{i \in \mathcal{C}} z_{inr}$$

$$w_{inr} = \begin{cases} 1 & \text{if } z_{inr} = U_{nr} \\ 0 & \text{otherwise} \end{cases} \qquad \text{choice}$$

Constraints

$$z_{inr} \leq U_{nr} \qquad \forall i, n, r$$

$$U_{nr} \leq z_{inr} + M_{nr}(1 - w_{inr}) \qquad \forall i, n, r$$

$$\sum_i w_{inr} = 1 \qquad \forall n, r$$

$$w_{inr} \leq y_{inr} \qquad \forall i, n, r$$

Capacity

Capacity cannot be exceeded $\Rightarrow y_{inr} = 1$

$$\sum_{m=1}^{n-1} w_{imr} \leq (c_i - 1)y_{inr} + (n-1)(1 - y_{inr}) \; \forall i > 0, n > c_i, r$$

Capacity has been reached $\Rightarrow y_{inr} = 0$

$$c_i(y_{in} - y_{inr}) \leq \sum_{m=1}^{n-1} w_{imr}, \; \forall i > 0, n, r$$

Family of models

Constraints

- Set of linear constraints characterizing choice behavior
- Can be included in any relevant optimization problem.

Examples

- Profit maximization
- Facility location

Difficulties

- big M constraints
- large dimensions

Profit maximization

Profit

If p_{in} is the price paid by individual to purchase option i, the revenue generated by this option is

$$\frac{1}{R}\sum_{r=1}^{R}\sum_{n=1}^{N} p_{in} w_{inr}.$$

Linearization

If $a_{in} \leq p_{in} \leq b_{in}$, we define $\eta_{inr} = p_{in} w_{inr}$, and the following constraints:

$$\begin{aligned} a_{in} w_{inr} &\leq \eta_{inr} \\ \eta_{inr} &\leq b_{in} w_{inr} \\ p_{in} - (1 - w_{inr}) b_{in} &\leq \eta_{inr} \\ \eta_{inr} &\leq p_{in} - (1 - w_{inr}) a_{in} \end{aligned}$$

A case study

Challenge

- Take a choice model from the literature.
- It cannot be logit.
- It must involve heterogeneity.
- Show that it can be integrated in a relevant MILP.

A case study

Challenge

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Parking choice

- [Ibeas et al., 2014]

Parking choices [Ibeas et al., 2014]

Alternatives

- Paid on-street parking
- Paid underground parking
- Free street parking

Model

- $N = 50$ customers
- $\mathcal{C} = \{\text{PSP}, \text{PUP}, \text{FSP}\}$
- $\mathcal{C}_n = \mathcal{C} \quad \forall n$
- $p_{in} = p_i \quad \forall n$
- Capacity of 20 spots
- Mixture of logit models

General experiments

Uncapacitated vs Capacitated case

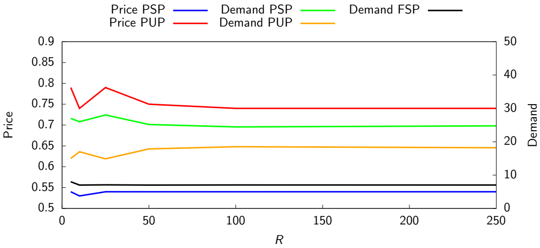
- Maximization of revenue
- Unlimited capacity
- Capacity of 20 spots for PSP and PUP

Price differentiation by population segmentation

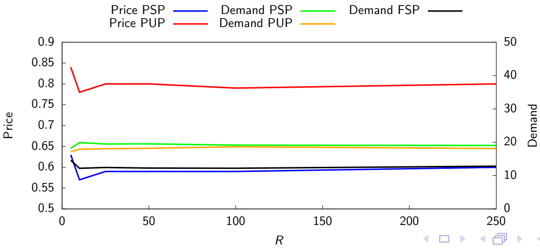
- Reduced price for residents
- Two scenarios
  1. Subsidy offered by the municipality
  2. Operator is forced to offer a reduced price

