Modeling car ownership and usage: a dynamic discrete-continuous choice modeling approach

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Abstract

In this research we specify a *dynamic discrete-continuous choice model (DDCCM)* of car ownership and usage. This model embeds a discrete-continuous choice model into a dynamic programming (DP) framework. The DDCCM allows to jointly model the transaction type, the annual driving distance, the fuel type, the car ownership status and the car state (i.e. the purchase of a new or second-hand car) relative to each car in a household's fleet. Moreover we model these decisions at the household level and for up to two cars per household. In this paper, the methodology is illustrated by an example of application where parameters are postulated.

Key words

Dynamic discrete-continuous choice model, dynamic programming, discrete choice models, car ownership and usage, constant elasticity of substitution.

Number of words

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1 Introduction

The purchase of a car within a household and its usage result from a complex decision process. In addition to their own socio-economic features, households' decisions are affected by various other factors, from the vehicle market supply to governmental regulations. Moreover car ownership decisions change over years due to variations in these factors. Identifying them is therefore highly important in terms of policy making since its allows to understand of the dynamics of fuel type shares and consumption.

Cars are durable goods. Indeed individuals keep their vehicles for several years and their expectations about the future affect their current decisions. Though many models for car ownership or usage have been developed in the transportation literature, most of them are static models and do not account for the forward-looking behavior of agents. However as highlighted by Cirillo and Xu (2011), it is essential to account for this aspect in the context of car acquisition. Recently, a few studies have started integrating that aspect. Among them, Xu (2011) develops a dynamic discrete choice model (DDCM) to explain car acquisition decisions and choice of fuel type and apply it to stated preferences data collected in Maryland. Schiraldi (2011) calibrates a DDCM on car register data in Italy to analyze the influence of scrappage policies. Aggregate socio-economic information is introduced into the model to capture heterogeneity of preferences. Other modeling approaches have been considered in order to jointly model car ownership and usage. For instance, Gillingham (2012) models cars' monthly mileage conditional on vehicle type. Other researches focus on applying a dynamic programming mixed logit (DPMXL) approach (Schjerning, 2008) to model vehicle type choice, usage and replacement decisions (according to discussions and unpublished work by Anders Munk-Nielsen[1]). The particularity of their model is that it can handle both discrete and continuous decision variables.

In this research, we specify a *dynamic discrete-continuous choice model (DDCCM)* that jointly models car usage and replacement decisions, including choice of ownership status and fuel type. Such a model embeds a discrete-continuous choice model into a dynamic programming framework. We propose the following novel features. First, we model choices at a household level and account for the fact that each household can have at most two cars. In terms of car usage it cannot be assumed that in two-car households, individuals choose the annual driving distances of the two cars independently. Therefore, in order to account for this dependency, we consider a *constant elasticity of substitution (CES) utility* for the choice of annual driving distances. Second, we model an extensive choice variable, consisting of the joint choice of the transaction type (e.g. adding, replacing a car, etc.), the annual distance that each car will be driven, the fuel type of each car, the decision to choose a company car and to select a new or second-hand car.

At our disposal we have the Swedish car and population registers, where detailed vehicle and socio-economic information is available at a disaggregate level for the years 1998 to 2008. This type of data is ideal for the proposed modeling approach since we can follow individuals, households and cars over time. It moreover allows to analyze and predict the dynamics of car holdings and usage for the whole population over several years.

This paper is an extension of the research presented at the Swiss Transport Research Conference (Glerum et al., 2013), where the methodological framework of this DDCCM was already introduced. The novel part of this paper consists of providing an illustration of the application of the DDCCM, resulting from the implementation of a major component of the model, i.e. the value iteration algorithm.

The paper is structured as follows. Section 2 presents the motivations for the current research and provides a description of the available data. Section 3 presents the methodological framework of the DDCCM. Section 4 provides an illustration of the application of the model. Section 5

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concludes the paper by outlining the next steps of this research.

2 Background and data

The purpose of this research is to develop a model of car ownership and usage in Sweden, where there is a mix of policies in place that act on different actors on the local, regional, and national level.

During recent years, there has been an increasing focus on energy efficiency of new cars and also on more energy efficient transport modes. Current transport policy in Sweden highlights public transport and bicycling, and the attractiveness of these modes affect the car ownership as such. For the purpose of this paper, we emphasize a few stylized facts about the Swedish car fleet and associated policies. First, in Sweden there is a strong tradition of buying large, powerful and heavy cars. The car fleet is one of the heaviest in Europe. In order to fulfil climate goals, a number of policies have been put in place over the last few years aiming at accelerating the introduction of clean cars in the fleet. Meanwhile, the definition of clean car has also evolved from being alternative fuel cars only, to any car that meet a specific standard specified in terms of CO_2 emissions and fuel consumption (from year 2005).

The new car sales show a strong demand shift in response to these policies. In recent years there has also been a significant increase of small diesel cars (Hugosson and Algers, 2012; Kageson, 2012). An important characteristic of the Swedish market for new cars is the high proportion of company owned cars, which is a consequence of the fringe benefit taxation system. This poses a number of challenges to our model which will be addressed below.

