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Abstract

One of the primary modes for worldwide trade of goods is maritime transport, which is exposed to rapid growth. In order to handle the increase of the traffic in ports, optimization of port operations has been studied and implemented in the past. In container terminals, significant amount of work has been done in the field of large scale optimization. In published literature, the main focus lies in integration of berth allocation and quay crane assignment, since these problems are strongly related. On the other hand, literature of integration of yard assignment and berth allocation is relatively less studied and limited to container terminals. In this research, we study two crucial optimization problems of berth allocation and yard assignment in context of bulk ports. We discuss how these problems are interrelated and can be integrated and solved as a single large scale optimization problem. More importantly, we highlight the differences in operations between bulk ports and container terminals which calls the need to devise specific solutions for bulk ports. The objective is to minimize the total service times of vessels berthing at the port. We propose an exact solution algorithm based on column generation to solve the combined problem. The master problem is formulated as a set-partitioning problem, and the subproblem to identify columns with negative reduced costs is solved as a mixed integer program.

Keywords

integrated planning, berth allocation, yard assignment, mixed integer programming, column generation

1 Introduction

Maritime transportation is a major channel of international trade. In the last decade, the shipping tonnage for dry bulk and liquid bulk cargo has risen by 52% and 48% respectively. The total volume of dry bulk cargoes loaded in 2008 stood at 5.4 billion tons, accounting for 66.3 per cent of total world goods loaded (UNCTAD, 2009). The proper planning and management of port operations in view of the ever growing demand represents a big challenge. A bulk port terminal is a zone of the port where sea-freight docks on a berth and is stored in a buffer area called yard for loading, unloading or transshipment of cargo. In general, the bulk terminal managers are faced with the challenge of maximizing efficiency both along the quay side and the yard. From the past research, it is well established that operations research methods and techniques can be successfully used to optimize port operations and enhance terminal efficiency. However while significant contributions have been made in the field of large scale optimization for container terminals, relatively little attention has been directed to bulk port operations.

Bulk terminal operations planning can be divided into two decision levels depending on the time frame of decisions: Tactical Level and Operational Level. Tactical level decisions involve medium to short term decisions regarding resource allocation such as port equipment and labor, berth and yard management, storage policies etc. In practice, these decisions could be based on "rules of thumb" in which the experience of the port managers plays an important role, or alternately more scientific approaches based on operations research methods etc could be in use. The operational level involves making daily and real time decisions such as crane scheduling, yard equipment deployment and last minute changes in response to disruptions in the existing schedule. This research focuses on the tactical level decision planning for the integrated berth and yard management in the context of bulk ports. We focus in particular on two crucial optimization problems in context of bulk port terminals: The Berth Allocation Problem (BAP) and the Yard Assignment Problem.

The tactical berth allocation problem refers to the problem of assigning a set of vessels to a given berthing layout within a given time horizon. There could be several objectives such as minimization of the service times to vessels, minimization of port stay time, minimization of number of rejected vessels, minimization of deviation between actual and planned berthing schedules etc. There are several spatial and temporal constraints involved in the BAP, which lead to a multitude of BAP formulations. The temporal attributes include the vessel arrival process, start of service, handling times of vessels, while the spatial attributes relate to the berth layout, draft restrictions and others. In a container terminal, all cargo is packed into containers, and thus there is no need for any specialized equipment to handle any particular type of cargo. In contrast in bulk ports, depending on the vessel requirements and cargo properties, a wide variety of equipment is used for discharging or loading operations. For example, liquid bulk is generally discharged using pipelines which are installed at only certain sections along the quay. Similarly, a vessel may require the conveyor facility to load cargo from a nearby factory outlet to the vessel. Thus, the cargo type on the vessel needs to be explicitly taken into consideration while modeling the berth allocation problem in bulk ports. The tactical yard assignment problem refers to decisions that concern the storage location and the routing of materials. This affects the travel distance between the assigned berth to the vessel and storage location of the cargo type of the vessel on the yard, and furthermore determines the storage efficiency of the yard. Thus, the problems of berth allocation and yard management are interrelated. The start times and end times of operations of vessels determine the workload distribution and deployment of yard equipment such as loading shovels, wheel loaders etc in the yardside. Moreover, berthing locations of vessels determine the storage locations of specific cargo types to specific yard locations, which minimize the total travel distance between the assigned berthing positions to the vessels and the yard locations storing the cargo type for the vessel. Similarly, the yard assignment of specific cargo types has an impact on the best berthing assignment for vessels berthing at the port. In this paper, we present an integrated model for the dynamic, hybrid berth allocation problem and yard assignment in context of bulk ports. To the best of our knowledge, very few scholars have attempted to investigate this problem in context of container terminals, while there is no published literaure for bulk ports. We present an exact solution algorithm based on column generation to solve the combined large scale problem.

2 Literature Review

From the past OR literature on container terminal operations, it is well established that integrated planning of operations can allow port terminals to reduce congestion, lower delay costs and enhance efficiency. Significant contribution has been made in the field of large scale optimization and integrated planning of operations in container terminals. Bulk ports on the other hand have recieved almost no attention in operations research literature. The integrated berth allocation and quay crane assignment or scheduling problem has been studied in the past by Park and Kim (2003), Meisel and Bierwirth (2006), Imai *et al.* (2008a), Meisel and Bierwirth (2008), and more recently by Giallombardo *et al.* (2010) and Vacca (2011) for container terminals. Comprehensive literature surveys on container terminal operations can be found in Steenken *et al.* (2004), Stahlbock and Voss (2008), Bierwirth and Meisel (2010).

