The Ideal Train Timetabling Problem

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Abstract

The aim of this paper is to analyze and to improve the current planning process of the passenger railway service.

At first, the state-of-the-art in research is presented. However, given the recent changes in legislature allowing competitors in the railway industry, the current way of planning is not sufficient anymore. The original planning is based on the accessibility/mobility concept provided by one carrier, whereas the competitive market consists of several carriers that are driven by the profit.

Moreover, the current practice does not define the ideal timetables and thus it is assumed that they evolve incrementally, based on a historical data (train occupation, ticket sales, etc.).

And thus, we introduce a definition of an ideal timetable that is expressed using the passenger cost. In order to create the timetables itself, we propose to insert the Ideal Train Timetabling Problem (ITTP) that is solved for each Train Operating Company (TOC) separately, into the planning process. The ITTP approach incorporates the passenger demand in the planning and its aim is to minimize the passenger cost(s).

The outcome of the ITTP is the ideal timetables (including connections between the trains and weighted by the demand), which then serve as an input for the traditional Train Timetabling Problem (TTP). The TTP takes into account wishes of each TOC (the ideal timetables) and creates global feasible timetable for the given railway network, while minimizing the changes of the TOCs' wishes.

The ITTP is in line with the new market structure and it can produce both: non-cyclic and cyclic timetables. The model is tested on the data provided by the Israeli Railways (IR). The instance consists of a full demand OD Matrix of an average working day in Israel during 2008. The results are compared to the current timetable of IR. Due to the large complexity of the model, it is solved using the Column Generation methodology.

Keywords

Railway Optimization, Timetabling, Demand, Ideal Timetable, Passenger Utility

1 Introduction

The time of dominance of one rail operating company (usually the national carrier) over the markets in Europe is reaching to an end. With the new EU regulation, the track management and train operating companies (TOC) have to be separated subjects. Thus allowing competition (private sector) to enter the market.

Up to this point, the national carriers were subsidized by local governments and their purpose was to offer the accessibility and mobility to the public (passengers). On the other hand, the goal of the private sector is to generate revenue. In transportation sector, revenue is generated by transporting passengers or goods. The prospective passengers and goods are considered to be the demand to capture, hence TOCs' goal is to maximize the captured demand.

In case of goods, the demand is more flexible and its market is more or less already open for the private sector, unlike for passengers. The main driver of goods' demand is cost and in some cases also the trip time. On the other hand, the passenger demand is also sensitive to the time of the departure related to the trip purpose (weekday peak hours for work or school, weekends for leisure, *etc.*) and others (comfort, perception, *etc.*).

With the change of the market, the TOCs have to adapt to a new business model. In this paper, we describe the current way of planning of the passenger railway service, and discuss, how the demand is taken into account in the current planning process (Section 2). After the analysis, we introduce current literature on the topic (Section 3) and elaborate on a new problem to be inserted in the planning process, in a way that the new objective is properly taken into account (Section 4). Due to the complexity of the problem, we decompose the model and solve it using column generation methodology (Section 5). At the end of the paper, description of the case study is shown (Section 6) and we finalize the paper with conclusions and future work (Section 7).

2 Railway Planning

In this section, we present the current state-of-the-art of the research in the planning of passenger railway service. Since planning a railway operation is a complex task, due to the large solution space, it is partitioned into several problems that are solved sequentially (Caprara *et al.* (2007)). These problems and the sequence in which they are solved, can be seen on Figure 1.

The logical first step, in the planning of passenger railway service, should be the railway network

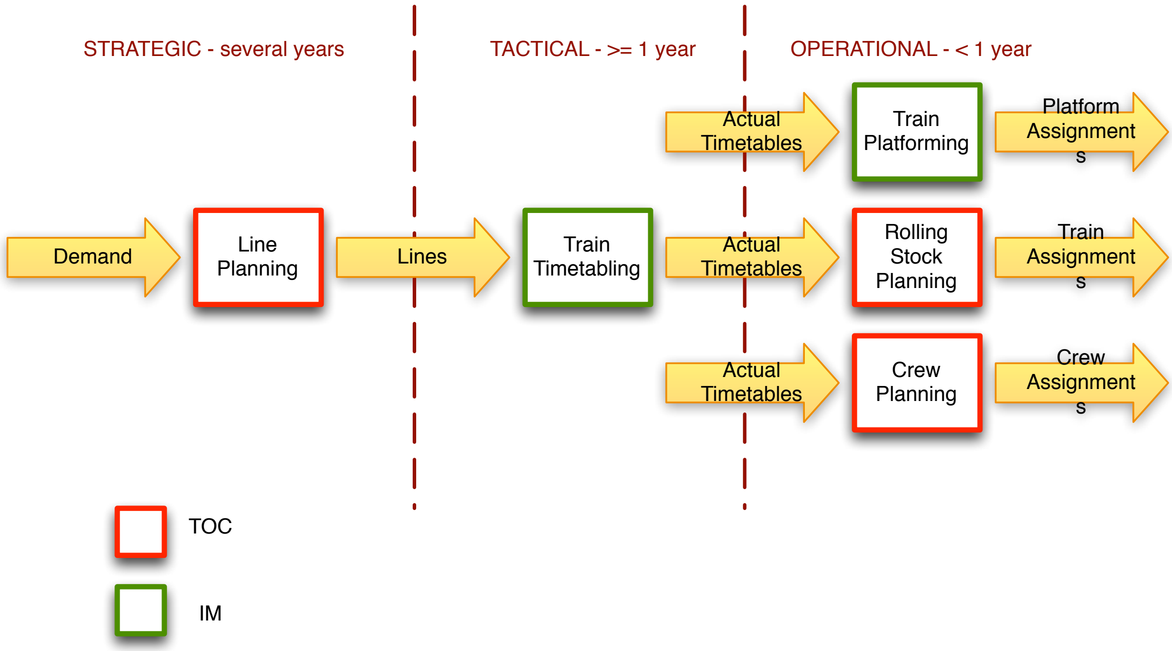

Figure 1: *Planning overview of railway operation*

design. However, since most of the railway infrastructure has been already built (starting in the early 19th century) and only small parts of the network are being build nowadays, it is often omitted from the planning process as such. Moreover, the decision, what new parts to add in the network, is often political and handled by the local authorities. The planning horizon then starts with the line design followed by the timetable design, rolling stock and crew scheduling and train platforming (Caprara *et al.* (2007)).

