Disaggregate Demand Models and Optimization

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Outline

1. Introduction
2. Microeconomics
3. The logit model
4. Profit optimization, facility location
5. Activity-based models
6. Conclusion
Demand models

- Supply = infrastructure
- Demand = behavior, choices
- Congestion = mismatch
Demand models

- Usually in OR:
  - optimization of the supply
  - for a given (fixed) demand
Aggregate demand

- Homogeneous population
- Identical behavior
- Price \((P)\) and quantity \((Q)\)
- Demand functions: \(P = f(Q)\)
- Inverse demand: \(Q = f^{-1}(P)\)
Disaggregate demand

- Heterogeneous population
- Different behaviors
- Many variables:
  - Attributes: price, travel time, reliability, frequency, etc.
  - Characteristics: age, income, education, etc.
- Complex demand/inverse demand functions.
Demand-supply interactions

Operations Research
- Given the demand...
- configure the system

Behavioral models
- Given the configuration of the system...
- predict the demand
Demand-supply interactions

Multi-objective optimization

Minimize costs

Maximize satisfaction
Microeconomics in a nutshell
In this lecture...

Microeconomics in a nutshell → Logit model
In this lecture...

Microeconomics in a nutshell

Profit maximization, facility location

Logit model
Microeconomics in a nutshell

Profit maximization, facility location

Logit model

Activity-based models
In this lecture...

**Optimization**

- Microeconomics in a nutshell
- Profit maximization, facility location
- Logit model
- Activity-based models
In this lecture...

**Optimization**
- Microeconomics in a nutshell
- Profit maximization, facility location

**Discrete choice**
- Logit model
- Activity-based models
In this lecture...

- Microeconomics in a nutshell
- Profit maximization, facility location
- Activity-based models
- Logit model
- Optimization
- Discrete choice

Tutorial
In this lecture...

**Tutorial**
- Microeconomics in a nutshell

**Research**
- Profit maximization, facility location

**Optimization**

**Discrete choice**
- Logit model
- Activity-based models
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Microeconomic consumer theory

Homo economicus
Rational and narrowly self-interested economic actor who is optimizing her outcome

Preference operators $\succ$, $\sim$, and $\succeq$

- $a \succ b$: $a$ is preferred to $b$,
- $a \sim b$: indifference between $a$ and $b$,
- $a \succeq b$: $a$ is at least as preferred as $b$. 
Microeconomic consumer theory

Rationality

- Completeness: for all bundles $a$ and $b$,

\[ a \succ b \text{ or } a \prec b \text{ or } a \sim b. \]

- Transitivity: for all bundles $a$, $b$ and $c$,

\[ \text{if } a \succeq b \text{ and } b \succeq c \text{ then } a \succeq c. \]

- “Continuity”: if $a$ is preferred to $b$ and $c$ is arbitrarily “close” to $a$, then $c$ is preferred to $b$. 

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Microeconomic consumer theory

Continuous choice set

- Consumption bundle

\[ Q = \begin{pmatrix} q_1 \\ \vdots \\ q_L \end{pmatrix}, \quad p = \begin{pmatrix} p_1 \\ \vdots \\ p_L \end{pmatrix} \]

- Budget constraint

\[ p^T Q = \sum_{\ell=1}^{L} p_\ell q_\ell \leq I. \]

- Decision variables: quantities.
Utility function

- Parameterized function:
  \[
  \tilde{U} = \tilde{U}(q_1, \ldots, q_L; \theta) = \tilde{U}(Q; \theta)
  \]

- Consistent with the preference indicator:
  \[
  \tilde{U}(Q_a; \theta) \geq \tilde{U}(Q_b; \theta)
  \]
  is equivalent to
  \[
  Q_a \succeq Q_b.
  \]

- Unique up to an order-preserving transformation
Microeconomic consumer theory

Optimization problem

\[
\max_Q \tilde{U}(Q; \theta)
\]

subject to

\[
p^T Q \leq I, \ Q \geq 0.
\]