The dynamic, hybrid berth allocation problem in context of bulk ports is studied by Umang *et al.* (2012). The berth allocation problem in container terminals has been widely studied in the past. Imai *et al.* (1997), Imai *et al.* (2001), Imai *et al.* (2008b), Imai *et al.* (2003), Monaco and Sammarra (2007), Buhrkal *et al.* (2011), Zhou and Kang (2008), Han *et al.* (2010), Cordeau *et al.* (2005), Mauri *et al.* (2008) propose methods to solve the discrete berth allocation problem. The continuous berth allocation problem is studied by Li *et al.* (1998), Guan *et al.*

(2002), Park and Kim (2003), Guan and Cheung (2004), Park and Kim (2002), Kim and Moon (2003), Lim (1998), Tong *et al.* (1999), Imai *et al.* (2005) and Chang *et al.* (2008). The berth allocation problem with hybrid layout is addressed by Moorthy and Teo (2006), Dai *et al.* (2008), Nishimura *et al.* (2001) and Cheong *et al.* (2010), and position-dependent handling times are considered by Cordeau *et al.* (2005) and Imai *et al.* (2007) for indented berths.

Yard management in container terminals involves several tactical and operational level decision problems. Scheduling and deployment of yard cranes is addressed by Cheung *et al.* (2002), Zhang *et al.* (2002), Ng and Mak (2005), Ng (2005) and Jung and Kim (2006). Storage and space allocation, stacking and re-marshalling strategies have been studied by Kim and Kim (1999), Kim *et al.* (2003), Lee *et al.* (2006) and few others. Nishimura *et al.* (2009) investigate the storage plan for transshipment hubs, and propose an optimization model to minimize the sum of the waiting time of feeders and the handling times for transshipment containers flow. Transfer operations that consist of routing and scheduling of internal trucks, straddle carriers and AGV's have been studied by Liu *et al.* (2004), Vis *et al.* (2005), and Cheng *et al.* (2005) among others. Works on integrated problems related to yard management in container terminals include Bish *et al.* (2001) and Kozan and Preston (2006) who propose the integration of yard allocation and container transfers, whereas Chen *et al.* (2007) and Lau and Zhao (2007) study the integrated scheduling of handling equipment in a container terminal. In the following, we discuss in more detail some articles relevant to our study.

Moorthy and Teo (2006) discuss the concepts of berth template and yard template in context of transshipment hubs in container shipping. They study the delicate trade-off between the level of service as indicated by the vessel waiting times and the operational cost for moving containers between the yard and quay in a container terminal. A robust berth allocation plan is developed using sequence pair approach, with the objective to minimize the total expected delays and connectivity cost that is related to the distance between the berthing positions of vessels belonging to the same transshipment group.

Cordeau *et al.* (2007) study the Service Allocation Problem (SAP), a tactical problem arising in the yard management of Gioia Tauro Terminal. The SAP is a yard management problem that deals with dedicating specific areas of the yard and the quay to the services or route plans of shipping companies which are planned in order to match the demand for freight transportation. The objective of the SAP is the minimization of container rehandling operations in the yard and it is formulated as a Generalized Quadratic Assignment Problem (GQAP, see e.g. Cordeau et al. (2006), and Hahn et al. (2008)). An evolutionary heuristic is developed to solve larger instances obtained from the real port data.

More recently, Zhen *et al.* (2011) propose a mixed integer model to simultaneously solve the tactical berth template and yard template planning in transshipment hubs. The objective is to minimize the sum of service cost derived from the violation of the vessels expected turnaround time intervals and operation cost related to the route length of transshipment container flows in yard. A heuristic algorithm is developed to solve large scale instances within reasonable time and numerical experiments are conducted on instances from real world data to validate the efficiency of the proposed algorithm.

To the best of our knowledge, operations research problems have received almost no attention thus far in the context of bulk port terminals. In context of container terminals, the major focus in the field of large scale optimization has been on studying the integrated berth allocation and quay crane scheduling or assignment problem, while very few studies have been attempted to study the integrated berth and yard template planning. In this research we study the integrated modeling of berth and yard management in context of bulk ports, and highlight the specific issues in bulk port operations that necessitate the need to devise specific solutions for bulk terminals.

3 Problem Statement

In this section we elaborate on the background for the integrated berth and yard assignment problem in context of bulk ports. A schematic representation of a bulk port terminal is shown in Figure 1. We consider a set of vessels N, to be berthed on a continuous quay of length L over a time horizon H. We consider dynamic vessel arrivals and a hybrid berth layout in which the quay boundary is discretized into a set M of sections of variable lengths. The dynamic, hybrid berth allocation problem in bulk ports is studied by Umang *et al.* (2012), in which two alternate exact solution approaches and a heuristic approach are proposed to solve the problem. In the present work, we extend the berth allocation problem to account for the assignment of different yard locations to specific cargo types and vessels berthing at the port. Thus, unlike the BAP model in which the unit handling times for given section along the quay k and cargo type w were provided as input parameters to the model, in the integrated framework the assignment of cargo locations to specific cargo types and vessels are also decision variables.