In the Line Planning Problem (LPP), the main input is demand per Origin/Destination (in the form of the OD Matrix) and the global railway infrastructure (network). Based on this input, selection of the most suitable lines from the pool of pre-processed potential lines, connecting these origins and destinations, is undertaken. Apart from that, expected frequencies and capacities of the lines are selected as well. This problem is handled by the TOC and it is usually solved every few years (the infrastructure and the OD matrix do not change fast). For more information about the LPP, please refer to the latest paper surveying the problem - Schöbel (2012).

The Train Timetabling Problem (TTP) exists in two settings: non-cyclic (Caprara *et al.* (2002)) and cyclic (Peeters (2003)). The difference between the two is, that trains with the cyclic timetables leave the stations at the beginning of every cycle, *i.e.* if the cycle is one hour, the trains then leave the station every hour in the same minute. The two options also differ in their respective inputs: the pre-created ideal timetables (consisting of the desired times and the time windows, that give degree of freedom to the problem, in order to find a feasible solution) for the non-cyclic version; frequencies and fixed departures for some of the trains in the cyclic version.

To the best of our knowledge, how to create the ideal timetables (non-cyclic timetabling), is nowhere to be found in the published literature. Even a definition, of what such an ideal timetable would be, is non existent. In case of the cyclic timetabling, the model only searches for a feasible solution, without considering what could be the best start of the cycle. We assume, that the common practice in the industry is to use the "historical" timetables and modify them in every new planning of timetables, using the given TOCs' train occupation data.

We believe, that the lack of the definition of the ideal timetables and how to create them, is a major gap, caused by the lack of a competition in the previous railway market settings. We assume, that not taking the passengers' wishes into account, lead to the decrease of the railway mode share in the transportation market.

And thus we propose to insert an additional section in the planning horizon called the Ideal Train Timetabling Problem (ITTP). In the ITTP, we introduce a definition of the ideal timetable (a definition of an ideal timetable, to the best of our knowledge, does not exist, even though the ideal timetables are used in the non-cyclic TTP) as follows: the ideal timetable, consists of train schedules, such that the cost, associated with traveling by train, of all of the passengers' is minimized (Section 4). Such a timetable would benefit both, passengers and the TOC in the respective manner: it would fit passengers' wishes, which would lead to the increase of the demand and to increase the TOCs' profit.

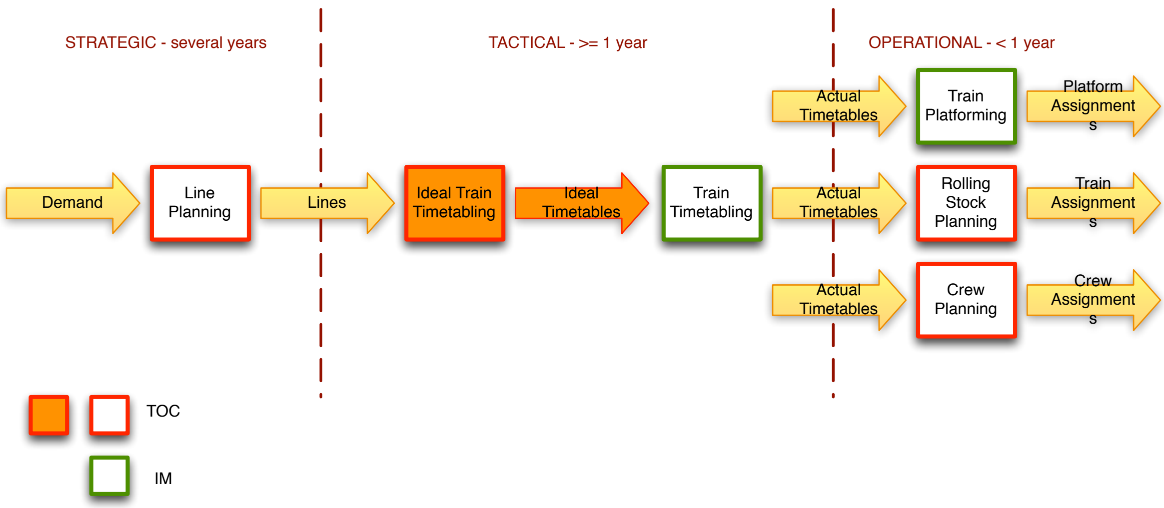

Figure 2: *Modified overview of railway operation*

The ITTP is using the output of the LPP and serves as an input to the traditional TTP and hence, it is placed between the two respective problems (Figure 2). The driver of this problem is the passenger demand. The model will allow timetables of the line to take the form of the non-cyclic or cyclic schedule. Moreover, we introduce a demand induced connections. The connections between the trains are not pre-set, but are subject to the demand. In the literature the connections are handled only in the cyclic version of the TTP, where the connections are always induced, without a proper reasoning.

Returning to the traditional TTP: the model is modifying the timetables for all scheduled trains, such that safety regulations in the railway network are maintained. The problem is minimizing the shifts by taking into account each timetable's cost or profit. As the ITTP will provide, not only the ideal timetables, but also their costs, it is then automatically ready to be integrated into the current planning process.

The design of the TTP suggests, that it serves mainly to the Infrastructure Manager (IM) to secure the safety and the feasibility of the network, whilst maximizing the wishes of TOCs. This problem is solved for every new timetable, *i.e.* typically every year (in Europe usually in December, EU directive from 2004 obliges all TOCs in Europe to do so on the same day; in some countries (France, Great Britain) twice a year).

The TTP in general is not able to solve all conflicts, specifically within the train stations, where a microscopic approach is needed. To handle these conflicts, the Train Platforming Problem (TPP) is solved (Caprara *et al.* (2007)). The TPP takes as input the designed actual timetables for train stations and creates the routings through the stations. This problem is considered operational (routings can be changed throughout the operation of the actual timetable) and it is handled by the IM.