One of the reasons for few dynamic modeling studies on car related choices may be that data is difficult to obtain. In this study we use the register data over the whole Swedish population that combines the population and car registers for the years 1998 to 2008. These registers are based on individuals and we have extensive socio-economic data such as net income, home and work locations, type of employment in addition to characteristics of each owned car (make, model, fuel type, fuel consumption, age, etc.) and the annual mileage from odometer readings. In addition we have information on all households types except for unmarried individuals living together without children. Part of this data (without car characteristics) was used by Pyddoke (2009).

We observe a variability in the data with corresponding demand shifts which will be important for the identification of parameters related to policy variables (such as registration taxes or fuel prices) inducing the shifts. Moreover, since geographic information is available at a detailed level, this allows to analyze the impact of policy changes at a regional level.

3 The dynamic discrete-continuous choice modeling framework

In this chapter, we present the framework of the DDCCM. We start by stating the main assumptions on which the model is based. Then we describe the model structure, from the base components to the specification of the full model. One of the key elements of the choice variable is the annual mileage of each car and we explain in detail its specification. We end the section by discussing a possible estimation method of the model.

3.1 Main assumptions

The DDCCM is formulated as a discrete-continuous choice model that is embedded into a *dynamic programming (DP)* framework. We model the joint decision of vehicle transactions, mileage, fuel type, use of a company car (if available) and purchase of a new or second-hand car, based on the following assumptions.

- Decisions are made at a household level. In addition, we assume that each household can have at most two cars, since a very small share of the Swedish households has more than two cars.
- The choice of vehicle transaction, fuel type(s), use of a company car(s) and selection of (a) new versus second-hand car(s) is *strategic*, that is, we assume that households take into account the future utility of the choice of these variables in their decision process.
- We consider an infinite-horizon problem to account for the fact that households make long-term decisions in terms of car transactions, choice of car ownership status, fuel type and car state (new versus second-hand). For example, individuals are assumed to strategically choose the fuel type of the car they purchase according to their expectation of fuel prices in the next years, or they decide to purchase one only car at present but already know that they might add another car in the future years.
- The choice of mileage(s) is conditional on the choice of the discrete decision variables (i.e. the transaction type, the type of ownership, the fuel type and the car state).
- The choice of mileage(s) is *myopic*, that is, households do not take into account the future utility of the choice of the current annual driving distance(s) in their decision process. We make the simplifying assumption that when households decide how much they will drive their car for the upcoming year, they only consider the utility of this choice for that particular year, but that they do not account for whether the residual value of their car is affected by usage.

3.2 Definition of the components

The DP framework is based on four fundamental elements: the *state space*, the *action space*, the *transition function* and the *instantaneous utility*. In this section, we describe each of these in detail.

The *state space* S is constructed based on the following variables:

- The *age* $y_{c,t}$ of car c at year t. We set an upper bound for the age $\bar{Y}$, assuming that above this upped bound, changes in age do not affect the utility or transition from one state to another. This implies that we have $y_{c,t} \in Y = \{0, 1, \dots, \bar{Y}\}$.
- The *car ownership status* $I_{c,t}$. It consists of a discrete variable indicating whether car c is owned privately (level 1), by sole proprietorship[2] (level 2) or by another type of company (level 3) at year t. We have $I_{c,t} \in I_C = \{0, 1, 2, 3\}$, where level 0 indicates the absence of car c.
- The *fuel type* $f_{c,t}$ of car c at year t. A car c can have one of the three following fuel types $f_{c,t}$: petrol, diesel or other fuel types (Flexi fuel ethanol, CNG, hybrid, plug-in hybrid and electric car) , denoted by 1, 2 and 3, respectively. Therefore we have $f_{c,t} \in F = \{0, 1, 2, 3\}$, where level 0 indicates the absence of a car.

Each state $s_t \in S$ can hence be represented as

$$s_t = (y_{1,t}, I_{1,t}, f_{1,t}, y_{2,t}, I_{2,t}, f_{2,t}). \tag{1}$$

Due to the fact that we only have information about the age of the car and its fuel type for privately-owned cars and cars owned by sole proprietorship, we do not represent age and fuel type

[2]By *sole proprietorship*, we mean a form of business that legally has no separate existence from its owner (Source: http://www.entrepreneur.com).

for company cars. Therefore, if we have $I_{c,t} = 3$, then this implies that we also have $y_{c,t} = 0$ and $f_{c,t} = 0$, respectively.

For households who have access to company cars, the size of the state space can be computed as

$$
\begin{aligned}
|S| &= (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1)^2 && (2)\\
&+ (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1) && (3)\\
&+ 1. && (4)
\end{aligned}
$$

The first term (2) consists of the number of possible states for two-car households. The element $|Y| \times (|I_C| - 2) \times (|F| - 1)$ of (2) is the number of states for housholds with privately owned cars or cars owned by sole proprietorship, while the element 1 of (2) is the number of states for households with company cars. For these households, we indeed only have the information of whether a company car is chosen or not. The exponent 2 stands for the two cars in the household. The second term (3) is the number of possible states for one-car housholds and the last term (4) stands for the absence of cars in a household. Assuming that cars can be at maximum 10 years old and given the above definitions of I_C and F, the size of the state space reaches the reasonable size of $3'783$.