A major difference between bulk port and container terminal operations is the need to explicitly account for the cargo type on the vessel in bulk ports. Depending on the vessel requirements and cargo types, a wide variety of specialized equipment such as conveyors an pipelines are used for discharging or loading operations. In contrast in a container terminal, all cargo is packed into containers, and thus there is no need for any specialized equipment to handle any particular type of cargo. Furthermore in bulk ports, depending on the cargo properties, there may be additional restrictions on the storage of specific cargo types in the yard which forbids two or more cargo types to be stored in adjacent yard locations to avoid intermixing.

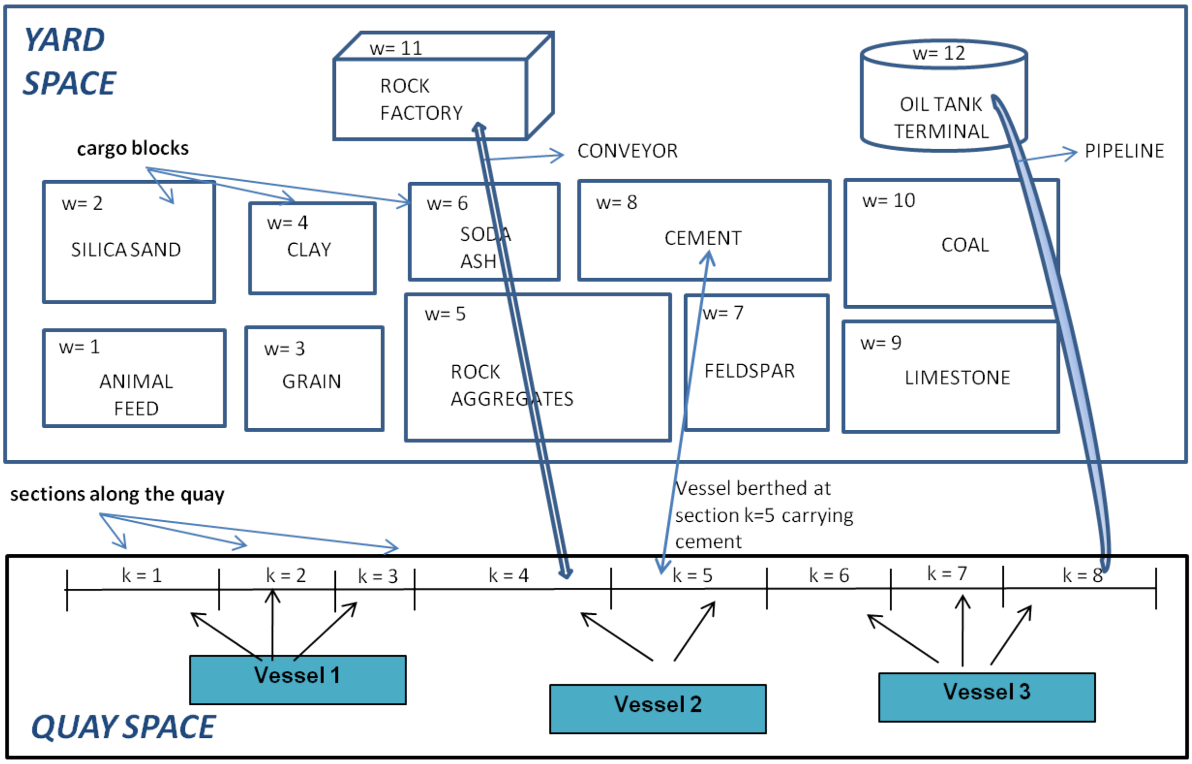

Figure 1: *Schematic representation of a bulk port terminal*

As discussed in Umang *et al.* (2012), in our model the main assumption in the computation of handling times is that all sections occupied by the berthed vessel are being operated on simultaneously. The amount of cargo handled at each section is assumed proportional to the section length. The handling time of the vessel is the time taken to load or discharge the section whose operation finishes last. The unit processing or handling time of a given vessel h_{ik}^{w} has a fixed component dependent on the number of quay cranes operating on the vessel, and a variable component which is dependent on the distance between the section k occupied by the vessel along the quay and the storage location of the cargo type w of the vessel on the yard. In the integrated model, we assume that a given vessel can discharge (load) cargo that can be transferred to (from) multiple yard locations. Thus, the distance measure used to calculate this variable component of handling time is the weighted average of the distance where the weights are equal to the cargo quantities that are transferred to (from) each yard location assigned to the vessel. It is further assumed that every yard location $p \in P$ has a dedicated cargo type $w \in W$, or alternately the yard location p is not assigned to any cargo type. Thus, the cargo assignment to a specific yard location does not vary over time.