Demand function

- Solution of the optimization problem
- KKT optimality conditions
- Quantity as a function of prices and budget

\[
Q^* = f(I, p; \theta)
\]
Indirect utility

Substitute the demand function into the utility

\[ U(I, p; \theta) = \tilde{U}(Q^*, \theta) = \tilde{U}(f(I, p; \theta), \theta) \]

Indirect utility
Maximum utility that is achievable for a given set of prices and income

In discrete choice...
- only the indirect utility is used
- therefore, it is simply referred to as “utility”
Microeconomic theory of discrete goods

Expanding the microeconomic framework

- Continuous goods
- and discrete goods

The consumer

- selects the quantities of continuous goods: \( Q = (q_1, \ldots, q_L) \)
- chooses an alternative in a discrete choice set \( i = 1, \ldots, j, \ldots, J \)
- discrete decision vector: \( (y_1, \ldots, y_J) \), \( y_j \in \{0, 1\} \), \( \sum_j y_j = 1 \).

Note

- In theory, one alternative of the discrete choice combines all possible choices made by an individual.
- In practice, the choice set will be more restricted for tractability.
Utility maximization

Utility

\[ \tilde{U}(Q, y, \tilde{z}^T y; \theta) \]

- **Q**: quantities of the continuous good
- **y**: discrete choice
- **\( \tilde{z}^T \) = (\tilde{z}_1, \ldots, \tilde{z}_i, \ldots, \tilde{z}_J) \in \mathbb{R}^{K \times J}**: \( K \) attributes of the \( J \) alternatives
- **\( \tilde{z}^T y \in \mathbb{R}^K \)**: attributes of the chosen alternative
- **\( \theta \)**: vector of parameters
Utility maximization

Optimization problem

\[
\max_{Q, y} \tilde{U}(Q, y, \tilde{z}^T y; \theta)
\]

subject to

\[
\begin{align*}
p^T Q + c^T y & \leq I \\
\sum_j y_j & = 1 \\
y_j & \in \{0, 1\}, \forall j.
\end{align*}
\]

where \( c^T = (c_1, \ldots, c_i, \ldots, c_J) \) contains the cost of each alternative.

Solving the problem

- Mixed integer optimization problem
- No optimality condition
- Impossible to derive demand functions directly
Solving the problem

Step 1: condition on the choice of the discrete good

- Fix the discrete good, that is select a feasible $y$.
- The problem becomes a continuous problem in $Q$.
- Conditional demand functions can be derived:

$$q_{\ell}\mid y = f(I - c^T y, p, \tilde{z}^T y; \theta),$$

or, equivalently, for each alternative $i$,

$$q_{\ell}\mid i = f(I - c_i, p, \tilde{z}_i; \theta).$$

- $I - c_i$ is the income left for the continuous goods, if alternative $i$ is chosen.
- If $I - c_i < 0$, alternative $i$ is declared unavailable and removed from the choice set.
Solving the problem

Conditional indirect utility functions
Substitute the demand functions into the utility:

$$U_i = U(I - c_i, p, \tilde{z}_i; \theta) \text{ for all } i \in \mathcal{C}.$$ 

Step 2: Choice of the discrete good

$$\max_y U(I - c^T y, p, \tilde{z}^T y; \theta)$$

- Enumerate all alternatives.
- Compute the conditional indirect utility function $U_i$.
- Select the alternative with the highest $U_i$.
- Note: no income constraint anymore.
Simple example: mode choice

Attributes

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Attributes</th>
<th>Travel time ($t$)</th>
<th>Travel cost ($c$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car (1)</td>
<td>$t_1$</td>
<td>$c_1$</td>
<td></td>
</tr>
<tr>
<td>Bus (2)</td>
<td>$t_2$</td>
<td>$c_2$</td>
<td></td>
</tr>
</tbody>
</table>

Utility

$$
\tilde{U} = \tilde{U}(y_1, y_2),
$$

where we impose the restrictions that, for $i = 1, 2$,

$$
\begin{cases} 
  1 & \text{if travel alternative } i \text{ is chosen,} \\
  0 & \text{otherwise}; 
\end{cases}
$$

and that only one alternative is chosen: $y_1 + y_2 = 1$. 
Simple example: mode choice

Utility functions

\[ U_1 = -\beta_t t_1 - \beta_c c_1, \]
\[ U_2 = -\beta_t t_2 - \beta_c c_2, \]

where \( \beta_t > 0 \) and \( \beta_c > 0 \) are parameters.