Uncapacitated vs Capacitated case

Uncapacitated



Capacitated



Computational time

	Uncapacitated case				Capacitated case			
R	Sol time	PSP	PUP	Rev	Sol time	PSP	PUP	Rev
5	2.58 s	0.54	0.79	26.43	12.0 s	0.63	0.84	25.91
10	3.98 s	0.53	0.74	26.36	54.5 s	0.57	0.78	25.31
25	29.2 s	0.54	0.79	26.90	13.8 min	0.59	0.80	25.96
50	4.08 min	0.54	0.75	26.97	50.2 min	0.59	0.80	26.10
100	20.7 min	0.54	0.74	26.90	6.60 h	0.59	0.79	26.03
250	2.51 h	0.54	0.74	26.85	1.74 days	0.60	0.80	25.93

Facility location

Data

- U_{in}: exogenous,
- C_i: fixed cost to open a facility,
- c_i: operational cost per customer to run the facility.

Objective function

$$\min \sum_{i \in \mathcal{C}_k} C_i y_i + \frac{1}{R} \sum_r \sum_i \sum_n c_i w_{inr}$$

Benders decomposition

$$\min \sum_{i \in \mathcal{C}_k} C_i y_i + \frac{1}{R} \sum_r \sum_i \sum_n c_i w_{inr}$$

subject to

$$\max_w U_{nr} = \sum_i U_{inr} w_{inr}$$

$$\sum_i w_{inr} \leq 1$$

$$w_{inr} \leq y_i$$

$$w_{inr} \geq 0$$

$$w_{inr}, y_i \in \{0, 1\}.$$

Benders decomposition

Customer subproblem: fix y_i^*

$$\max_w U_{nr} = \sum_i U_{inr} w_{inr}$$

subject to

$$\sum_i w_{inr} = 1$$
$$w_{inr} \leq y_i^*$$
$$w_{inr} \geq 0.$$

Property

Totally unimodular: no integrality constraint is required.

Benders decomposition

Primal

$$\min_{w} U = -\sum_i U_i w_i$$

subject to

$$\sum_i w_i = 1$$

$$w_i \leq y_i^* \qquad \forall i$$

$$w_i \geq 0.$$

Dual

$$\max_{\lambda,\mu} \lambda + \sum_i \mu_i y_i^*$$

subject to

$$\lambda + \mu_i \leq -U_i \qquad \forall i$$

$$\mu_i \leq 0 \qquad \forall i$$

Bender decomposition

Ongoing work

- Exploit the duality results to generate cuts for the master problem.
- Investigate the use of Benders for other problems.
  - profit maximization,
  - maximum likelihood estimation of the parameters.

Road map

Optimization

Microeconomics in a nutshell

Discrete choice

Logit and MEV models

Profit maximization, facility location

Activity-based models

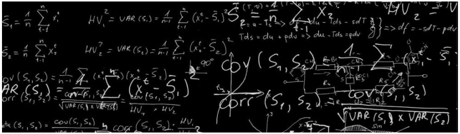
Outline

Introduction

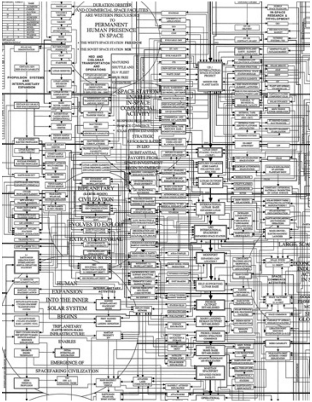
- Travel demand is derived from activity demand.
- Activity demand is influenced by socio-economic characteristics, social interactions, cultural norms, basic needs, etc. [Chapin, 1974]
- Activity demand is constrained in space and time [Hägerstraand, 1970].