Based on the preceding discussion, the unit handling time h_{ik}^{w} for vessel i with cargo type w occupying section k along the quay includes the time taken to transfer unit quantity of cargo between the cargo location on the yard and section k, and the time taken to load (or unload) the cargo from the quay side to the vessel. In equation 1, these are denoted by β_k and α_k respectively. Thus we have,

$$h_{ik}^{w} = \alpha_{ik}^{w} + \beta_{ik}^{w} \quad (1)$$

$$\alpha_{ik}^{w} = T/n_{ik}^{w} \quad (2)$$

$$\beta_{ik}^{w} = V_w r_k^i \quad (3)$$

In equation 2, T is the crane handling rate for loading or discharging operations, and n_{ik}^{w} is the number of cranes operating in section k on vessel i for cargo type w. β_{ik}^{w} is the time taken to transfer a unit quanity of cargo between the cargo location w on the yard and the section k for vessel i, which is assumed to be a linear function of the weighted average distance r_k^i between the section k and all cargo locations assigned to vessel i. The parameter V^w depends on the rate of transfer of cargo type w. Thus for example, if a vessel is using the conveyor facility to load rock aggregates from the rock factory directly into the vessel, the parameter V_w is equal to the cargo transfer rate for the conveyor facility, and if there are no additional cranes operating on the vessel, the parameter α_{ik}^{w} which is provided as an input parameter to the model is equal to zero. In practice, the fixed specialized equipment facilites such as conveyors and pipelines are dedicated to handling certain cargo types. For example, liquid bulk is transferred using pipelines, and rock aggregates are transferred using conveyor facility. Thus, the assignment of these cargo types to the specific locations of these facilities in the port may be predefined in the model. The objective of the integrated optimization model that we solve is to minimize the sum of the service times of all vessels, which includes the handling or processing times and the berthing delays for all vessels berthing at the port.

4 Mathematical Formulation

In this section, we present an mixed integer linear programming formulation for the integrated berth allocation and yard assignment in bulk ports.

Input parameters The following input data is assumed available:-

$N =$	set of vessels
$M =$	set of sections
$P =$	set of cargo locations
$W =$	set of cargo types
$H =$	set of time steps
$W_i =$	set of cargo type(s) to be loaded or discharged from vessel i indexed from w=1 to w=$\|W_i\|$
$\bar{P}(p) =$	set of cargo locations neighbouring cargo location p
$\bar{W}(w) =$	set of cargo types that cannot be stored adjacent to cargo type w
$i =$	1,...,$\|N\|$ vessels berthing at the port
$k =$	1,...,$\|M\|$ sections along the quay
$p =$	1,...,$\|P\|$ cargo locations on the yard
$w =$	1,...,$\|W\|$ cargo types on the yard
$t =$	1,...,$\|H\|$ time steps in the planning horizon
$A_i =$	expected arrival time of vessel i
$D_i =$	draft of vessel i
$L_i =$	length of vessel i
$Q_i =$	quantity of cargo for vessel i
$d_k =$	draft of section k
$\ell_k =$	length of section k
$b_k =$	starting coordinate of section k
$\alpha_{ik}^{w} =$	deterministic component of handling time for cargo type w of vessel i berthed at section k $\forall w \in W_i$;
$L =$	total length of quay
$V_w =$	constant dependent on rate of transfer of cargo type w
$r_k^p =$	distance between cargo location p and section k
$R_w =$	maximum amount of cargo of type w that can be handled in a single time step
$B =$	large positive constant
$F =$	maximum number of cargo locations that can be assigned to a single vessel
$\rho_{i\ell k} =$	fraction of cargo handled at section k when vessel i is berthed at starting section ℓ

$$\delta_{i\ell k} = \begin{cases} 1 & \text{if vessel } i \text{ starting at section } \ell \text{ touches section } k; \\ 0 & \text{otherwise.} \end{cases}$$

Decision Variables The following decision variables are used in the model:-

m_i integer ≥ 0, represents the starting time of handling of vessel $i \in N$;

c_i integer ≥ 0, represents the total handling time of vessel $i \in N$;

$h_{ik}^w =$ handling time for unit quantity of cargo type w for vessel i berthed at section k $\forall w \in W_i$;

$\beta_{ik}^w =$ variable component of handling time of vessel i with cargo type w berthed at section k along the quay;

$\lambda_{ip} =$ amount of cargo handled by vessel i at cargo location p;

$\eta_i =$ number of cargo locations assigned to vessel i;

$r_k^i =$ weighted average distance between vessel i occupying section k and all cargo locations assigned to the vessel;

s_k^i binary, equals 1 if section $k \in M$ is the starting section of vessel $i \in N$, 0 otherwise;

x_{ik} binary, equals 1 if vessel $i \in N$ occupies section $k \in M$, 0 otherwise;

y_{ij} binary, equals 1 if vessel $i \in N$ is berthed to the left of vessel $j \in M$ without any overlapping in space, 0 otherwise;

z_{ij} binary, equals 1 if handling of vessel $i \in N$ finishes before the start of handling of vessel $j \in N$, 0 otherwise;

π_w^p binary, equals 1 if cargo type w is stored at cargo location p;

ω_t^{ip} binary, equals 1 if vessel i is being handled at location p at time t;

ϕ_{ip} binary, equals 1 if vessel i uses cargo location p;

θ_{it} binary, equals 1 if vessel i is being handled at time t;