Equivalent specification

\[ U_1 = -(\beta_t/\beta_c) t_1 - c_1 = -\beta t_1 - c_1 \]
\[ U_2 = -(\beta_t/\beta_c) t_2 - c_2 = -\beta t_2 - c_2 \]

where \( \beta > 0 \) is a parameter.

Choice

- Alternative 1 is chosen if \( U_1 \geq U_2 \).
- Ties are ignored.
Simple example: mode choice

Choice

Alternative 1 is chosen if

\[-\beta t_1 - c_1 \geq -\beta t_2 - c_2\]

or

\[-\beta(t_1 - t_2) \geq c_1 - c_2\]

Alternative 2 is chosen if

\[-\beta t_1 - c_1 \leq -\beta t_2 - c_2\]

or

\[-\beta(t_1 - t_2) \leq c_1 - c_2\]

Dominated alternative

- If \(c_2 > c_1\) and \(t_2 > t_1\), \(U_1 > U_2\) for any \(\beta > 0\)
- If \(c_1 > c_2\) and \(t_1 > t_2\), \(U_2 > U_1\) for any \(\beta > 0\)
Simple example: mode choice

Trade-off

- Assume $c_2 > c_1$ and $t_1 > t_2$.
- Is the traveler willing to pay the extra cost $c_2 - c_1$ to save the extra time $t_1 - t_2$?
- Alternative 2 is chosen if

$$-\beta(t_1 - t_2) \leq c_1 - c_2$$

or

$$\beta \geq \frac{c_2 - c_1}{t_1 - t_2}$$

- $\beta$ is called the *willingness to pay* or *value of time*
Simple example: mode choice

\[ c_1 + \beta t_1 = c_2 + \beta t_2 \]

Alt. 1 is dominant

Alt. 2 is preferred

Alt. 1 is chosen

Alt. 2 is chosen
Behavioral validity of the utility maximization?

Assumptions

Decision-makers

- are able to process information
- have perfect discrimination power
- have transitive preferences
- are perfect maximizer
- are always consistent

Relax the assumptions

Use a probabilistic approach: what is the probability that alternative $i$ is chosen?
Random utility model

Probability model

\[ P(i|C_n) = \Pr(U_{in} \geq U_{jn}, \forall j \in C_n), \]

Random utility

\[ U_{in} = V_{in} + \varepsilon_{in} = \beta^T X_{in} + \varepsilon_{in}. \]

Similarity with linear regression

\[ Y = \beta^T X + \varepsilon \]

Here, \( U \) is not observed. Only the choice is observed.
Derivation

Joint distributions of $\varepsilon_n$

- Assume that $\varepsilon_n = (\varepsilon_{1n}, \ldots, \varepsilon_{Jn})$ is a multivariate random variable
- with CDF
  
  $$F_{\varepsilon_n}(\varepsilon_1, \ldots, \varepsilon_J)$$

- and pdf
  
  $$f_{\varepsilon_n}(\varepsilon_1, \ldots, \varepsilon_J) = \frac{\partial^J F}{\partial \varepsilon_1 \cdots \partial \varepsilon_J}(\varepsilon_1, \ldots, \varepsilon_J).$$

The random utility model: $P_n(i|\mathcal{C}_n) =$

$$\int_{-\infty}^{+\infty} \frac{\partial F_{\varepsilon_{1n}, \varepsilon_{2n}, \ldots, \varepsilon_{Jn}}}{\partial \varepsilon_i} \left( \ldots, V_{in} - V_{(i-1)n} + \varepsilon, \varepsilon, V_{in} - V_{(i+1)n} + \varepsilon, \ldots \right) d\varepsilon$$
Random utility model

- The general formulation is complex.
- We can derive specific models based on simple assumptions.
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Road map

Optimization

Microeconomics in a nutshell

→

Discrete choice

Logit model
Error term

Random utility

\[ U_{in} = V_{in} + \varepsilon_{in}. \]

Assumptions about the distribution

- **Probit**: central limit theorem: the sum of many i.i.d. random variables approximately follows a normal distribution.

