

Scheduling of daily activities: an optimization approach

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Outline

- 1 Introduction
- 2 Model
- 3 Mixed integer optimization problem
- 4 Examples



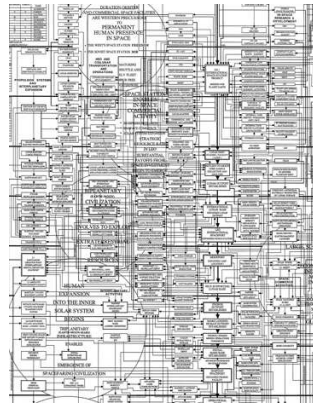
Introduction



- Travel demand is derived from activity demand.
- Activity demand is influenced by socio-economic characteristics, social interactions, cultural norms, basic needs, etc. [Chapin, 1974]
- Activity demand is constrained in space and time [Hägerstrand, 1970].

Rule-based models

$$\begin{aligned} \bar{S}_1 &= \frac{1}{n} \sum_{i=1}^n X_i^1 & HV_1^2 &= VAR(S_1) = \frac{1}{m} \sum_{j=1}^m (\bar{x}_j - \bar{S}_1)^2 & \bar{S}_1^{(j)} &= \frac{1}{n} \sum_{i=1}^n X_i^{1(j)} \\ && HV_1^2 &= VAR(S_1) = \frac{1}{m} \sum_{j=1}^m (\bar{x}_j - \bar{S}_1)^2 & \text{Tels + dus + pake + dus - Tels + dus} &\} \Rightarrow d_f = -d_T - \\ \bar{S}_2 &= \frac{1}{n} \sum_{i=1}^n X_i^2 & HV_2^2 &= VAR(S_2) = \frac{1}{m+1} \sum_{j=1}^{m+1} (x_j^2 - \bar{S}_2)^2 & \text{Tels + dus + pake + dus - Tels + dus} &\} \Rightarrow d_f = -d_T - \\ cov(S_1, S_2) &= \frac{1}{n+1} \sum_{i=1}^n (X_i^1 - \bar{S}_1)(X_i^2 - \bar{S}_2) & & & & \\ VAR(S_1, S_2) &= \frac{VAR(S_1) + VAR(S_2)}{\sqrt{VAR(S_1) + VAR(S_2)}} & & & & \\ \cos(\hat{S}_1, \hat{S}_2) &= \frac{cov(S_1, S_2)}{\sqrt{VAR(S_1) + VAR(S_2)}} & & & & \\ \cos(\hat{S}_1, \hat{S}_2) &= \frac{cov(S_1, S_2)}{\sqrt{VAR(S_1) + VAR(S_2)}} & & & & \end{aligned}$$



State of the art: econometric approach

[Pinjari et al., 2011]

- ... individuals make their activity-travel decisions to maximize the utility derived from the choices they make.
- These model systems usually consist of a series of ... discrete choice models ... that are used to predict ... individuals' activity-travel decisions.
- these model systems employ econometric systems of equations ... to capture relationships between ... socio-demographics and ... attributes on the one hand and the observed activity-travel decision outcomes on the other.



State of the art: econometric approach

[Pinjari et al., 2011]: main criticisms

- *individuals are not necessarily fully rational utility maximizers*
- *the approach does not explicitly model the underlying decision processes and behavioral mechanisms that lead to observed activity-travel decisions.*



Research question

Relax the *series of discrete choice models* approach

- The interactions of all decisions is complex.
- Sequence of models is most of the time arbitrary.

Integrated approach

Develop a model involving all activity-based decisions:

- activity participation,
- activity pattern,
- location choice,
- time of day,
- duration.

Research objectives

- Integrated approach based on first principles.
- Theoretical framework: utility maximization.
- Individuals solve a scheduling problem.
- Important aspects: trade-offs on activity duration.



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First principles



- Each individual n has a time-budget (a day).
- Each activity i considered by n is associated with a utility U_{in} .
- Individuals schedule their activities as to **maximize** the total utility, subject to their time-budget constraint.

Further assumptions



Individuals are **time sensitive**

- Have a desired *start time*, *duration* and/or *end time* for each activity
- Deviations from their desired times in the scheduling process decrease the utility function

Time



- Time horizon: 24 hours.
- Discretization: T time intervals.
- Trade-off between model accuracy and computational time.

Space



- Discrete and finite set S of locations, indexed by s .
- Trips between location are modeled exogenously.
- For each (s_o, s_d) , $\rho(s_o, s_d)$ is the travel time.
- Extensions to include mode and route choices are possible.

Activities

Definition: Activity

An activity requires a trip to/from a given location.