Formulation

$$\min \sum_{i}(m_i - A_i + c_i) \tag{4}$$

$$\text{s.t. } m_i - A_i \geq 0 \qquad \forall i \epsilon N \tag{5}$$

$$\sum_{k\epsilon M}(s_k^j b_k) + B(1-y_{ij}) \geq \sum_{k\epsilon M}(s_k^i b_k) + L_i \qquad \forall i,j\epsilon N, i \neq j \tag{6}$$

$$m_j + B(1-z_{ij}) \geq m_i + c_i \qquad \forall i\epsilon N, \forall j\epsilon N, i \neq j \tag{7}$$

$$y_{ij} + y_{ji} + z_{ij} + z_{ji} \geq 1 \qquad \forall i\epsilon N, \forall j\epsilon N, i \neq j \tag{8}$$

$$\sum_{k\epsilon M} s_k^i = 1 \qquad \forall i\epsilon N \tag{9}$$

$$\sum_{k\epsilon M}(s_k^i b_k) + L_i \leq L \qquad \forall i\epsilon N \tag{10}$$

$$\sum_{p\epsilon M}(\delta_{i\ell k} s_\ell^i) = x_{ik} \qquad \forall i\epsilon N, \forall k\epsilon M \tag{11}$$

$$(d_k - D_i)x_{ik} \geq 0 \qquad \forall i\epsilon N, \forall k\epsilon M \tag{12}$$

$$c_i \geq h_{ik}^w \rho_{i\ell k} Q_i - B(1-s_\ell^i) \qquad \forall i\epsilon N, \forall l\epsilon M, \forall k\epsilon M, \forall w\epsilon W_i \tag{13}$$

$$h_{ik}^w = \alpha_{ik}^w + \beta_{ik}^w \qquad \forall w\epsilon W_i, \forall k\epsilon M \tag{14}$$

$$\beta_{ik}^w = V_w r_k^i \qquad \forall i\epsilon N, \forall w\epsilon W_i, \forall k\epsilon M \tag{15}$$

$$r_k^i = \sum_{p\epsilon P}(r_k^p \lambda_{ip})/Q_i \qquad \forall i\epsilon N, \forall k\epsilon M \tag{16}$$

$$\sum_{p\epsilon p} \phi_{ip} \leq F \qquad \forall i\epsilon N \tag{17}$$

$$\pi_w^p + \pi_u^q \leq 1 \qquad \forall w\epsilon W, \forall u\epsilon \bar{W}(w), \forall p\epsilon P, \forall q\epsilon \bar{P}(p) \tag{18}$$

$$\sum_{i\epsilon N} \omega_t^{ip} \leq 1 \qquad \forall p\epsilon P, \forall t\epsilon H \tag{19}$$

$$\sum_{w\epsilon W} \pi_w^p \leq 1 \qquad \forall p\epsilon P \tag{20}$$

$$\phi_{ip} \leq \pi_w^p \qquad \forall i\epsilon N, \forall w\epsilon W_i, \forall p\epsilon P \tag{21}$$

$$\omega_t^{ip} \geq \phi_{ip} + \theta_{it} - 1 \qquad \forall i\epsilon N, \forall p\epsilon P, \forall t\epsilon H \tag{22}$$

$$\omega_t^{ip} \leq \phi_{ip} \qquad \forall i\epsilon N, \forall p\epsilon P, \forall t\epsilon H \tag{23}$$

$$\omega_t^{ip} \leq \theta_{it} \qquad \forall i\epsilon N, \forall p\epsilon P, \forall t\epsilon H \tag{24}$$

$$\sum_{t\epsilon H} \theta_{it} = c_i \qquad \forall i\epsilon N \tag{25}$$

$$t + B(1-\theta_{it}) \geq m_i + 1 \qquad \forall i\epsilon N, \forall t\epsilon H \tag{26}$$

$$t \leq m_i + c_i + +B(1-\theta_{it}) \qquad \forall i\epsilon N, \forall t\epsilon H \tag{27}$$

$$Q_i = \sum_{p\epsilon P} \lambda_{ip} \qquad \forall i\epsilon N \tag{28}$$

$$\lambda_{ip} \leq \phi_{ip} Q_i \qquad \forall i\epsilon N, \forall p\epsilon P \tag{29}$$

$$\phi_{ip} \leq \lambda_{ip} \qquad \forall p\epsilon P \tag{30}$$

$$\lambda_{ip} \leq \sum_{w\epsilon W_i} \sum_{t\epsilon H}(R_w \omega_t^{ip} + B(1-\pi_w^p)) \qquad \forall i\epsilon N, \forall p\epsilon P \tag{31}$$

$$s_k^i \,\epsilon\, \{0,1\} \qquad \forall i \epsilon N, \forall k \epsilon M \tag{32}$$

$$x_{ik} \,\epsilon\, \{0,1\} \qquad \forall i \epsilon N, \forall k \epsilon M \tag{33}$$

$$y_{ij} \,\epsilon\, \{0,1\} \qquad \forall i, j \epsilon N \tag{34}$$

$$z_{ij} \,\epsilon\, \{0,1\} \qquad \forall i, j \epsilon N \tag{35}$$

$$\pi_w^p \,\epsilon\, \{0,1\} \qquad \forall p \epsilon P, \forall w \epsilon W \tag{36}$$

$$\omega_t^{ip} \,\epsilon\, \{0,1\} \quad \forall i \epsilon N, \forall p \epsilon P, \forall t \epsilon H \tag{37}$$

$$\phi_{ip} \,\epsilon\, \{0,1\} \qquad \forall i \epsilon N, \forall p \epsilon P \tag{38}$$

$$\theta_{it} \,\epsilon\, \{0,1\} \qquad \forall i \epsilon N, \forall t \epsilon H \tag{39}$$