- **Logit**: Gumbel theorem: the maximum of many i.i.d. random variables approximately follows an Extreme Value distribution: \( EV(\eta, \mu) \).
Logit model

\[ P_n(i|C_n) = \frac{y_{in} e^{V_{in}}}{\sum_{j=1}^{J} y_{jn} e^{V_{jn}}}. \]

Why “logit”?

If \( U_{in} \) and \( U_{jn} \) are EV distributed, \( U_{in} - U_{jn} \) follows a logistic distribution.

Availability of alternatives

\[
y_{in} = \begin{cases} 
1 & \text{if } i \in C_n, \\ 
0 & \text{otherwise.} 
\end{cases}
\]

\( y_{in}=1 \) if alternative \( i \) is available to individual \( n \).
Example

Two alternatives

\[ V_{0n} = 0 \]
\[ V_{1n} = -10 \times \text{price} + 3 \]

Choice probability

\[ P_n(1 | \text{price}) = \frac{e^{-10 \times \text{price} + 3}}{e^{0} + e^{-10 \times \text{price} + 3}} = \frac{e^{-10 \times \text{price} + 3}}{1 + e^{-10 \times \text{price} + 3}} \]
The logit model

Example

Choice probability

$P_n(1|\text{price})$

Price

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

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Beyond logit

- Other distributional assumptions can be used.
- Logit is not always consistent with observed behavior.
- Trade-off between model complexity and behavioral realism.
- Examples: Multivariate Extreme Value models, mixtures models, hybrid choice models.
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A simple example

Data
- \( C \): set of movies
- Population of \( N \) individuals
- Utility function:
  \[
  U_{in} = \beta_{in} p_{in} + f(z_{in}) + \varepsilon_{in}
  \]

Decision variables
- What movies to propose? \( y_{in} \)
- What price? \( p_{in} \)
Profit maximization

Data

- Two alternatives: my theater \((m)\) and the competition \((c)\)
- We assume an heterogeneous population of \(N\) individuals

\[
U_{cn} = 0 + \varepsilon_{cn}
\]
\[
U_{mn} = \beta_n p_m + c_{mn} + \varepsilon_{mn}
\]

- \(\beta_n < 0\)
- Logit model: \(\varepsilon_{mn}\) i.i.d. EV
Heterogeneous population

Two groups in the population

\[ U_{mn} = \beta_n p_m + c_{mn} + \varepsilon_{mn} \]

\( n = 1: \) Young fans: 
\( \frac{2}{3} \)
\( \beta_1 = -10, \ c_{m1} = 3 \)

\( n = 2: \) Others: \( \frac{1}{3} \)
\( \beta_1 = -0.9, \ c_{1m} = 0 \)
Demand

- Total demand
- Young fans
- Others

Price vs. Demand graph showing how demand changes with price for different categories.
Demand and revenues

A graph showing the relationship between price and demand, with different demand segments labeled: Total demand, Young fans, Others. The x-axis represents price, ranging from 0 to 2, while the y-axis represents demand, ranging from 0 to 1. The graph also shows the comparison between revenues and demand across different price points.
Optimization

Profit maximization
- Non linear
- Non convex

Facility location
- Discrete
The main idea
The main idea

Linearization

- Hopeless to linearize the logit formula (we tried...)
- Anyway, we want to go beyond logit.
The main idea

Linearization

- Hopeless to linearize the logit formula (we tried...)
- Anyway, we want to go beyond logit.

First principles

Each customer solves an optimization problem
The main idea

Linearization

- Hopeless to linearize the logit formula (we tried...)
- Anyway, we want to go beyond logit.