Activities

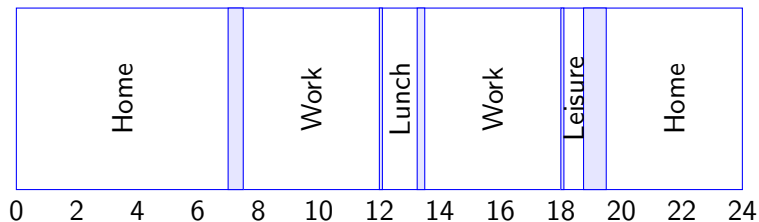


- Set A of activities.
- Location s_a .
- Starting time x_a , $0 \leq x_a \leq T$.
- Duration: $\tau_a \geq 0$.
- Feasible time interval: $[\gamma_a^-, \gamma_a^+]$ (e.g. opening hours).
- “Home”: same, except for boundary conditions.

Modeling location choice

An activity that can take place at m locations is modeled as a set B_k of m activities with a unique location.

Scheduling



Categories



- [Castiglione et al., 2014]: mandatory, maintenance, discretionary.
- Flexible, somewhat flexible, not flexible.

Category

Activities that share the same preference profile.



Preferences

Preferences

- desired starting time x_a^* ,
- desired duration τ_a^* .

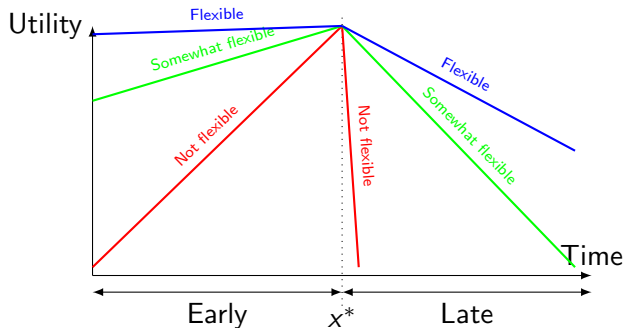
Penalties

- Starting early [Small, 1982]:
 $\theta_e \max(x_a^* - x_a, 0)$.
- Starting late [Small, 1982]:
 $\theta_\ell \max(x_a - x_a^*, 0)$.
- Shorter activity: $\theta_{ds} \max(\tau_a^* - \tau_a, 0)$.
- Longer activity: $\theta_{dl} \max(\tau_a - \tau_a^*, 0)$.



Preferences

Parameters depend on the category type



Disutility of travel



Traveling is part of the activity

- Travel from a to a^+ contributes to U_a : t_a .
- Exception: last activity of the day (home).
- In this version, travel choices are exogenous.

Utility function

An individual n derives the following utility from performing activity a , with a schedule flexibility k :

$$\begin{aligned}
 U_{an} = & \theta_e \max(x_a^* - x_a, 0) \\
 & + \theta_\ell \max(x_a - x_a^*, 0) \\
 & + \theta_{ds} \max(\tau_a^* - \tau_a, 0) \\
 & + \theta_{d\ell} \max(\tau_a - \tau_a^*, 0) \\
 & + \theta_{tt} t_a \\
 & + c_{an} + \sigma_{an} \varepsilon_{an},
 \end{aligned}$$

where ε_{an} is a random term with mean 0 and variance 1.



Utility function



Error terms

- Rely on simulation.
- Draw ξ_{anr} , $r = 1, \dots, R$.
- Optimization problem for each r .
- Utility: U_{anr} .

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Decision variables for individual n and draw r

For each (potential) activity a :

- Activity participation: $w_{anr} \in \{0, 1\}$.
- Starting time: $x_{anr} \in \{0, \dots, T\}$.
- Duration: $\tau_{anr} \in \{0, \dots, T\}$.
- Scheduling: $z_{abnr} \in \{0, 1\}$: 1 if activity b immediately follows a .
- Travel time: t_{anr} : travel time from a to the next activity.



