Coordination between discrete choice models to accelerate DTA

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Day-to-day DTA simulation

- One day (simulation iteration):
  1. Every traveler **chooses** a travel pattern.
  2. All travelers **execute** their travel patterns (network loading).
  3. All travelers **observe** the resulting network conditions.
- Eventually, the process attains stationarity.
- One wishes to attain stationarity as quickly as possible.
- Recipe: Decide who gets to replan in a given day.

Notation (person level)

- (All of this in a given simulation iteration.)
- Choices.
  - n is one of N decision makers.
  - λ_n is a 0/1 replanning indicator.
  - Δu_n is n's *anticipated* utility gain from replanning.
  - Δu_n^* is n's subsequently *realized* utility change.
- Network slots.
  - i is a (link, time interval) tuple – a "slot".
  - Δx_{ni} is ...
    - +1 if n intends to use slot i and did not do so before;
    - -1 if n intends to not use slot i and did do so before;
    - 0 in all other cases.

Notation (population level)

- *Anticipated* utility change:

$$\Delta u(\boldsymbol{\lambda}) = \sum_n \lambda_n \Delta u_n$$

- *Realized* utility change:

$$\Delta u^*(\boldsymbol{\lambda}) = \sum_n \Delta u_n^*(\boldsymbol{\lambda})$$

- Slot usage change:

$$\Delta x^2(\boldsymbol{\lambda}) = \sum_i \left(\sum_n \lambda_n \Delta x_{ni} \right)^2$$

Recipe

- In a given iteration, there exists a $\beta \geq 0$ such that

$$\beta \cdot (\Delta u(\boldsymbol{\lambda}) - \Delta u^*(\boldsymbol{\lambda})) \quad \leq \quad \Delta x^2(\boldsymbol{\lambda})$$

  for all possible choices of $\boldsymbol{\lambda} \in \{0, 1\}^N$.
- Yields a lower bound on the realized utility change:

$$\beta \cdot \Delta u^*(\boldsymbol{\lambda}) \quad \geq \quad \beta \cdot \Delta u(\boldsymbol{\lambda}) - \Delta x^2(\boldsymbol{\lambda}).$$

- Select in every iteration a $\boldsymbol{\lambda}$ that maximizes this lower bound.
- (Track β over simulation iterations by MSA.)

Replanner selection problem

- Solve in simulation iteration k the problem

$$\max_{(\lambda_n)\in\{0,1\}^N} \beta \sum_n \lambda_n \Delta u_n^{(k)} - \sum_i \left(\sum_n \lambda_n \Delta x_{ni}^{(k)} \right)^2 .$$

- (Not sure if this has a behavioral counterpart.)
- Simplifying this into a continuum model with link-additive utilities, this turns into the projection method for VIs.
- Binary: Some decision makers may never get to replan.

Replanner selection problem with aging

- Solve in simulation iteration k the problem

$$\max_{(\lambda_n)\in\{0,1\}^N} \quad \beta \sum_n \lambda_n \Delta u_n^{(k)} - \sum_i \left(\sum_n \lambda_n \frac{\Delta x_{ni}^{(k)}}{a_n^{(k)}} \right)^2$$

  where the age

$$a_n^{(k)} = \begin{cases} 1 & \text{if } n \text{ replanned in iteration } k-1 \\ a_n^{(k-1)} + 1 & \text{otherwise} \end{cases}$$

  counts for how long a person has not replanned.
- By aging, everyone eventually gets to replan.
- (Behaviorally, a kind of impatience.)

Asymptotic analysis

- Assumptions:
  - n is exogenous observer with given Δu_n and Δx_{ni}.
  - Large population (continuous limit) approximation.
  - Stationary conditions.
- Slot change similarity of n and the (infinite) population:

$$s_n = \sum_i \Delta x_{ni} \cdot \left(\frac{1}{N} \sum_{m=1}^{N} \lambda_m \frac{\Delta x_{mi}}{a_m} \right)$$

- Resulting stationary replanning rate of commodity n:

$$\lambda_n \propto \frac{\Delta u_n}{s_n}.$$

Utility over iterations (all-of-Stockholm scenario)