We note that not all households have access to company cars and some states of S are then unavailable.

The *action space* A is constructed based on the following variables:

- The *transaction* h_t in the household composition of the car fleet at year t. Every year, the household can choose to increase, decrease or replace all or part of the fleet, or not to do anything. We additionally make the simplifying assumption that a household cannot purchase more than one car per time period. The enumeration (see Figure 1) leads to nine possible transactions. Therefore we have $h_t \in H = \{1, \ldots, 9\}$.
- The *annual mileage* $\tilde{m}_{c,t} \in \mathbb{R}^+$ for each car c.
- The choice $\tilde{I}_{c,t} \in I_C$ of *car ownership status*.
- The *fuel type* $\tilde{f}_{c,t} \in F$.
- The *car state* $\tilde{r}_{c,t}$, i.e. the decision to purchase a new or second-hand car. We hence have $\tilde{r}_{c,t} \in R = \{0, 1, 2\}$, where level 0 means that no car has been bought, level 1 means that car c is bought new and level 2 means that car c is bought second-hand.

Note that if $I_{c,t} = 1, 2$, then we model the choice of mileage $\tilde{m}_{c,t}$, fuel type $\tilde{f}_{c,t}$ and car state $\tilde{r}_{c,t}$ for a car c for the next year. However, for $I_{c,t} = 3$, no information on the choice of mileage, fuel type or state of the car (new or second-hand) for the next year is available from the data. Therefore we do not model such decisions.

Each action $a_t \in A$ can be represented as

$$a_t = (h_t, \tilde{m}_{1,t}, \tilde{I}_{1,t}, \tilde{f}_{1,t}, \tilde{r}_{1,t}, \tilde{m}_{2,t}, \tilde{I}_{2,t}, \tilde{f}_{2,t}, \tilde{r}_{2,t}). \tag{5}$$

From some particular states s_t, not all actions are available. Hence, we implicitly have $a_t \in A(s_t)$ and the total number of actions must be obtained by enumerating all possible actions from each particular state. Table 1 summarizes the number of actions that can be attained for households with 0, 1 or 2 cars, depending on the type of transaction which is chosen. For example, a 1-car household that decides to increase the fleet of 1 car has the choice between 3 types of car ownership

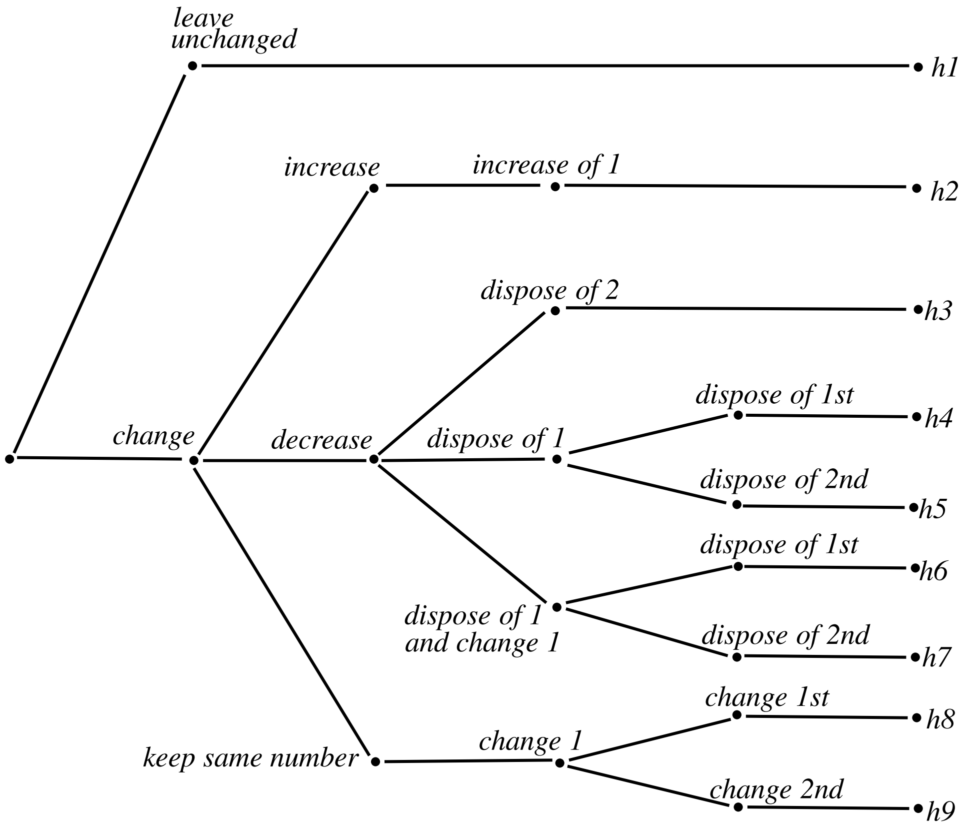


Figure 1: The nine possible transactions in a household fleet

status, 3 types of fuel and 2 types of car state, leading to 18 possible actions. In the row 'Sum', the total number of possible actions for households with respectively 0, 1 or 2 cars are reported.