First principles

Each customer solves an optimization problem

Solution

Use the utility and not the probability
A linear formulation

Utility function

\[ U_{in} = V_{in} + \varepsilon_{in} = \sum_k \beta_k x_{ink} + f(z_{in}) + \varepsilon_{in}. \]

Simulation

- Assume a distribution for \( \varepsilon_{in} \)
- E.g. logit: i.i.d. extreme value
- Draw \( R \) realizations \( \xi_{inr}, r = 1, \ldots, R \)
- The choice problem becomes deterministic
Scenarios

Draws

- Draw \( R \) realizations \( \xi_{inr}, r = 1, \ldots, R \)
- We obtain \( R \) scenarios

\[
U_{inr} = \sum_k \beta_k x_{ink} + f(z_{in}) + \xi_{inr}.
\]

- For each scenario \( r \), we can identify the largest utility.
- It corresponds to the chosen alternative.
Capacities

- Demand may exceed supply
- Each alternative \( i \) can be chosen by maximum \( c_i \) individuals.
- An exogenous priority list is available.
- Can be randomly generated, or according to some rules.
- The numbering of individuals is consistent with their priority.
Choice set

Variables

\[ y_i \in \{0, 1\} \quad \text{operator decision} \]
\[ y_{in}^d \in \{0, 1\} \quad \text{customer decision (data)} \]
\[ y_{in} \in \{0, 1\} \quad \text{product of decisions} \]
\[ y_{inr} \in \{0, 1\} \quad \text{capacity restrictions} \]

Constraints

\[ y_{in} = y_{in}^d y_i \quad \forall i, n \]
\[ y_{inr} \leq y_{in} \quad \forall i, n, r \]
Utility

Variables

\[ U_{inr} \]
\[ z_{inr} = \begin{cases} U_{inr} & \text{if } y_{inr} = 1 \\ \ell_{nr} & \text{if } y_{inr} = 0 \end{cases} \]
\((\ell_{nr} \text{ smallest lower bound})\)

Constraint: utility

\[ U_{inr} = V_{in} - \beta_{in} p_{in} + q_d(x_d) + \xi_{inr} \forall i, n, r \]
Utility (ctd)

Constraints: discounted utility

\[ \ell_{nr} \leq z_{inr} \quad \forall i, n, r \]
\[ z_{inr} \leq \ell_{nr} + M_{inr}y_{inr} \quad \forall i, n, r \]
\[ U_{inr} - M_{inr}(1 - y_{inr}) \leq z_{inr} \quad \forall i, n, r \]
\[ z_{inr} \leq U_{inr} \quad \forall i, n, r \]
Choice

Variables

\[ U_{nr} = \max_{i \in C} z_{inr} \]

\[ w_{inr} = \begin{cases} 1 & \text{if } z_{inr} = U_{nr} \\ 0 & \text{otherwise} \end{cases} \]

Constraints

\[ z_{inr} \leq U_{nr} \quad \forall i, n, r \]

\[ U_{nr} \leq z_{inr} + M_{nr}(1 - w_{inr}) \quad \forall i, n, r \]

\[ \sum_{i} w_{inr} = 1 \quad \forall n, r \]

\[ w_{inr} \leq y_{inr} \quad \forall i, n, r \]
Capacity

Capacity cannot be exceeded $\Rightarrow y_{inr} = 1$

$$\sum_{m=1}^{n-1} w_{imr} \leq (c_i - 1)y_{inr} + (n - 1)(1 - y_{inr}) \forall i > 0, n > c_i, r$$

Capacity has been reached $\Rightarrow y_{inr} = 0$

$$c_i(y_{in} - y_{inr}) \leq \sum_{m=1}^{n-1} w_{imr}, \forall i > 0, n, r$$
Family of models

Constraints
- Set of linear constraints characterizing choice behavior
- Can be included in any relevant optimization problem.