Objective function

Additive utility

$$\max \sum_{a \in A} w_{anr} U_{anr}$$



Constraints

Time budget

$$\sum_a \tau_{anr} = T, \forall n, r.$$

Time windows

$$0 \leq \gamma_a^- \leq x_{anr} \leq x_{anr} + \tau_{anr} \leq \gamma_a^+ \leq T, \forall a, n, r.$$



Constraints

Precedence constraints

$$z_{abnr} + z_{banr} \leq 1, \forall a, b, n, r.$$

Single successor/predecessor

$$\sum_{b \in A \setminus \{a\}} z_{abnr} = w_{anr}, \forall a, n, r,$$

$$\sum_{b \in A \setminus \{a\}} z_{banr} = w_{anr}, \forall a, n, r.$$



Constraints

Travel time

$$t_{anr} = \sum_{b \in A} z_{abnr} \rho(s_a, s_b).$$

Consistent timing

$$(z_{abnr} - 1)T \leq x_{anr} + \tau_{anr} + t_{anr} - x_b \leq (1 - z_{abnr})T, \forall a, b, n, r.$$

Mutually exclusive duplicates

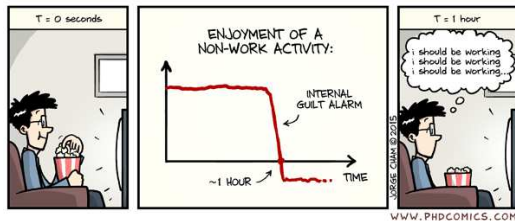
$$\sum_{a \in B_k} w_{anr} = 1, \forall k, n, r.$$

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Internal data set



Data collection

- One-week survey on planned daily schedules for 10 individuals.
- Planning across **week** and **day**.
- Resulting data (unavailable in traditional trip diaries):
 - Set of all considered activities and locations.
 - Preferences in terms of start times and duration.
 - Flexibilities for start times and duration.

Data set

- **Weekly plan:** filled on Sunday, considerations for next Mon-Sun
- Considered activities (*Which out-of-home activities do you plan to do this week?*):
 - Choice between 9 categories + 1 “other” option
 - Frequency of the activity
- Set of transportation modes (for travel time computations)
- Routine preferences:
 - Minimal daily duration at home (Mon-Fri and Sat-Sun)
 - Typical daily duration at work
 - Earliest departure from home (Mon-Fri and Sat-Sun)
 - Latest arrival at home (Mon-Fri and Sat-Sun)



Data set

- **Daily plan:** filled each day (Sun-Sat), considerations for the next day
- Considered activities (*Which activities do you plan to do tomorrow?*)
- Preferred times:
 - Start time: absolute value or relative, e.g. “after work”
 - End time
 - Duration
- Considered location(s) and feasible time windows for each
- Flexibility (early, late, short, long):
 - -1: not flexible
 - 0: moderately flexible (threshold value to be specified in minutes)
 - 1: flexible
- Other constraints (e.g. drop-off at home after grocery shopping)



Data set

- **Model input:**

- All possible activities for the week (Mon-Fri and Sat-Sun)
- All considered locations and travel times matrix (for the preferred mode(s), computed using Google Maps)
- Individual flexibilities and preferred times for each activity

- **Output:**

- Optimal schedule for 1 day (Mon-Fri or Sat-Sun)



Individual 1 (weekday)

- Considered activities $\rightarrow$ location(s):

work	$\rightarrow$	1 (home), 2
education	$\rightarrow$	1
errands	$\rightarrow$	2, 3
fitness	$\rightarrow$	4
leisure/social	$\rightarrow$	5
lunch	$\rightarrow$	2

- Timing preferences and flexibility from daily schedules
- Preference for home time from week schedule



Individual 1 (weekday)

Optimal schedules generated for random draws of $\varepsilon_{a_n} \sim \mathcal{N}(0, 1)$



Individual 2 (weekday)

- Considered activities $\rightarrow$ location(s):

work	$\rightarrow$	1 (home), 2
errands	$\rightarrow$	3, 4, 5
fitness	$\rightarrow$	1
leisure/social	$\rightarrow$	2, 6
lunch	$\rightarrow$	2



Individual 2 (weekday)

Optimal schedules generated for random draws of $\varepsilon_{a_n} \sim \mathcal{N}(0, 1)$



Conclusions

Achievements so far

- Formulation of the model.
- Applied on a simple case.
- The results make sense.
- We are able to draw from a distribution of activity schedules.

Challenges

- Use of real data.
- Parameter estimation.



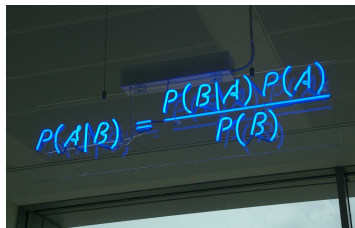
Conclusions



Real data

Challenges: classical RP issues

Conclusions



$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Parameter estimation

- Prior: $f(\beta)$.
- Data: $Y = (i_n, x_n)_{n=1}^N$.
- Likelihood: $L(Y|\beta)$.
- Parameters:

$$f(\beta|Y) \propto L(Y|\beta)f(\beta).$$

Challenges

- Metropolis-Hastings algorithm.
- Calculation of the likelihood.



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