Given that a household is in a state s_t and has chosen an action a_t, the *transition function* $f(s_{t+1}|s_t,a_t)$ is defined as the rule mapping s_t and a_t to the next state s_{t+1}. In our case, s_{t+1} can be inferred deterministically from s_t and a_t, implying that $f(s_{t+1}|s_t,a_t)$ only takes two values: 1 when s_t and a_t lead to action s_{t+1} and 0 otherwise.

Assuming that $a_t^D = (h_t, \tilde{I_{1,t}}, \tilde{f_{1,t}}, \tilde{r_{1,t}}, \tilde{I_{2,t}}, \tilde{f_{2,t}}, \tilde{r_{2,t}})$ gathers the discrete components of an action a_t and $a_t^C = (\tilde{m}_{1,t}, \tilde{m}_{2,t})$ gathers the continuous components, the *instantaneous utility* can be defined as:

$$u(s_t, a_t^C, a_t^D, x_t, \theta) = v(s_t, a_t^C, a_t^D, x_t, \varepsilon_C(a_t^C), \theta) + \varepsilon_D(a_t^D), \tag{6}$$

where variable x_t contains socio-economic information relative to the household, θ is a vector of parameters to be estimated. Expression $v(s_t, a_t^C, a_t^D, x_t, \varepsilon_C(a_t^C), \theta)$ is a deterministic term, $\varepsilon_D(a_t^D)$ is a random error term for the discrete actions and $\varepsilon_C(a_t^C)$ captures the randomness inherent to the continuous decision(s). Similarly as proposed by Rust (1987), the instantaneous utility has an additive-separable form.

Transaction name	0 car	1 car	2 cars
h_1: leave unchanged	1	1	1
h_2: increase 1	18	18	-
h_3: dispose 2	-	-	1
h_4: dispose 1st	-	1	1
h_5: dispose 2nd	-	-	1
h_6: dispose 1st and change 2nd	-	-	18
h_7: dispose 2nd and change 1st	-	-	18
h_8: change 1st	-	18	18
h_9: change 2nd	-	-	18
Sum	19	38	76

Table 1: Number of possible actions for households with 0, 1 or 2 cars (in the action space generated by the discrete components of the choice variable).

3.3 Formulation of the discrete-continuous choice model in a dynamic programming framework

As in a DDCM case (see e.g. Aguirregabiria and Mira, 2010; Cirillo and Xu, 2011), the value function of the DDCCM is defined as:

$$\begin{aligned} V(s_t,x_t,\theta) &= \max_{a_t\in A}\{u(s_t,a_t,x_t,\theta)+\beta\sum_{s_{t+1}\in S}\bar{V}(s_{t+1},x_{t+1},\theta)f(s_{t+1}|s_t,a_t)\} && (7)\\ &= \max_{a_t\in A}\{v(s_t,a_t^C,a_t^D,x_t,\varepsilon_C(a_t^C),\theta)+\varepsilon_D(a_t^D)+\beta\sum_{s_{t+1}\in S}\bar{V}(s_{t+1},x_{t+1},\theta)f(s_{t+1}|s_t,a_t)\} && (8)\end{aligned}$$

In order to obtain a version of the Bellman equation that does not depend on the random utility error term $\varepsilon_D(a_t^D)$, we consider the *integrated value function* $\bar{V}(s_t,x_t,\theta)$, given as follows:

$$\bar{V}(s_t,x_t,\theta)=\int V(s_t,x_t,\theta)dG_{\varepsilon_D}(\varepsilon_D(a_t^D)) \tag{9}$$

where G_{ε_D} is the CDF of ε_D.

In the case where all actions are discrete and the random terms $\varepsilon_D(a_t^D)$ are i.i.d. extreme value, it corresponds to the logsum (see e.g. Aguirregabiria and Mira, 2010). We aim at finding a closed-form formula in the case where the choices are both discrete and continuous too. In fact, a closed-form formula is possible in the special case where the choice of mileage of each car in the household is assumed myopic. This implies that individuals choose how much they wish to drive their car(s) every year, without accounting for the expected discounted utility of this choice for the following years[3].