The Rolling Stock Planning Problem (RSPP) is taking care of a fleet of a TOC, *i.e.* what is the train composition (numbers of 1st and 2nd class coaches) to be able to satisfy the demand and published timetable without exceeding the available rolling stock (Caprara *et al.* (2007)). This problem is as well operational and in the jurisdiction of the TOC.

The last step in the planning horizon is the Crew Planning Problem, which assigns crew to the scheduled trains, subject to union rules and other woking restrictions. The goal is to minimize the size of the crew needed for a global daily operation of the service. This problem is operational and handled by the TOC.

For more detailed description of the complete railway planning horizon, please refer to Caprara *et al.* (2007) or to Huisman *et al.* (2005).

In this section, we have identified several drawbacks of the current state-of-the-art planning process. The cause (of these drawbacks) is the recent market change in the railway industry, *i.e.* moving from the accessibility/mobility concept provided by one carrier to the competitive market consisting of several carriers, that are driven by the profit. The building stone of a competitive market is the demand. And thus in this paper, we propose a new planning phase (ITTP) based on the demand to create the most attractive timetables for the passengers (both cyclic and non-cyclic) including the connections between the trains in the network (also based on the demand).

3 Literature Review

The state-of-the-art literature is mostly focused on the traditional planning problems and considers the demand only in the initial phase (*i.e.* the LPP). In order to be able to insert the ITTP in the planning horizon, we have surveyed the LPP and the TTP (both non-cyclic and cyclic versions). Apart from that, we have also looked into the literature on demand interaction (outside of the traditional planning scope) in the passenger railway service (namely: revenue management, dynamic pricing, discrete choice models, *etc.*).

3.1 Line Planning Problem

The literature is basically divided into two partitions by passenger based and cost based objective function. One of the first models, that maximizes the direct travelers, can be found in Bussieck *et al.* (1997b). The model is maximizing the amount of direct passengers and uses one binary decision variable (1 – if the line is selected to be in the solution; 0 – otherwise). In order to solve the model efficiently, valid inequalities are introduced. This model is also used by Hooghiemstra *et al.* (1999). The phd thesis Bussieck (1997) extends the methodologies of solving the direct passengers objective and moreover evaluates the minimization of operational costs (and its techniques) as defined by Claessens.

In Claessens and van Dijk (1995) and Claessens *et al.* (1998) different approach is presented: instead of maximizing direct travelers, the minimization of costs is the objective. Additional decision variables on frequency and length of the trains (in terms of the number of carriages) are used. Unfortunately this leads to a non-linear model and thus linear reformulation is developed instead (one decision variable representing the combination of the above). Another linearizations and additional techniques to solve this model are shown in Zwaneveld (1997), Goossens *et al.* (2001) and Bussieck *et al.* (2004). Goossens *et al.* (2006) also works with the cost optimal model and extends the approach by introducing multiple line types (Goossens (2004) further extends the method). Both of the above types of the model (passenger and cost based) are then presented again in Bussieck *et al.* (1997a).

In Barber *et al.* (2008), another type of the model maximizing passenger coverage is presented. The main difference, comparing to the others, is that the lines are constructed from scratch, instead of using the set of preprocessed lines.

Different kind of model, minimizing the passengers' travel time is presented in Pfetsch and Borndörfer (2005), Schöbel and Scholl (2006) and Borndörfer *et al.* (2007, 2008).

Lastly, the LPP is integrated with TTP in Kaspi and Raviv (2013). The model is minimizing the total time passengers spend in the network. As a solution method cross-entropy metaheuristic is used. The latest paper surveying the published literature on LPP is Schöbel (2012).

3.2 Train Timetabling Problem

3.2.1 Non-Cyclic

Most of the models, on the non-cyclic timetabling, in the published literature, formulate the problem either as MILP or ILP. The MILP model uses continuous time, whereas the ILP model discretizes the time. Due to the complexity of the problem, many heuristic approaches are considered.

Brannlund *et al.* (1998) use discretized time and solve the problem with lagrangian relaxation of the track capacity constraints. The model is formulated as an ILP. Caprara *et al.* (2002, 2006), Fischer *et al.* (2008) and Cacchiani *et al.* (2012) also use lagrangian relaxation of the same constraints to solve the problem. In Cacchiani *et al.* (2008), column generation approach is tested. The approach tends to find better bounds than the lagrangian relaxation. In Cacchiani *et al.* (2010a), several ILP re-formulations are tested and compared. In Cacchiani *et al.* (2010b), the ILP formulation is adjusted, in order to be able to schedule extra freight trains, whilst keeping the timetables of the passengers' trains fixed. In Cacchiani *et al.* (2013), dynamic programming, to solve the clique constraints, is used.

In Carey and Lockwood (1995), a heuristic, that considers one train at a time and solves a MILP, based on the already scheduled trains, is introduced. Higgins *et al.* (1997) then show several more heuristics to solve the MILP model.

Oliveira and Smith (2000) and Burdett and Kozan (2010), re-formulate the problem as job-shop scheduling. Erol (2009), Caprara (2010) and Harrod (2012), survey different types of models for the TTP.

3.2.2 Cyclic

One of the first papers, dealing with cyclic timetables is Serafini and Ukovich (1989). The paper brings up the topic of cyclic scheduling based on the Periodic Event Scheduling Problem (PESP). The problem is solved via proposed algorithm.

In Nachtigall and Voget (1996) model for minimization of the waiting times in the railway network, whilst keeping the cyclic timetables (based on PESP), is solved using branch and bound and in Nachtigall (1996) using genetic algorithms. Another algorithm, based on constraint generation, to solve the PESP formulation is presented in Odijk (1996). In Lindner and Zimmermann (2000), branch and bound algorithm is also applied to solve the PESP. In Kroon and Peeters (2003), variable trip times are considered. Peeters (2003) further elaborates on PESP and in Liebchen (2004) implementation of the symmetry in the PESP model is discussed. In Liebchen and Mohring (2002), the PESP attributes are analyzed on the case study of Berlin's underground. Lindner and Zimmermann (2005) propose to use decomposition based branch and bound algorithm to solve the PESP.