Literature

Econometric models



Rule-based models



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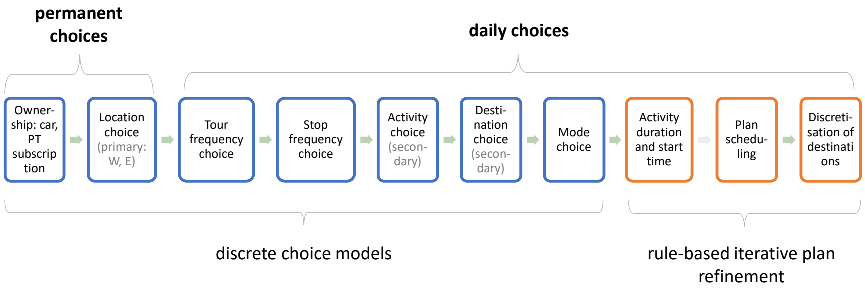
State of the art: econometric approach

[Bhat, 2005]

- Multiple Discrete Continuous Extreme Value
- Based on first principles.
- Decision-maker solves an optimization problem, with a time budget.
- Several alternatives may be chosen.
- Model derived from KKT conditions.

State of practice

Sequence of decisions Source: [Scherr et al., 2020]



Research question

Relax the *series of discrete choice models* approach

- The interactions of all decisions is complex.
- Sequence of models is most of the time arbitrary.

Integrated approach

Develop a model involving many decisions:

- activity participation,
- activity location,
- activity duration,
- activity scheduling,
- travel mode,
- travel path.

Research objectives

- Integrated approach based on first principles.
- Theoretical framework: utility maximization.
- Individuals solve a scheduling problem.
- Important aspects: trade-offs on activity sequence, duration and starting time.
- Again, we replace the error terms by draws.

Decision variables for individual n and draw r

For each (potential) activity a:

- Activity participation: $w_{anr} \in \{0, 1\}$.
- Starting time: $x_{anr} \in \{0, \ldots, T\}$.
- Duration: $\tau_{anr} \in \{0, \ldots, T\}$.
- Scheduling: $z_{abnr} \in \{0, 1\}$: 1 if activity b immediately follows a.

Objective function

Additive utility

$$\max \sum_{a \in A} w_{anr} U_{anr} + \theta_t \sum_{a \in A} \sum_{b \in A} z_{abnr} \rho(s_a, s_b, m_a, p_a).$$

Constraints

Time budget

$$\sum_{a \in A} w_{anr} \tau_{anr} + \sum_{a \in A} \sum_{b \in A} z_{abnr} \rho(s_a, s_b, m_a, p_a) = T, \ \forall n, r.$$

Time windows

$$0 \leq \gamma_a^- \leq x_{anr} \leq x_{anr} + \tau_{anr} \leq \gamma_a^+ \leq T, \ \forall a, n, r.$$

Constraints

Precedence constraints

$$z_{abnr} + z_{banr} \leq 1, \ \forall a, b, n, r.$$

Single successor/predecessor

$$\sum_{b \in A \setminus \{a\}} z_{abnr} = w_{anr}, \ \forall a, n, r,$$

$$\sum_{b \in A \setminus \{a\}} z_{banr} = w_{anr}, \ \forall a, n, r.$$

Constraints

Consistent timing

$$(z_{abnr} - 1)T \leq x_{anr} + \tau_{anr} + t_{anr} - x_{bnr} \leq (1 - z_{abnr})T, \ \forall a, b, n, r.$$

where

$$t_{anr} = \sum_{b \in A} z_{abnr} \rho(s_a, s_b, m_a, p_a).$$

Other constraints...

see [Pougala et al., 2021] for details

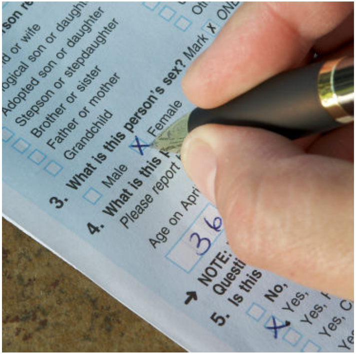
- mode of transportation
- route
- car availability
- etc.

Optimization problem

Simulation-based optimization

- For each realization of the error terms, we have an optimal schedule.
- It includes all the choice dimensions (activity participation, location, duration, scheduling, and mode and route).
- We can generate an empirical distribution of chosen schedules.