Under the hypothesis of myopicity of the choice of annual driving distance(s), the integrated

[3] This assumption was also made in the unpublished work by Anders Munk-Nielsen, University of Copenhagen. We make this reasonable hypothesis here too.

value function is obtained as follows:

$$\begin{aligned}
\bar{V}(s_t,x_t,\theta) &= \int V(s_t,x_t,\theta)dG_\varepsilon(\varepsilon_D(a_t^D)) \\
&= \int \max_{a_t\in A}\{u(s_t,a_t,x_t,\theta,\varepsilon(a_t))+\beta\sum_{s_{t+1}\in S}\bar{V}(s_{t+1},x_{t+1},\theta)f(s_{t+1}|s_t,a_t)\}dG_\varepsilon(\varepsilon_D(a_t^D)) \\
&= \int \max_{a_t^D}\{\max_{a_t^C}\{v(s_t,a_t^C,a_t^D,x_t,\varepsilon_C(a_t^C),\theta)\}+\varepsilon_D(a_t^D)+\beta\sum_{s_{t+1}\in S}\bar{V}(s_{t+1},x_{t+1},\theta)f(s_{t+1}|s_t,a_t)\}dG_\varepsilon(\varepsilon_D(a_t^D)) \\
&= \log\sum_{a_t^D}\exp\{\max_{a_t^C}\{v(s_t,a_t^C,a_t^D,x_t,\varepsilon_C(a_t^C),\theta)\}+\beta\sum_{s_{t+1}\in S}\bar{V}(s_{t+1},x_{t+1},\theta)f(s_{t+1}|s_t,a_t)\} \qquad (10)
\end{aligned}$$

Similarly as in the case of a DDCM, the value function is obtained by iterating on Equation (10).

3.4 Choice of optimal mileage(s)

We assume that expression $v(s_t,a_t^C,a_t^D,x_t,\varepsilon_C(a_t^C),\theta)$ of Equation (10) is the sum of the utility of the discrete actions v_t^D and the utility of the continuous actions v_t^C:

$$v(s_t,a_t^C,a_t^D,x_t,\varepsilon_C(a_t^C),\theta) = v_t^D(s_t,a_t^D,x_t,\theta)+v_t^C(s_t,a_t^D,a_t^C,x_t,\varepsilon_C(a_t^C),\theta) \qquad (11)$$

By assumption, each household can have at maximum two cars. This implies that for two-car households, the annual mileage of each car must be decided every year. Expression $v(s_t,a_t^C,a_t^D,x_t,\varepsilon_C(a_t^C),\theta)$ of Equation (10) must hence be maximized with respect to the two annual driving distances. Given the additive form of Equation (11), we only need to maximize expression $v_t^C(s_t,a_t^D,a_t^C,x_t,\varepsilon_C(a_t^C),\theta)$ with respect to a_t^C.

However, if a household owns two cars, we observe from the data that one car is generally driven more than the other one, i.e. one is used for long distances while the other is used for shorter trips. Hence, we can assume that the choice that the household actually makes is not the independent choices of how much each car will be driven, but rather the repartition of the total mileage that it plans to drive across the two cars.

This motivates the use of a *constant elasticity of substitution (CES)* utility function for the choice of mileage(s), since it allows to evaluate the rate of substitution of mileages $m_{1,t}$ and $m_{2,t}$:

$$v_t^C(s_t,a_t^D,a_t^C,x_t,\varepsilon_C(a_t^C),\theta) = (m_{1,t}^{-\rho}+\alpha\cdot m_{2,t}^{-\rho})^{-1/\rho} \qquad (12)$$

Parameter ρ is the elasticity of substitution of v_t^C. Expression α represents the weight of the mileage of one car relative to the other. It is a function of socio-economic characteristics about the household x_t and a random term $\varepsilon_C(a_t^C)$:

$$\alpha := \exp\{\gamma x_t - \varepsilon_C(a_t^C)\}. \qquad (13)$$

Here, γ is a vector of parameters to estimate. To simplify the problem, we will assume in a first stage that α is only a constant to estimate. This implies that we assume no error term $\varepsilon_C(a_t^C)$. We will later relax this assumption and assume that α has the form of Equation (13).

The choice of $m_{1,t}$ and $m_{2,t}$ must be made such that the budget constraint of the household holds:

$$p_{1,t}m_{1,t}+p_{2,t}m_{2,t} = \mathrm{Inc}_t, \qquad (14)$$

where $p_{c,t} := \mathrm{cons}_{c,t}\cdot\mathrm{pl}_{c,t}$ is the cost per km of driving car $c\in\{1,2\}$ in SEK/km, that is the product of the car consumption $\mathrm{cons}_{c,t}$ and the price of a liter of fuel $\mathrm{pl}_{c,t}$ for that car. Variable Inc_t is the share of the household's annual income which is used for expenses related to car fueling.

The above formulation of the CES utility fonction with the budget constraint has the following advantages. First, the constraint enables us to solve the maximization problem according to one dimension only. Such an approach has been considered by Zabalza (1983), in a context of trade-off between leisure and income. Second, the use of a CES function is also convenient, since the elasticity of substitution is directly obtained from the estimate of parameter ρ.