Kroon *et al.* (2007) and Shafia *et al.* (2012), deal with robustness of cyclic timetables. Liebchen and Mohring (2004) propose to integrate network planning, line planning and rolling stock scheduling into the one periodic timetabling model (based on PESP). Caimi *et al.* (2007) and Kroon *et al.* (2014) introduce flexible PESP – instead of the fixed times of the events, time windows are provided.

3.3 Demand Related

Apart of the classical problems (shown on Figure 1), other additional techniques like revenue management, dynamic pricing or discrete choice models can be used to affect the demand. Especially the revenue management, which has been proven effective in the airline industry.

In Ben-Khedher *et al.* (1998), the decision tool RailCap, used by the french national carrier SNCF, is described. The main responsibility of the tool is to adjust the train capacity (by adding new unit to the train, drop empty extra units or open them for reservations on double-unit trains, open an optional train to reservations and assign it an itinerary-compatible fleet type) based on the current reservations, ODs and forecasted demand. The tool is maximizing the expected incremental profit subject to operational constraints (mainly availability of the rolling stock and its routing through the network).

In Chierici *et al.* (2004), model to maximize the demand captured by train is presented. The resulting timetable is cyclic (coming from constraints). The model integrates modal choice logit with 3 alternatives - bus, train and car (utility consists of travel time, monetary cost, walking time, average waiting time and comfort). Coefficients are estimated by a revealed preference. Since the MILP is non-convex 2 methods are tested: branch and bound and heuristic approach. Both are tested on a regional network in Italy, with real schedules as input. It is shown that with the current schedule, only 4 % of the population can choose train.

In Lythgoe and Wardman (2002), analysis of the demand for travel from and to airport and a formulation of a discrete choice model are introduced. Whelan and Johnson (2004) are showing a discrete choice model to decrease the overcrowding on the trains by adding a special ticket costs, when at the same time not reducing the total amount of passengers transported, *i.e.* smoothing the demand along the time horizon. In Cordone and Redaelli (2011), integration of the modal choice model and classical cyclic TTP is presented.

In Li *et al.* (2006), simulation framework based on the dynamic pricing is discussed. In Crevier *et al.* (2012) the aim is to cover the demand with maximizing the profit using different pricing strategies. The model is using preset booked schedules, which are then utilized on an operational level.

Abe *et al.* (2007) describes the revenue management (RM) in the railway industry with case studies of RM around the world. Comparison with the RM in airline industry is elaborated as well. In Bharill and Rangaraj (2008), revenue management for Indian Railways is described. Armstrong and Meissner (2010) shows the overview of RM in the railway industry (both freight and passenger). Wang *et al.* (2012) describes a MILP model for RM.

4 Ideal Train Timetabling Problem

The aim of this problem is to define and to provide the ideal timetables as an input for the traditional TTP. It is not well said in the TTP, what ideal means. It is only briefly mentioned, that supposedly, those are the timetables, that bring the most profit to the TOCs (this assumption is in line with the competitive market). Generally speaking, the more of the demand captured, the higher the profit. Thus the ITTP's goal is to design TOC's timetables, such that the captured passenger demand is maximized (objective, but not the form of the objective function).

The input of the ITTP is the demand that takes the form of the amount of passengers that want to travel between OD pair $i \in I$ and that want to arrive to their destination at their ideal time $t' \in T^i$. Apart of that, there is a pool of lines $l \in L$ along with the lines' frequencies expressed as the available train units $v \in V^l$ (both results of the LPP) and the set of paths between every OD pair $p \in P^i$. The path is called an ordered sequence of lines to get from an origin to a destination including details such as the running time from the origin of the line to the origin of the OD pair h_i^{pl} (where $l = 1$), the running time from an origin of the OD pair to a transferring point between two lines r_i^{pl} (where $l = 1$), the running time from the origin of the line to the transferring point in the path h_i^{pl} (where $l > 1$), the running time from one transferring point to another r_i^{pl} (where $l > 1$ and $l < |L^p|$) and the running time from the last transferring point to a destination of the OD pair r_i^{pl} (where $l = |L^p|$). Note that the index p is always present as different lines using the same track might have different running times.

Part of the ITTP is the routing of the passengers through the railway network. Using a decision variable $x_i^{t'p}$, we secure that each passenger (it') can use exactly one path. Similarly, within the path, passenger can use exactly one train on every line in the path (decision variable $y_i^{t'plv}$). These decision variables, among others, allow us to backtrace the exact itinerary of every passenger. The timetable is understood as a set of departures for every train on every line d_v^l. The timetable can take form of a non-cyclic or a cyclic version (depending if the cyclicity constraints are active, see Section 4.1).

Since the input demand is static, the intuitive objective function would be to minimize the total travel time of every passenger. However, the information about the ideal arrival time t' is present and hence to maximize the demand, we have to combine the total travel time and the timeliness of the arrival to the destination.

To express the timeliness, we borrow the concept of the scheduled delay from the traffic flow theory (see Arnott *et al.* (1990)). The logic behind it, is as follows: if a passenger arrives to his/her destination on his/her ideal time, then his/her scheduled delay ($s_i^{t'p}$) is equal to zero, otherwise linear delay functions are applied. There are two cases:

- **Being early –** $s_i^{t'p} = (t - t') \cdot f_2$
- **Being late –** $s_i^{t'p} = (t' - t) \cdot f_1$

where t is the time of the arrival into the destination. We assume that the scheduled delay is perceived differently, when being late and early. The calibration of the ratio between being late and early is a subject for further analysis (in our example in Figure 3, we use $f_1 = 2$ and $f_2 = 1$). The scheduled delay is different for every OD pair i with different ideal times t' using different paths p.