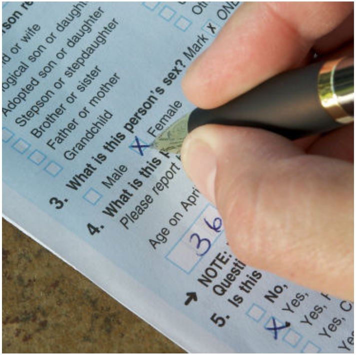
Real data



Dataset

- 2015 Swiss Mobility and Transport Microcensus.
- Daily trip diaries for 57'000 individuals.
- Records of activities, visited location, mode/path choice.

Real data

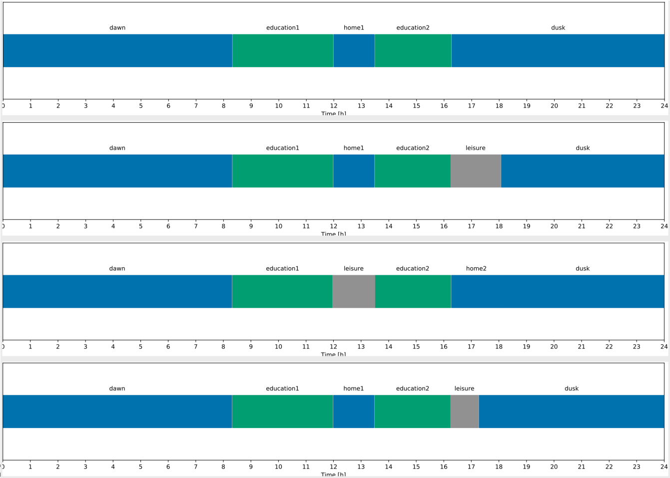


Assumptions

- Desired start times and durations are the recorded ones.
- Feasible time windows: percentiles start and end times from out of sample distribution.
- Only the recorded locations are considered.
- Uniform flexibility profile across population.

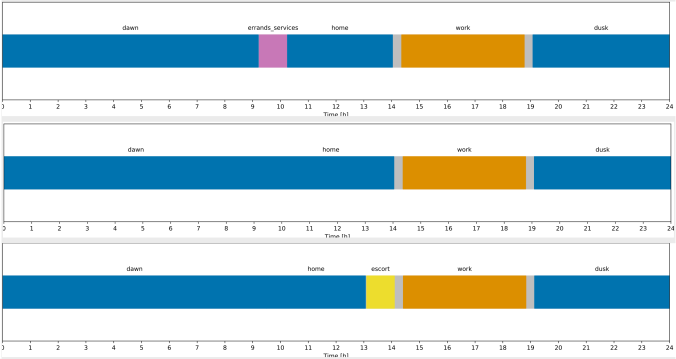
Individual 1 (weekday)

Optimal schedules generated for random draws of ε_{a_n}



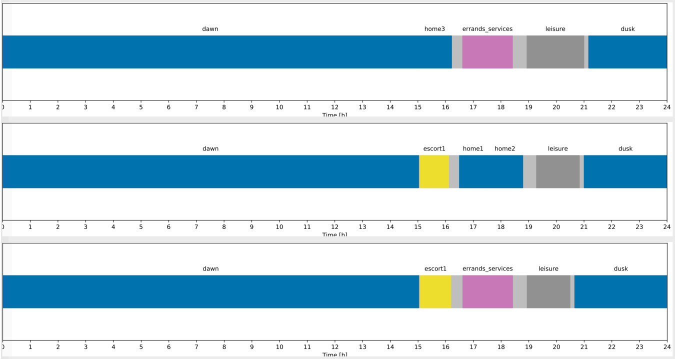
Individual 2 (weekday)

Optimal schedules generated for random draws of ε_{a_n}



Individual 3 (weekday)

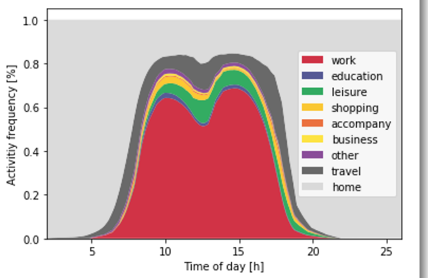
Optimal schedules generated for random draws of ε_{a_n}