The optimal value of mileage $m_{1,t}$ is obtained by solving the following maximization problem:

$$\max_{m_{1,t},m_{2,t}} v_t^C, \text{ such that } p_{1,t}m_{1,t} + p_{2,t}m_{2,t} = \text{Inc}_t \tag{15}$$

Assuming that we know what share of the household's income is spent on fuel[4], we can obtain an analytical solution for $m_{2,t}$:

$$m_{2,t}^* = \frac{\text{Inc}_t \cdot p_{2,t}^{(-1/(\rho+1))}}{p_{2,t}^{(\rho/(\rho+1))} + p_{1,t}^{(\rho/(1+\rho))}\alpha^{(-1/(\rho+1))}}. \tag{16}$$

We can then infer the value of the optimal mileage for the other car:

$$\begin{aligned} m_{1,t}^* &= \frac{\text{Inc}_t}{p_{1,t}} - \frac{p_{2,t}}{p_{1,t}} m_{2,t}^* && (17) \\ &= \frac{\text{Inc}_t}{p_{1,t}} - \frac{p_{2,t}}{p_{1,t}} \cdot \frac{\text{Inc}_t \cdot p_{2,t}^{(-1/(\rho+1))}}{p_{2,t}^{(\rho/(\rho+1))} + p_{1,t}^{(\rho/(1+\rho))}\alpha^{(-1/(\rho+1))}} && (18) \end{aligned}$$

Consequently, we obtain the optimal value for the deterministic utility of the continuous actions:

$$v_t^{C*} = \left(\left(\frac{\text{Inc}_t \cdot p_{2,t}^{(-1/(\rho+1))}}{p_{2,t}^{(\rho/(\rho+1))} + p_{1,t}^{(\rho/(1+\rho))}\alpha^{(-1/(\rho+1))}} \right)^{-\rho} + \alpha \cdot \left(\frac{\text{Inc}_t}{p_{1,t}} - \frac{p_{2,t}}{p_{1,t}} \cdot \frac{\text{Inc}_t \cdot p_{2,t}^{(-1/(\rho+1))}}{p_{2,t}^{(\rho/(\rho+1))} + p_{1,t}^{(\rho/(1+\rho))}\alpha^{(-1/(\rho+1))}} \right)^{-\rho} \right)^{-1/\rho} \tag{19}$$

Then v_t^{C*} can be inserted back in Equation (11). The Bellman equation (10) becomes:

$$\bar{V}(s_t, x_t, \theta) = \log \sum_{a_t^D} \left\{ \exp\{v_t^D(s_t, a_t^D, x_t, \theta) + v_t^{C*}(s_t, a_t^D, a_t^{C*}, x_t, \theta)\} + \beta \sum_{s_{t+1} \in S} \bar{V}(s_{t+1}, x_{t+1}, \theta) f(s_{t+1}|s_t, a_t) \right\}, \tag{20}$$

where $a_t^{C*} = (m_{1,t}^*, m_{2,t}^*)$.

The integrated value function $\bar{V}$ is then computed by value iteration.

3.5 Model estimation

The parameters of the DDCCM are obtained by maximizing the following likelihood function

$$\mathscr{L} = \prod_{n=1}^{N} \prod_{t=1}^{T_n} P(a_{n,t}^D | s_{n,t}, x_{n,t}, \theta), \tag{21}$$

where N is the total population size, T_n is the number of years household n is observed and $P(a_{n,t}^D | s_{n,t}, x_{n,t}, \theta)$ is the probability that household n chooses a particular discrete action $a_{n,t}^D$ at

[4]For example, from 2006 to 2009, households in Sweden spent between 7.3 and 8.1 percent of their income on the operation of motor-cars (Source: Statistics Sweden).

time t. This probability is obtained as follows:

$$P(a_{n,t}^{D}|s_{n,t},x_{n,t},\theta)=\frac{v_{n,t}^{D}(s_{n,t},a_{n,t}^{D},x_{n,t},\theta)+v_{n,t}^{C*}(s_{n,t},a_{n,t}^{D},a_{n,t}^{C*},x_{n,t},\theta)+\beta\sum_{s_{n,t+1}\in S}\bar{V}(s_{n,t+1},x_{n,t+1},\theta)f(s_{n,t+1}|s_{n,t},a_{n,t})}{\sum_{a_{n,t}^{\tilde{D}}}\left\{v_{n,t}^{D}(s_{n,t},a_{n,t}^{\tilde{D}},x_{n,t},\theta)+v_{n,t}^{C*}(s_{n,t},a_{n,t}^{\tilde{D}},a_{n,t}^{\tilde{C}*},x_{n,t},\theta)+\beta\sum_{s_{n,t+1}\in S}\bar{V}(s_{n,t+1},x_{n,t+1},\theta)f(s_{n,t+1}|s_{n,t},\tilde{a_{n,t}})\right\}} \tag{22}$$

The simplest way to estimate this type of model is using the nested fixed point algorithm proposed by Rust (1987) where the DP problem is solved for each iteration of the non-linear optimization algorithm searching of the parameter space. Our DP problem is quite simple because of the transition function being deterministic. Still, we are in the case of a large state and action space and we expect that it is computationally infeasible to estimate the model using the nested fixed point algorithm. Instead we will adopt the approach by Aguirregabiria and Mira (2002) which reduces the number of times the DP problem needs to be solved.

4 Illustrative example

As described in the previous sections, the discrete-continuous choice model is embedded into a dynamic programming framework in order to account for the expected discounted utility of each action. In this section, we present an illustration of the results of the value iteration algorithm, for imposed parameter values.