Examples
- Profit maximization
- Facility location

Difficulties
- big $M$ constraints
- large dimensions
Profit maximization

Profit
If $p_{in}$ is the price paid by individual to purchase option $i$, the revenue generated by this option is

$$
\frac{1}{R} \sum_{r=1}^{R} \sum_{n=1}^{N} p_{in} w_{inr}.
$$

Linearization
If $a_{in} \leq p_{in} \leq b_{in}$, we define $\eta_{inr} = p_{in} w_{inr}$, and the following constraints:

$$
\begin{align*}
    a_{in} w_{inr} & \leq \eta_{inr} \\
    \eta_{inr} & \leq b_{in} w_{inr} \\
    p_{in} - (1 - w_{inr}) b_{in} & \leq \eta_{inr} \\
    \eta_{inr} & \leq p_{in} - (1 - w_{inr}) a_{in}
\end{align*}
$$
A case study

Challenge

- Take a choice model from the literature.
- It cannot be logit.
- It must involve heterogeneity.
- Show that it can be integrated in a relevant MILP.
A case study

Challenge

- Take a choice model from the literature.
- It cannot be logit.
- It must involve heterogeneity.
- Show that it can be integrated in a relevant MILP.

Parking choice

- [Ibeas et al., 2014]
Parking choices [Ibeas et al., 2014]

Alternatives
- Paid on-street parking
- Paid underground parking
- Free street parking

Model
- \( N = 50 \) customers
- \( C = \{\text{PSP}, \text{PUP}, \text{FSP}\} \)
- \( C_n = C \quad \forall n \)
- \( p_{in} = p_i \quad \forall n \)
- Capacity of 20 spots
- Mixture of logit models
General experiments

Uncapacitated vs Capacitated case
- Maximization of revenue
- Unlimited capacity
- Capacity of 20 spots for PSP and PUP

Price differentiation by population segmentation
- Reduced price for residents
- Two scenarios
  1. Subsidy offered by the municipality
  2. Operator is forced to offer a reduced price
Uncapacitated vs Capacitated case

Uncapacitated

Capacitated
## Computational time

| $R$ | **Uncapacitated case** | | | **Capacitated case** | | |
|-----|------------------------|--:|---:|------------------------|--:|---:|---:|
| 5   | Sol time | 2.58 s | PSP | 0.54 | PUP | 0.79 | Rev | 26.43 | Sol time | 12.0 s | PSP | 0.63 | PUP | 0.84 | Rev | 25.91 |
| 10  | Sol time | 3.98 s | PSP | 0.53 | PUP | 0.74 | Rev | 26.36 | Sol time | 54.5 s | PSP | 0.57 | PUP | 0.78 | Rev | 25.31 |
| 25  | Sol time | 29.2 s | PSP | 0.54 | PUP | 0.79 | Rev | 26.90 | Sol time | 13.8 min | PSP | 0.59 | PUP | 0.80 | Rev | 25.96 |
| 50  | Sol time | 4.08 min | PSP | 0.54 | PUP | 0.75 | Rev | 26.97 | Sol time | 50.2 min | PSP | 0.59 | PUP | 0.80 | Rev | 26.10 |
| 100 | Sol time | 20.7 min | PSP | 0.54 | PUP | 0.74 | Rev | 26.90 | Sol time | 6.60 h | PSP | 0.59 | PUP | 0.79 | Rev | 26.03 |
| 250 | Sol time | 2.51 h | PSP | 0.54 | PUP | 0.74 | Rev | 26.85 | Sol time | 1.74 days | PSP | 0.60 | PUP | 0.80 | Rev | 25.93 |
Facility location

Data
- $U_{in}$: exogenous,
- $C_i$: fixed cost to open a facility,
- $c_i$: operational cost per customer to run the facility.

Objective function

$$\min \sum_{i \in C_k} C_i y_i + \frac{1}{R} \sum_r \sum_i \sum_n c_i W_{inr}$$
Benders decomposition

\[
\begin{align*}
\min & \quad \sum_{i \in C_k} C_i y_i + \frac{1}{R} \sum_r \sum_i \sum_n c_i w_{inr} \\
\text{subject to} & \\
\max & \quad U_{nr} = \sum_i U_{inr} w_{inr} \\
\sum_i w_{inr} & \leq 1 \\
w_{inr} & \leq y_i \\
w_{inr} & \geq 0 \\
w_{inr}, y_i & \in \{0, 1\}. 
\end{align*}
\]
Benders decomposition