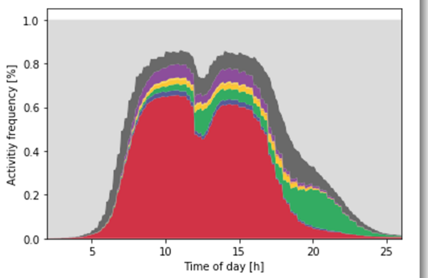
Validation

Activity profiles for full-time workers, Lausanne area

Simulation



Microcensus

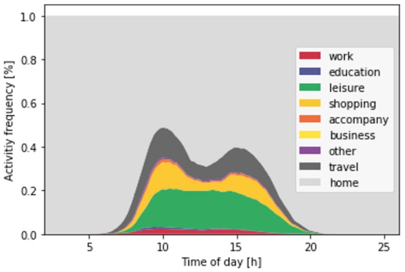


Source: SBB. Acknowledgment to Patrick Manser.

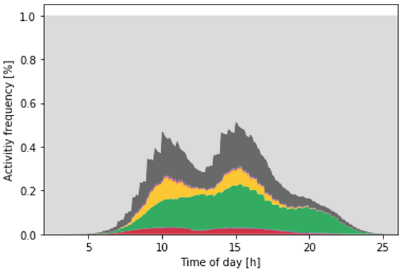
Validation

Activity profiles for individuals older than 65, Lausanne area

Simulation



Microcensus

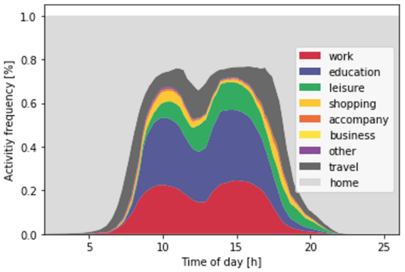


Source: SBB. Acknowledgment to Patrick Manser.

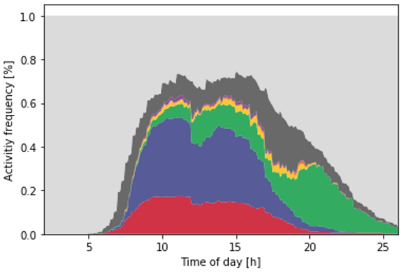
Validation

Activity profiles for students, Lausanne area

Validation



Microcensus

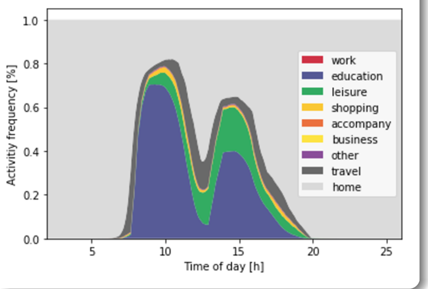


Source: SBB. Acknowledgment to Patrick Manser.

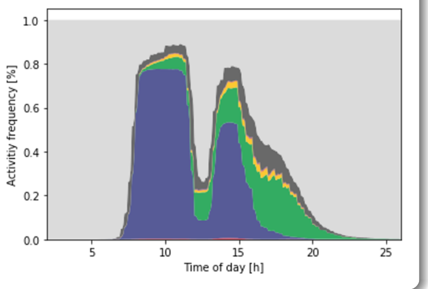
Validation

Activity profiles for primary school pupils, Lausanne area

Validation



Microcensus



Source: SBB. Acknowledgment to Patrick Manser.

Activity-based models

Ongoing work

- Synthetic population
- Estimation of the parameters
- Social interactions

Outline

Conclusion

	Optimization	**Discrete choice**
Tutorial	Microeconomics in a nutshell	Logit and MEV models
Research	Profit maximization, facility location	Activity-based models

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- Stefano Bortolomiol,
- Tim Hillel,
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- Meritxell Pacheco,
- Janody Pougala,
- Shadi Sharif Azadeh,
- and many others...

Readings

- [Pacheco Paneque, 2020]
- [Pacheco et al., 2021]
- [Bortolomiol et al., forta]
- [Bortolomiol et al., fortb]
- [Pougala et al., 2021]

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