4.1 Example of specification

As an example, we consider a simple specification of the deterministic (instantaneous) utility relative to the choice of the discrete variables:

$$v_t^D(s_t,a_t^D,x_t,\theta)=\mathrm{C}(s_t)+\tau(a_t^D)+\beta_{\mathrm{Age}}(a_t^D,s_t)\cdot\max(\mathrm{Age1}_t,\mathrm{Age2}_t), \tag{23}$$

where $\mathrm{Age1}_t$ is the age of the first of the two cars and $\mathrm{Age2}_t$ is the age of the second car. Expressions $\mathrm{C}(s_t)$, $\tau(a_t^D)$ and $\beta_{\mathrm{Age}}(a_t^D,s_t)$ are parameters that would typically be estimated on data. We follow the approach proposed by Schiraldi (2011) and specify a constant $\mathrm{C}(s_t)$ relative to households owning at least one car and a transaction cost $\tau(a_t^D)$. The constant is included in order to capture differences of preferences between households owning at least one car and households without a car. The transaction cost is meant to capture the unobserved costs of actions involving the acquisition of a new car, i.e. actions with transactions h_2 (increase of 1), h_6 (dispose of 1st and change 2nd), h_7 (dispose of 2nd and change 1st), h_8 (change 1st) or h_9 (change 2nd).

In order to illustrate the application of the DDCCM, we postulate values for the parameters of Equation (23). The signs and values are chosen in order to match a priori expectations. For example, in a one-car household, the older the car is, the more likely the household is to dispose of it. Hence, we give a positive sign to parameter β_{Age}. Assuming that owning at least one car has a positive impact on choice (as in Schiraldi, 2011, we set C to 5 if the household owns at least a car and to 0 otherwise. The other chosen parameter values are reported in Table 2. Parameter β_{Age} depends on the size of the household, on whether the first or the second car is the oldest and on the transaction type. The transaction cost τ varies according to the different transactions types.

Likewise, we choose values for the parameters of the deterministic (instantaneous) utility relative to the choice of mileage. As described in Section 3.4, we assume that α is a constant and fix its value to 0.3. The elasticity of substitution ρ is set to 0.5. Moreover we fix the discount factor in the Bellman equation (20) to 0.7.

		β_{Age}			τ
Transaction name	Case	0 car	1 car	2 cars	all households
h_1: leave unchanged		0	-1	-1	0
h_2: increase 1		0	0	-	-3
h_3: dispose 2		-	-	1	0
h_4: dispose 1st	1st car is oldest	-	1.5	1.5	0
	2nd car is oldest	-	-	0	0
h_5: dispose 2nd	1st car is oldest	-	-	0	0
	2nd car is oldest	-	-	1.5	0
h_6: dispose 1st and change 2nd		-	-	0	-4
h_7: dispose 2nd and change 1st		-	-	0	-4
h_8: change 1st	1st car is oldest	-	1.5	1.5	-4
	2nd car is oldest	-	-	0	-4
h_9: change 2nd	1st car is oldest	-	-	0	-4
	2nd car is oldest	-	-	1.5	-4

Table 2: Parameters for the deterministic (instantaneous) utility relative to the discrete actions

4.2 Results from the value iteration

Using the parameters of Section 4.1, we iterate on the Bellman equation (20) to obtain the value function for an example of observation. The latter consists of a household with an annual income of $530'000$ SEK that spends about 8% of its resources on fuel (following the hypothesis described in Section 3.4).

The program is implemented in C++ and the running time to obtain the value function is about 2 minutes on a 20-core computer.

Figure 2 shows boxplots of the value function for ages of car ranging from 0 to 3[5]. As expected we observe that the value function decreases as the maximum of the ages of the two cars increases. This shows that the older the car is, the smaller its expected discounted utility becomes.

[5] We note that in this evaluation of the value function, we restricted the maximum of the age of the two cars to 3 years for a better visualization of the difference of the results, but the upper bound for age can be increased.