Since it is much more attractive to express the objective function in monetary units, for further estimation of the profit (not a subject of this paper) and full integration with the TTP (the objective is to minimize the timetable shifts subject to the profit), we multiply the value of the

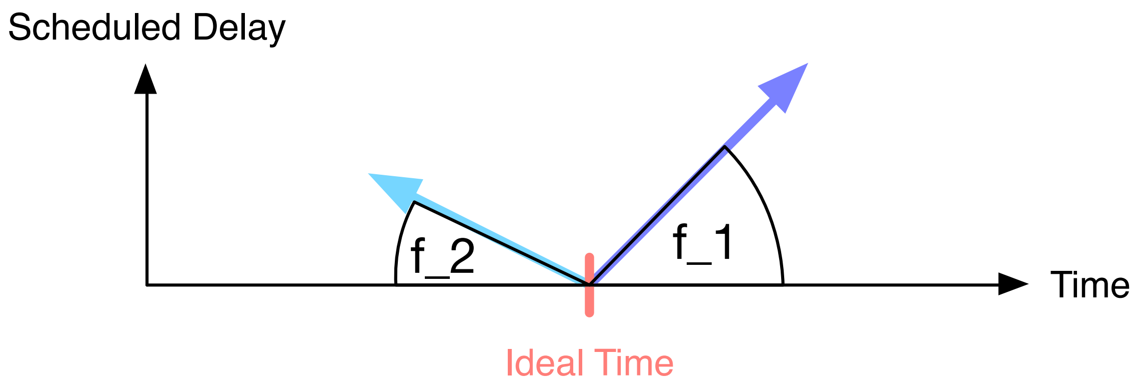

Figure 3: *Scheduled Delay Concept*

scheduled delay (in time units) by the *value of time* q_2 (monetary units per time unit). The same goes for the total travel time, where it is split into *in vehicle time* (time units) multiplied by the q_2 and *waiting time* (time units) multiplied by the *value of waiting time* q_1 (monetary units per time units). According to Wardman (2004) and Axhausen *et al.* (2008), the time spent waiting is perceived differently (two times more) than the time spent in vehicle.

Lastly, using a direct train, instead of a several trains with interchanges, is more attractive to the passengers (see Axhausen *et al.* (2008)). To take care of this attribute, we introduce a minimum transfer time m. Since it is as well a waiting time, it is multiplied by the same value of q_1. As we allow unlimited number of transfers, it is then multiplied by the size of the path minus one $(|L^p| - 1)$, as the transfers happen in-between two lines.

In the end, we can combine all of the above attributes into a one cost $C_i^{t'}$:

$$C_i^{t'} = \left(q_1 \cdot w_i^{t'} + q_1 \cdot m \cdot \sum_{p \in P} x_i^{t'p} \cdot (|L^p| - 1) + q_2 \cdot \sum_{p \in P} \sum_{l \in L^p} r_i^{pl} \cdot x_i^{t'p} + q_2 \cdot \sum_{p \in P} s_i^{t'p} \cdot x_i^{t'p} \right) \tag{1}$$

In the final form of the objective function, the above cost is weighted by the demand $D_i^{t'}$. As the problem is no longer concerned by the accessibility and mobility of the passengers, we don't need to take into account the maximum travel time between an origin and a destination.

Based on the above assumptions, we can formulate the ideal timetable:

> The ideal timetable consists of such train departures that the passengers' global

costs are minimized, *i.e.* the fastest most convenient path to get from the origin to the destination traded-off by a timely arrival to the destination for every passenger.

4.1 Mathematical Formulation

In this section, we present a mixed integer programming formulation for the ideal train timetabling problem.

Input Parameters Following is the list of parameters used in the model:

$i \in I$	–	set of origin-destination pairs
$t \in T$	–	set of time steps t in the planning horizon
$t' \in T^i$	–	set of ideal times for OD pair i
$l \in L$	–	set of operated lines
$v \in V^l$	–	set of available vehicles on line l
$p \in P^i$	–	set of possible paths between OD pair i
$l \in L^p$	–	set of lines in the path p
r_i^{pl}	–	running time between OD pair i on path p using line l
h_i^{pl}	–	time to arrive from the starting station of the line l to the origin of the pair i
$D_i^{t'}$	–	demand between OD i with ideal time t'
m	–	minimum transfer time
c	–	cycle
q_1	–	value of the waiting time
q_2	–	value of the in vehicle time
f_1	–	coefficient of being early
f_2	–	coefficient of being late

The lines and the set of available vehicles per line V^l is an output from the Line Planning Problem based on the selected frequencies within the problem.

Decision Variables Following is the list of decision variables used in the model:

$C_i^{t'}$	–	the total cost of the passengers with ideal time t' between OD pair i
$w_i^{t'}$	–	the total waiting time of the passengers with ideal time t' between OD pair i

$w_i^{t'p}$ – the total waiting time of the passengers with ideal time t' between OD pair i using path p

$w_i^{t'pl}$ – the waiting time of the passengers with ideal time t' between OD pair i on the line l that is part of the path p

$x_i^{t'p}$ – 1 – if the passengers with ideal time t' between OD pair i choose path p; 0 – otherwise

$s_i^{t'}$ – the final scheduled of the passengers with ideal time t' between OD pair i

$s_i^{t'p}$ – scheduled delay of the passengers with ideal time t' between OD pair i traveling on the path p

d_v^l – the departure time of a train v on the line l

$y_i^{t'plv}$ – 1 – if the passengers with ideal time t' between OD pair i on the path p take the train v on the line l; 0 – otherwise

z_v^l – frequency within cyclicity

Model The mathematical formulation then looks as follows:

$$\min \sum_{i\in I} \sum_{t'\in T^i} D_i^{t'} \cdot C_i^{t'} \tag{2}$$

$$\begin{aligned} C_i^{t'} = q_1 \cdot w_i^{t'} + q_1 \cdot m \cdot \sum_{p\in P} x_i^{t'p} \cdot (|L^p| - 1) \\ +q_2 \cdot \sum_{p\in P} \sum_{l\in L^p} r_i^{pl} \cdot x_i^{t'p} + q_2 \cdot \sum_{p\in P} s_i^{t'} \qquad \forall i \in I, \forall t' \in T^i, \end{aligned} \tag{3}$$

$$\sum_{p\in P^i} x_i^{t'p} = 1, \qquad \forall i \in I, \forall t' \in T^i, \tag{4}$$

$$\sum_{v\in V^l} y_i^{t'plv} = 1, \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p, \tag{5}$$

$$w_i^{t'} \geq w_i^{t'p} - M \cdot \left(1 - x_i^{t'p}\right), \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \tag{6}$$

$$w_i^{t'p} = \sum_{l\in L^p\setminus 1} w_i^{t'pl}, \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \tag{7}$$