The objective function (4) minimizes the total service time of all vessels berthing at the port. Constraint (5) ensures that vessels can be serviced only after their arrival. Constraints (6)-(8) are the non-overlapping restrictions for any two vessels berthing at the port. Note that the constraints (6)-(7) have been linearized by using a large positive constant B. Constraints 9)-(11) ensure that each vessel occupies only as many number of sections as determined by its length and the starting section occupied by the vessel. Constraints (12) ensure that the draft of the vessel does not exceed the draft of any occupied section. Constraints (13) are used to determine the total handling time for any given vessel by which is equal to the time taken to process the section whose operation finishes last. Constraints (14) determine the unit handling time of a vessel i at a given section k as the sum of the fixed component dependent on the number of cranes operating in the section and the variable component given by constraint (15) which is dependent on the weighted average distance between the section k and all cargo locations assigned to the vessel. The average distance weighted over the cargo quantities transferred between each individual cargo location assigned to the vessel and the section occupied by the vessel is calculated by constraints (16). Constraints (17) impose an upper bound on the maximum number of cargo locations that can be assigned to any single vessel. Constraints (18) ensure that two cargo types that cannot be stored together are not assigned to adjacent yard locations. Constraints (19) state that a given cargo location at a given time can be used by at most one vessel to avoid congestion. Constraints (20) state that a given yard location can be assigned to at most one cargo type. Constraints (21) ensure that a vessel is assigned to a yard location only if that yard location stores the cargo type on the vessel. Constraints (22) - (24) control the values of the binary decision variable ω_t^{ip} which should take value equal to 1, if and only if both the binary variables ϕ_{ip} and θ_{it} are equal to 1. Similarly, constraints (25) - (27) contol the values of the binary decision variable θ_{it} which should be equal to 1 at all times between the start and end of berthing of the vessel along the quay. Constraints (28)-(30)

state that the total cargo quantity to be loaded (discharged) is equal to the sum of the cargo quantities transferred from (to) all the cargo locations assigned to the vessel. Constraints (31) are capacity constraints to ensure that the amount of cargo transferred in a unit time does not exceed the maximum amount of cargo that can be handled as given by parameter R_w for cargo type w.

5 Solution Approach

Since the mixed programming formulation of the integrated model is extremely complex and unwieldy, it cannot be directly solved using CPLEX. Instead we propose an exact solution algorithm based on column generation using the branch and price framework to solve the problem. For mathematical justification of branch and price, please refer to Barnhart *et al.* (1998) and Feillet (2010). In general, this method decomposes the model into a master problem and sub problem, and hence reduces the solution space and enhances the convergence speed.

Algorithm 1: Branch and Price

```
Data: data file, Ω, finished - boolean, duals - float
Result: Ω1 ⊂ Ω, solution
begin
    Ω1 ← greedy(Ω)
    duals ← ∅
    solution ← ∅
    repeat
        duals ← solveMaster(Ω1)
        finished ← true
        for i ∈ N do
            temp ← solveSubProblem(i, duals)
            if reducedCost(temp) < 0 then
                Ω1 ∪ temp
                finished ← false
    until finished
    solution ← solveMaster(Ω1)
    if solution ∉ Z then
        solution ← branch&bound(solution)
    print solution
```

In our proposed algorithm (1), we obtain an initial feasible solution using a greedy heuristic which is passed on to the master problem in the first iteration. The greedy heuristic (algorithm 2) is designed to extract any feasible solution for the master problem. The heuristic searches through the space of all of the initial feasible columns where one column represents the assignment of a single vessel to section(s) and cargo location(s) in time. In each iteration, the restricted master problem is solved and the dual variables are reported to the sub problem, which is executed for each vessel separately. Thus, at every iteration $|N|$ subproblems equal to the number of vessels in the instance are solved to optimality. If neither of the sub problems yield a column with negative reduced cost, column generation is terminated and integrality of solution is checked. If the solution is not integral, branching on the current solution is needed, otherwise algorithm terminates by printing the optimal solution.