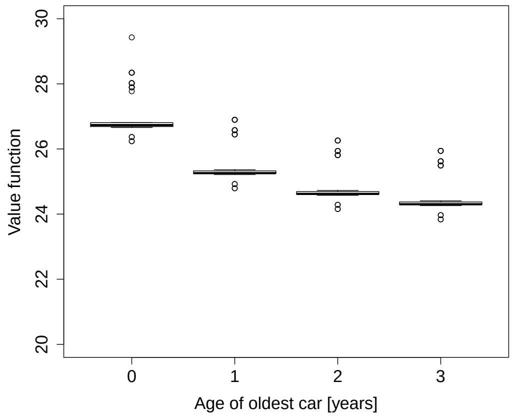
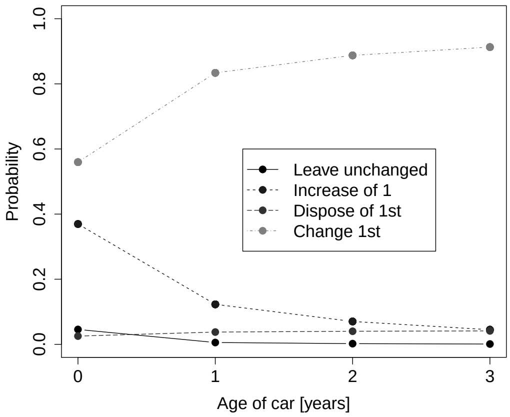
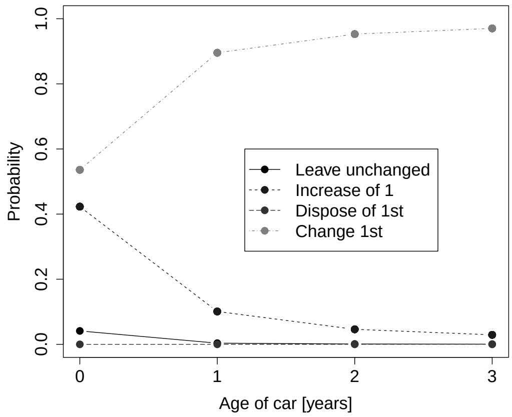


Figure 2: Value function

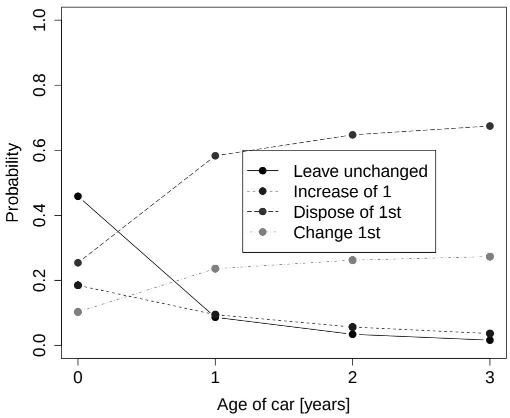
Probabilities of choosing a particular action can be computed using Equation (22). As an example, we evaluate these probabilities for the above mentioned household, assuming that it has a one private diesel car. We analyze the probabilities of leaving the household car fleet unchanged, adding one car, disposing of the only car and changing the only car, as a function of the age of the car (Figure 3). For comparison purposes, we do not only display the probabilities for the above specified model (Figure 3(d)), but also for a model with the same specification but without transaction cost (Figure 3(b)), and for static models with (Figure 3(c)) and without (Figure 3(a)) transaction cost. By static models, we denote models based on the assumption that households do not account for the future utility of the choices of transaction, type of contract, fuel type and state of the car.



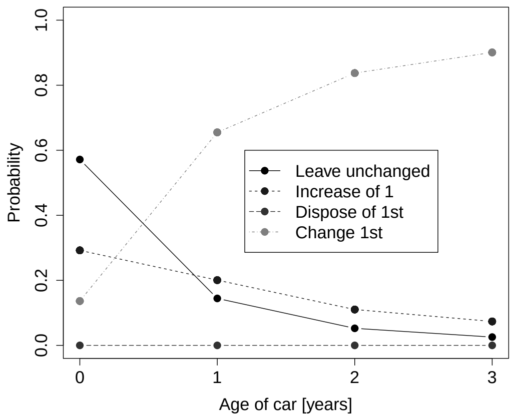
(a) Static – without transaction cost



(b) Dynamic – without transaction cost



(c) Static – with transaction cost



(d) Dynamic – with transaction cost

Figure 3: Choice probabilities for different transactions, as a function of the age of the car.

Figure 3 allows to analyze variations of particular probabilities as a function of the age of the car, e.g. the probability of replacing the only car in the household (denoted as changing 1st). As expected (from the imposed sign on the parameter) we note that the probability of changing the only car in the household increases as the age of the car increases.

Although we emphasize on the fact that no behavioral interpretation can be made on the model at this point since all parameters have chosen values, we want to highlight that considering a dynamic model can result in different choice probabilities than in the static case and it is a key aspect to investigate. Moreover the introduction of a transaction cost also affects the trade-offs between the transactions.

5 Conclusion

This paper presents a methodology designed to jointly model car ownership, usage and choice of fuel type together with an example of application. One of the main properties of the model is that it accounts for the forward-looking behavior of individuals, which is crucial in the case of demand for durable goods such as car, since the purchase of a car affects the utility of an individual for the present and future years of ownership (Schiraldi, 2011).

In order to obtain a realistic model, we account for households' decisions rather than individual ones. We specify a CES utility function to capture substitution patterns that occur when two-car households decide on the annual mileage of the two cars. We also consider a comprehensive choice variable, that accounts for decisions that are usually jointly made, such as car ownership, choice of fuel type and annual mileage.

The next steps in this research are (1) to validate the DDCCM by estimating it on synthetic data generated from distributions of attributes of observations in the Swedish register of cars and individuals and (2) to estimate it on the full register data. In a later stage, we will assess the impact of policies implemented during the years of the data on the dynamics of the Swedish fleet and perform a forecasting analysis of several policy scenarios that have already been defined in the planning process of the Swedish government for the upcoming years.

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