Customer subproblem: fix $y_i^*$

$$\max_w U_{nr} = \sum_i U_{inr} w_{inr}$$

subject to

$$\sum_i w_{inr} = 1$$

$$w_{inr} \leq y_i^*$$

$$w_{inr} \geq 0.$$
Benders decomposition

**Primal**

$$\min_w U = - \sum_i U_i w_i$$

subject to

$$\sum_i w_i = 1$$

$$w_i \leq y_i^* \quad \forall i$$

$$w_i \geq 0.$$  

**Dual**

$$\max_{\lambda, \mu} \lambda + \sum_i \mu_i y_i^*$$

subject to

$$\lambda + \mu_i \leq -U_i \quad \forall i$$

$$\mu_i \leq 0 \quad \forall i.$$
Profit optimization, facility location

Bender decomposition

Ongoing work

- Exploit the duality results to generate cuts for the master problem.
- Investigate the use of Benders for other problems.
  - profit maximization,
  - maximum likelihood estimation of the parameters.
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Introduction

- Travel demand is derived from activity demand.
- Activity demand is influenced by socio-economic characteristics, social interactions, cultural norms, basic needs, etc. [Chapin, 1974]
- Activity demand is constrained in space and time [Hägerstråand, 1970].
State of practice

Sequence of decisions

Source: [Scherr et al., 2020]
Research question

Relax the *series of discrete choice models* approach

- The interactions of all decisions is complex.
- Sequence of models is most of the time arbitrary.

Integrated approach

Develop a model involving many decisions:

- activity participation,
- activity location,
- activity duration,
- activity scheduling,
- travel mode,
- travel path.
Research objectives

- Integrated approach based on first principles.
- Theoretical framework: utility maximization.
- Individuals solve a scheduling problem.
- Important aspects: trade-offs on activity sequence, duration and starting time.
- Again, we replace the error terms by draws.
Real data

Dataset

- 2015 Swiss Mobility and Transport Microcensus.
- Daily trip diaries for 57’000 individuals.
- Records of activities, visited location, mode/path choice.
Real data

Assumptions

- Desired start times and durations are the recorded ones.
- Feasible time windows: percentiles start and end times from out of sample distribution.
- Only the recorded locations are considered.
- Uniform flexibility profile across population.
Individual 1 (weekday)

Optimal schedules generated for random draws of $\varepsilon_{an}$
Individual 2 (weekday)

Optimal schedules generated for random draws of $\varepsilon_{an}$
Individual 3 (weekday)

Optimal schedules generated for random draws of $\varepsilon_{an}$
Validation

Activity profiles for full-time workers, Lausanne area

Simulation

Microcensus

Source: SBB. Acknowledgment to Patrick Manser.
Validation

Activity profiles for individuals older than 65, Lausanne area

Simulation

Microcensus

Source: SBB. Acknowledgment to Patrick Manser.
Validation

Activity profiles for students, Lausanne area

Source: SBB. Acknowledgment to Patrick Manser.
Validation

Activity profiles for primary school pupils, Lausanne area

Validation

Microcensus

Source: SBB. Acknowledgment to Patrick Manser.
Activity-based models

Ongoing work

- Synthetic population
- Estimation of the parameters
- Social interactions
Outline

1. Introduction
2. Microeconomics
3. The logit model
4. Profit optimization, facility location
5. Activity-based models
6. Conclusion
Conclusion

**Microeconomics in a nutshell**

- **Optimization**
  - Profit maximization, facility location
- **Discrete choice**
  - Logit model
- Activity-based models

**Tutorial**
- Microeconomics in a nutshell

**Research**
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Readings

- [Pacheco Paneque, 2020]
- [Pacheco et al., 2021]
- [Bortolomiol et al., forta]
- [Bortolomiol et al., fortb]
- [Pougala et al., 2021]
Bibliography I


A general framework for the integration of complex choice models into mixed integer optimization. 

Capturing trade-offs between daily scheduling choices. 
Technical Report TRANSP-OR 210101, Transport and Mobility Laboratory, Ecole Polytechnique Fédérale de Lausanne, Lausanne, Switzerland.

Scherr, W., Manser, P., and Bützberger, P. (2020).  
Simba mobi: Microscopic mobility simulation for corporate planning. 
Transportation Research Procedia, 49:30–43.