$$\begin{aligned} w_i^{t'pl} \geq \left(\left(d_v^l + h_i^{pl}\right) - \left(d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m\right)\right) \\ -M \cdot \left(1 - y_i^{t'pl'v'}\right) - M \cdot \left(1 - y_i^{t'plv}\right), \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p : \\ l > 1, l' = l - 1, \forall v \in V^l, \forall v' \in V^{l'}, \end{aligned} \tag{8}$$

$$\begin{aligned} w_i^{t'pl} \leq \left(\left(d_v^l + h_i^{pl}\right) - \left(d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m\right)\right) \\ +M \cdot \left(1 - y_i^{t'pl'v'}\right) + M \cdot \left(1 - y_i^{t'plv}\right), \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p : \\ l > 1, l' = l - 1, \forall v \in V^l, \forall v' \in V^{l'}, \end{aligned} \tag{9}$$

$$s_i^{t'} \geq s_i^{t'p} - M \cdot \left(1 - x_i^{t'p}\right), \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \tag{10}$$

$$s_i^{t'p} \geq f_2 \cdot \left(\left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right) - t'\right) - M \cdot \left(1 - y_i^{t'p|L|v}\right), \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall v \in V^{|L|}, \tag{11}$$

$$s_i^{t'p} \geq f_1 \cdot \left(t' - \left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right)\right) - M \cdot \left(1 - y_i^{t'p|L|v}\right), \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall v \in V^{|L|}, \tag{12}$$

$$d_v^l - d_{v-1}^l = c \cdot z_v^l, \qquad \forall l \in L, \forall v \in V : v > 1, \tag{13}$$

$$w_i^{t'} \geq 0, \qquad \forall i \in I, \forall t' \in T^i, \tag{14}$$

$$w_i^{t'p} \geq 0, \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \tag{15}$$

$$w_i^{t'pl} \geq 0, \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p, \tag{16}$$

$$x_i^{t'p} \in (0, 1), \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \tag{17}$$

$$s_i^{t'p} \geq 0, \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \tag{18}$$

$$d_v^l \geq 0, \qquad \forall l \in L, \forall v \in V^l, \tag{19}$$

$$y_i^{t'plv} \in (0, 1), \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p, \forall v \in V^l, \tag{20}$$

$$z_v^l \in \mathbb{N}, \qquad \forall l \in L, \forall v \in V^l. \tag{21}$$

The objective function **(2)** is minimizing the passengers' costs. Constraint **(3)** calculates the cost of the solution. Constraints **(4)** secure that every passenger is using exactly one path to get from his/her origin to his/her destination. Similarly constraints **(5)** make sure that every passenger takes exactly one train on each of the lines in his/her path. Constraints **(6)** select the best path in terms of the waiting time. Constraints **(7)** add up all waiting times along the given path. Constraints **(8)** and **(9)** set the proper waiting time in the transferring stations. Constraints **(10)** select the best path in terms of the scheduled delay. Constraints **(11)** and **(12)** are complementary constraints (one at a time is active) that calculate the scheduled delay in passengers' destinations. Lastly, constraints **(13)** are handling the cyclicity of the created timetables (if removed, the created timetables would take non-cyclic form). Constraints **(14)-(21)** set the domains of decision variables.

5 Solution Approach

The mixed integer programming formulation of the ITTP has a large solution space, which makes the problem difficult to solve. Moreover, the presence of the big M constraints lead to a weak lower bound. Thus in the following section, we decompose the mixed integer model and formulate it as a set partitioning problem.

5.1 Set Partitioning Model

Let Ω be the set of feasible assignments of a demand between all OD pairs with all ideal times. Note that a feasible assignment represents the assignment of a single demand between a given OD pair with a given ideal time to a given path.

Input Parameters The following input parameters are used in the set partitioning model:

$a \in \Omega$	–	set of all possible assignments
$i \in I$	–	set of origin-destination pairs
$t \in T$	–	set of all time steps
$t' \in T^i$	–	set of times that there is a demand between OD pair i
$l \in L$	–	set of operated lines
c	–	cycle
D_a	–	demand using assignment a
n_l	–	number of available train units on line l
C_a	–	cost of the assignment a

$$B_a^{it'} = \begin{cases} 1 & \text{if OD pair } i \text{ at time } t' \text{ is assigned in assignment } a, \\ 0 & \text{otherwise.} \end{cases}$$

$$E_a^{lt} = \begin{cases} 1 & \text{if the assignment } a \text{ is using line } l \text{ at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

Decision Variables Following is the list of decision variables used in the model:

$$\lambda_a \quad - \quad \begin{cases} 1 & \text{if assignment } a \text{ is a part of the solution,} \\ 0 & \text{otherwise.} \end{cases}$$

$$x_l^t \quad - \quad \begin{cases} 1 & \text{if there is a train scheduled on line } l \text{ at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

Model

$$\min \sum_{a \in \Omega} C_a \cdot D_a \cdot \lambda_a \tag{22}$$

$$\sum_{a\in\Omega} B_a^{it'} \cdot \lambda_a = 1, \qquad \forall i \in I, \forall t' \in T^i, \tag{23}$$

$$\sum_{a\in\Omega} E_a^{lt} \cdot \lambda_a \le x_l^t, \qquad \forall l \in L, \forall t \in T, \tag{24}$$

$$\sum_{t\in T} x_l^t \le n_l, \qquad \forall l \in L, \tag{25}$$

$$\sum_{t''=t}^{min(t+c,T)} x_l^{t''} \le 1, \qquad \forall l \in L, \forall t \in T, \tag{26}$$

$$\lambda_a \in \{0,1\}, \qquad \forall a \in \Omega, \tag{27}$$

$$x_l^t \in \{0,1\}, \qquad \forall l \in L, t \in T. \tag{28}$$

The objective function **(22)** is minimizing the costs of passengers. Constraints **(23)** ensure that there is exactly one assignment for every passenger in the optimal solution. Constraints **(24)** link the scheduled trains with used assignments. Constraints **(25)** dictate that the amount of scheduled trains on every line does not exceed the amount of available physical trains. Constraints **(26)** are cyclicity constraints. Constraints **(27)** and **(28)** set the domains of decision variables.