Algorithm 2: Greedy Heuristic

Data: N - set of vessels, C - set of columns
Result: $inital\ solution$

begin
  $inital\ solution \leftarrow \emptyset$
  for $i \in N$ **do**
    for $j \in C$ **do**
      if $initial\ solution = \emptyset$ **then**
        $initial\ solution \leftarrow j$
        break
      else if $i \in j$ **and** j *is compatible with* $initial\ solution$ **then**
        $initial\ solution \cup j$
        break
  return $inital\ solution$

5.1 Master Problem

In our algorithm, the master problem is formulated as a set-partitioning model. Let Ω_1 be the pool of active columns in the master problem which represent a subset of all the feasible berthing assignments Ω. Here a feasible assignment represents the assignment of a single vessel at a given set of sections(s) for a specific time period and assigned to a set of specific cargo location(s) in the yard. Let c_a be the cost of assignment $a \in \Omega_1$. The following input parameters are used in the master problem:

$$A_a^i = \begin{cases} 1 & \text{if vessel } i \text{ is assigned in assignment } a, \\ 0 & \text{otherwise.} \end{cases}$$

$$B_a^{kt} = \begin{cases} 1 & \text{if section } k \text{ is occupied at time } t \text{ in assignment } a, \\ 0 & \text{otherwise.} \end{cases}$$

$$C_a^{lw} = \begin{cases} 1 & \text{if cargo } w \text{ is stored at location } l \text{ in assignment } a, \\ 0 & \text{otherwise.} \end{cases}$$

$$D_a^{lt} = \begin{cases} 1 & \text{if cargo location } l \text{ is handling assignment } a \text{ at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

And the parameter ct_w, which is the number of vessels carrying cargo type w. The master problem for the integrated berth allocation and yard assignment can then be formulated using the following set partitioning model:

$$minimize \sum_{a \in \Omega_1} c_a \cdot \lambda_a \tag{40}$$

$$\sum_{a \in \Omega_1} A_a^i \cdot \lambda_a = 1, \qquad \forall i \in N, \tag{41}$$

$$\sum_{a \in \Omega_1} B_a^{kt} \cdot \lambda_a \leq 1, \qquad \forall k \in M, \forall t \in H, \tag{42}$$

$$\sum_{a \in \Omega_1} C_a^{lw} \cdot \lambda_a - ct_w \cdot \mu_w^l \leq 0, \qquad \forall l \in L, \forall w \in W, \tag{43}$$

$$\sum_{w \in W} \mu_w^l \leq 1, \qquad \forall l \in L, \tag{44}$$

$$\mu_w^l + \mu_{\overline{w}}^{\bar{l}} \leq 1, \qquad \forall l \in L, \forall \bar{l} \in \overline{L}, \forall w \in W, \forall \overline{w} \in \overline{W}, \tag{45}$$

$$\sum_{a \in \Omega_1} D_a^{lt} \cdot \lambda_a \leq 1, \qquad \forall l \in L, \forall t \in H, \tag{46}$$

$$\lambda_p \geq 0, \qquad \forall a \in \Omega_1, \tag{47}$$

$$\mu_w^l \geq 0, \qquad \forall l \in L, \forall w \in W. \tag{48}$$

where λ_a indicates whether assignment a is selected ($\lambda_a = 1$) or not ($\lambda_a = 0$) in the solution and μ_w^l is an additional binary decision variable that indicates whether location $l \in L$ stores cargo type $w \in W$.

The objective function **(40)** minimizes the total service time of vessels berthing at the port. Constraint **(41)** ensures that each vessel has exactly one feasible berthing assignment in the final solution. Constraint **(42)** eliminates overlaps, stating that at most one vessel can occupy a given section at given time. Constraints **(43)** and **(44)** ensure that at most one cargo type can be stored in any location in the yard. Constraint **(45)** ensures that cargo types that cannot be stored together are not stored at adjacent locations in the yard. Constraint **(46)** ensures that at most one vessel can be handled at a cargo location at a given time. Constraints **(47)** and **(48)** state that both the decision variables λ_p and μ_w^l can only take positive values.

5.2 Sub-Problem

In each iteration, we solve $|N|$ subproblems, one for each vessel $i \in N$. In each subproblem, the objective is to identify the feasible column for that particular vessel which has the most negative reduced cost that should be added to the current pool of columns Ω_1 in the master problem. Note that the index $i \in N$ is removed from all decision variables and input parameters since the problem is solved separately for each given vessel $i \in N$. This is given by:

$$\begin{aligned} minimize\ (c + s - a) - (\alpha + \textstyle\sum_{k \in K} \sum_{t \in T} \beta_{kt} \cdot beta_{kt} + \\ \textstyle\sum_{l \in L} \sum_{t \in T} \gamma_{lt} \cdot gamma_{lt} + \sum_{l \in L} \sum_{w \in W} \delta_{lw} \cdot delta_{lw}) \end{aligned} \tag{49}$$

where:

- $\alpha, \beta_{kt}, \gamma_{lt}, \delta_{lw}$ are dual variables obtained from master problem
- decision variables connecting the duals:
  - $beta_{kt} \in (0,1)$ – 1 if vessel occupies section k at time t, 0 otherwise
  - $gamma_{lt} \in (0,1)$ – 1 if vessel uses location l at time t, 0 otherwise
  - $delta_{lw} \in (0,1)$ – 1 if cargo type w is stored at location l, 0 otherwise
- c is the handling time of the solution
- a is the time of arrival of the vessel
- s is the starting time of the solution

The input parameters used in the subproblem are:

- $fraction_{jk}$ – fraction of cargo handled at section k, if the starting section of the vessel is section j
- M – large enough number (set to 1 000 000)
- sc_j – starting coordinate of section j
- $length$ – length of the vessel
- ql – quay length