In this new formulation, we have reduced the amount of decision variables and got rid of the big M constraints. However, the solution space is now even bigger and thus in order to avoid the "explosion" of the solution space (time), we propose to solve the linear programming relaxation of the above problem using column generation, as described in the next section.

5.2 Column Generation

In the linear programming (LP) relaxation of the set partitioning problem the domains of λ_a and x_l^t are extended to $[0, 1]$. Despite the large number of variables it is possible to solve the LP relaxation using a column generation algorithm. In a column generation algorithm we maintain a restricted master problem (RMP) that only considers a small subset $\Omega_1 \subseteq \Omega$ of all the possible variables. New variables are added to Ω_1 until we can decide that no variable in $\Omega \setminus \Omega_1$ can improve the solution that results from only using the variables in Ω_1.

In the first iteration of column generation, the RMP is solved using the set Ω_1 consisting of passenger assignments in the initial feasible solution provided by the CPLEX after solving the original formulation using the current published timetables of a TOC. Thereafter, in each successive iteration of the column generation process, the following dual variables are passed to the subproblem for identifying feasible assignments with negative reduced cost:

Algorithm 1: Branch and Price

Data: data file, Ω, *finished* - boolean, *duals* - float

Result: $\Omega_1 \subset \Omega$, *solution*

```
1  begin
2      Ω1 ← initialSolution
3      duals ← ∅
4      solution ← ∅
5      repeat
6          duals ← solveMaster(Ω1)
7          finished ← true
8          for i ∈ N do
9              temp ← solveSubProblem(i, t′, duals)
10             if reducedCost(temp) < 0 then
11                 Ω1 ∪ temp
12                 finished ← false
13     until finished
14     solution ← solveMaster(Ω1)
15     if solution ∉ ℤ then
16         ub ← solveMaster(Ω1, integral)
17         if solution = ub then
18             break
19         solution ← branch&bound(solution)
20     print solution
```

$\alpha_i^{t'}$ – dual variables for constraint 23

β_l^t – dual variables for constraint 24

We do not need to consider the dual variables corresponding to constraints **(25)** and **(26)** since these constraints do not involve the λ_a variables and therefore the associated dual variables do not impact the reduced cost of the λ_a variables. Based on the dual variables from RMP, the subproblem generates new columns to enter the active pool of columns Ω_1 by calculating the most negative reduced cost column for each vessel separately in each iteration of the column generation process. When there are no columns with negative reduced cost for any subproblem to enter Ω_1, the column generation terminates.

The column generation in pseudocode can be seen in Lines (1) – (13) in Algorithm 1. For mathematical justification of column generation, please refer to Barnhart *et al.* (1998), Desaulniers *et al.* (2005) and Feillet (2010).

5.2.1 Sub-Problem

In each iteration of column generation, we solve a sub-problem for every OD pair i, every ideal time $t' \in T^i$ and every path $p \in P^i$. In each subproblem, the objective is to identify the feasible assignment for that particular OD pair with ideal time with the most negative reduced cost to be added to the current pool of active columns Ω_1 in the restricted master problem. Note that the index i and t' is removed from all decision variables, since it is solved separately for each OD pair i with ideal time t'.

Input Parameters Following is the list of parameters used in the model:

i	–	the origin destination pair
t'	–	ideal travel time for OD pair i
p	–	path of the sub-problem
$t \in T$	–	set of all time steps
$l \in L^p$	–	the sequence of lines used to get from the origin to the destination
r_l	–	running time of line l
h_l	–	running time to get from the starting station of the line l to the first station on the same line included in the current path
m	–	the minimum transfer time
q_1	–	the value of time spent waiting
q_2	–	the value of time spent in vehicle
f_1	–	coefficient of being early
f_2	–	coefficient of being late

Decision Variables Following is the list of decision variables used in the model:

$beta_l^t$	–	$\begin{cases} 1 & \text{if line } l \text{ is used at time } t, \\ 0 & \text{otherwise.} \end{cases}$
w	–	the total waiting time of the passengers
w_l	–	the waiting time of the passengers when transferring to line l
s	–	scheduled delay of the passengers
C	–	the cost of the passengers

Model

$$\min\ C - \left(\alpha + \sum_{l\in L}\sum_{t\in T}\beta_l^t \cdot beta_l^t\right) \tag{29}$$

$$C = \left(q_1 \cdot w + q_1 \cdot m \cdot (|L| - 1) + q_2 \cdot \sum_{l\in L} r^l + q_2 \cdot s\right) \tag{30}$$

$$\sum_{t\in T} beta_l^t = 1, \qquad l \in L^p, \tag{31}$$

$$w = \sum_{l\in L\backslash 1} w_l, \tag{32}$$

$$w_l \geq (t \cdot beta_l^t + h_l) - \left(t'' \cdot beta_{l-1}^{t''} + h_{l-1} + r_{l-1} + m\right), \qquad \forall l \in L^p : l > 1, \forall t, t'' \in T : t \geq t'' + h_{l-1} + r_{l-1} \tag{33}$$

$$w_l \leq (t \cdot beta_l^t + h_l) - \left(t'' \cdot beta_{l-1}^{t''} + h_{l-1} + r_{l-1} + m\right), \qquad \forall l \in L^p : l > 1, \forall t, t'' \in T : t \geq t'' + h_{l-1} + r_{l-1} \tag{34}$$

$$s \geq f_2 \cdot \left(t \cdot beta_{|L|}^t + h_{|L|}\right) - t', \qquad \forall t \in T, \tag{35}$$

$$s \geq f_1 \cdot t' - \left(t \cdot beta_{|L|}^t + h_{|L|}\right), \qquad \forall t \in T, \tag{36}$$

$$h \geq 0, \tag{37}$$

$$beta_l^t \in \{0, 1\}, \qquad \forall l \in L, \forall t \in T. \tag{38}$$

The objective function **(29)** is looking for a column with the most negative reduced cost. Constraint **(30)** calculates the cost of the solution. Constraints **(31)** make sure that passengers take exactly one train on each line in the path. Constraints **(32)** calculate the total waiting time. Constraints **(33)** and **(34)** set the proper waiting time in the transferring stations. Constraints **(35)** and **(36)** are complementary constraints (one at a time is active) that calculate the scheduled delay in passengers' destinations. Constraints **(37)** and **(38)** set the domains of decision variables.

6 Case Study

As our case study, we use the data from the Israeli Railways (IR), kindly provided by Mor Kaspi and Tal Raviv, who have cleaned the data and used them in their study (Kaspi and Raviv (2013)).

The data consist of an hourly demand from 6 a.m. to 1 a.m. for every origin destination pair in the IR network (Figure 4). The demand has been extracted from the ticket sells and train counts in the year 2008 and it is a representative sample of an average working day (over the whole

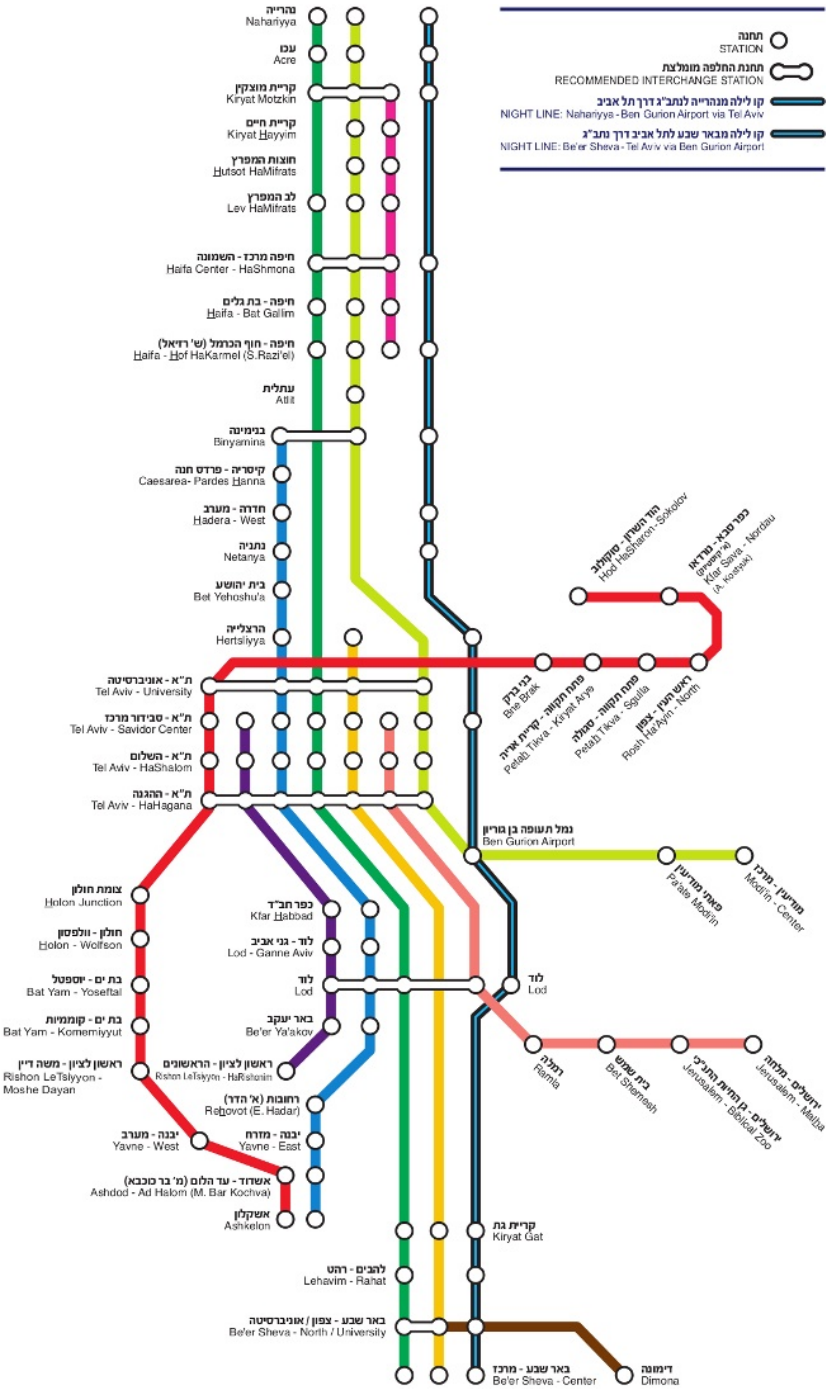

Figure 4: *Israeli Railways Network*

year of 2008) in Israel.

We further modify the data for our specific use. The demand is smoothed into minutes using the poisson process. Moreover, since the scheduled delay is related to the arrival into the destination, we shift the demand's ideal time from the origin to the destination, by adding up the shortest travel time for the specific OD (assuming that the passengers minimize their cost).

For the values of time, we are using the swiss values as estimated in Axhausen *et al.* (2008), due to the unavailability of the israeli values.

The IR network consists of 48 passenger stations, which adds up to 2256 OD pairs. As our benchmark, we are using the current IR timetables (year 2014), from which we have extracted 36 unidirectional lines (in our study we do not take into account different stopping patterns within the same lines) that are being operated by 389 trains (it is an odd number, as we do not take into account trains scheduled between 1 a.m. and 6 a.m., as no demand data exist for this period of time). The data processing is still undergoing, however we have not seen a path between an OD pair that would require more then one interchange. The size of the problem justifies the use of column generation.

7 Conclusions and Future Work

In this research, we survey the literature on the current planning horizon for the railway passenger service and we identify a gap in the planning horizon – demand based (ideal) timetables. We then introduce a definition of such an ideal timetable and formulate a mixed integer linear problem that can design such timetables. Since the proposed formulation is complex (large amount of decision variables, big M constraints), we propose to decompose the problem and solve it using column generation methodology.

At the current stage, the implementation is undergoing and hence no results are being provided. We plan to use a demand data obtained from the Israeli Railways as mentioned in the above section. The current timetables of IR are cyclic, thus we will be able to measure the cost savings between our cyclic timetables and the ones of IR. Moreover, using our non-cyclic version of the model, we will be also able to measure the actual cost of the cyclic timetables.

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