- o_{jk} – 1 if section k is occupied by the vessel, when section j is the starting section, 0 otherwise
- Z – maximum number of locations used by vessel
- w – cargo type carried on the vessel
- d_{kl} – distance between section k and location l
- $cranes_k$ – number of cranes in section k
- F – crane handling rate, V_w – cargo transfer rate

The decision variables used in the subproblem are:

- $ht_k \geq 0$ – handling time of section k
- $ss_j \in (0, 1)$ – 1 if section j is the starting section of the vessel
- $x_j \in (0, 1)$ – 1 if section j is occupied by the vessel
- $split_l \in (0, 1)$ – 1 if vessel uses location l
- $cs_l \geq 0$ – quantity of cargo stored at location l
- $td_k \geq 0$ – total average distance for section k
- $time_t \in (0, 1)$ – 1 if the vessel is at time t served, 0 otherwise

The subproblem is formulated as a mixed integer program as follows:

$$s - a \geq 0, \tag{50}$$

$$c \geq ht_k \cdot fraction_{jk} - M \cdot (1 - ss_j), \quad \forall j, k \in K, \tag{51}$$

$$\sum_{j \in K} ss_j = 1, \tag{52}$$

$$\sum_{j \in K} ss_j \cdot sc_j + length \leq ql, \tag{53}$$

$$\sum_{k \in K} o_{jk} \cdot ss_j = x_j, \quad \forall j \in K, \tag{54}$$

$$\sum_{l \in L} split_l \leq Z, \tag{55}$$

$$split_l \leq delta_{lw}, \quad \forall l \in L, \tag{56}$$

$$\sum_{l \in L} cs_l = quantity, \tag{57}$$

$$cs_l \leq split_l \cdot quantity, \quad \forall l \in L, \tag{58}$$

$$split_l \leq cs_l, \quad \forall l \in L, \tag{59}$$

$$td_k = \left(\sum_{l \in L} d_{kl} \cdot cs_l \right) / quantity, \quad \forall k \in K, \tag{60}$$

$$ht_k = F / cranes_k + V_w \cdot td_k, \quad \forall k \in K, \tag{61}$$

$$\sum_{t \in T} time_t = c, \tag{62}$$

$$t + M \cdot (1 - time_t) \geq s + 1, \quad \forall t \in T, \tag{63}$$

$$t \leq s + c + M \cdot (1 - time_t), \quad \forall t \in T, \tag{64}$$

$$beta_{kt} \geq x_k + time_t - 1, \quad \forall k \in K, \forall t \in T, \tag{65}$$

$$beta_{kt} \leq x_k, \quad \forall k \in K, \forall t \in T, \tag{66}$$

$$beta_{kt} \leq time_t, \quad \forall k \in K, \forall t \in T, \tag{67}$$

$$gamma_{lt} \geq split_l + time_t - 1, \quad \forall l \in L, \forall t \in T, \tag{68}$$

$$gamma_{lt} \leq split_l, \quad \forall l \in L, \forall t \in T, \tag{69}$$

$$gamma_{lt} \leq time_t, \quad \forall l \in L, \forall t \in T. \tag{70}$$

Constraint **(50)** ensures that vessels can only be served after their arrival. Constraint **(51)** calculates the total handling time of the vessel. Constraint **(52)** states that a vessel has exactly one starting section. Constraint **(53)** states that a vessel should be berthed such that it does not extend beyond the length of the quay. Constraint **(54)** determines if a particular section is occupied by the vessel. Constraints **(55)** impose an upper bound on the number of cargo locations that can be assigned to a single vessel. Constraint **(56)** ensures that a vessel is assigned to a yard location only if that yard location stores the cargo type on the vessel. Constraints **(57) - (59)** state that the total cargo quantity to be loaded (discharged) is equal to the sum of the cargo quantities transferred from (to) all the cargo locations assigned to the vessel. Constraint **(60)** calculates the weighted average distance over cargo quantities and constraint **(61)** calculates the handling time for given vessel and berthed section. Constraints **(62) - (64)** control the values of the binary decision variable $time_t$ ensuring they take value equal to 1 at all times when the vessel is berthed along the quay. Constraints **(63) - (65)** control the values of the binary decision variable $beta_{kt}$ to ensure that they take value equal to 1 if and only if both x_k and $time_t$ are equal to 1. Similarly, constraints **(66) - (68)** control the values of the binary decision variable $gamma_{lt}$ to ensure that they take value equal to 1 if and only if both $split_l$ and $time_t$ are equal to 1.

6 Conclusions and Future Work

In this paper, we present an integrated model for the unified optimization of berth allocation and yard assignment in bulk ports. To the best of our knowledge this problem has not been studied thus far in context of bulk ports, and our work makes an exploratory study on this crossover field. An exact solution algorithm based on column generation is proposed to solve the integrated model. The master problem is formulated as a set partitioning problem, and the subproblem is solved as a mixed integer programming problem. Simple numerical results based on real port data have also been conducted to validate the proposed algorithm, though results have not been included in this paper.

Currently we are working on applying the branch and bound framework to get integer solutions from the solution of the linear relaxation of the master problem which provides a lower bound on the exact solution. In future, we also want to validate the efficiency of the proposed algorithm on reasonably large sized instances inspired from real port data.

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