

# Are You The Beatles Or The Rolling Stones?

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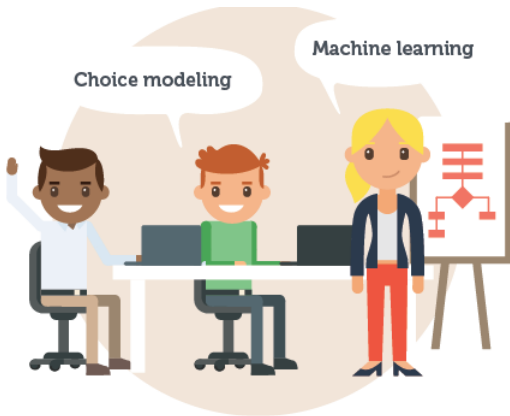


ARE YOU

THE BEATLES

OR

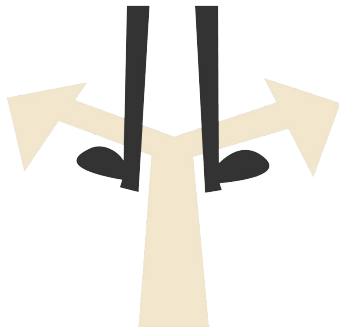
the Rolling  
Stones



*"The new and good ideas come from having a very broad and multidisciplinary range of interests."*

Robin Chase

# Discrete choice models



**Understand** and **predict** individual choice behavior  
using mathematical models

# Discrete choice models



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using mathematical models

# Discrete choice models

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Prediction accuracy + Interpretability

# Machine learning

## Neural networks

# Machine learning

Neural networks

Very good prediction accuracy

# Machine learning

Neural networks

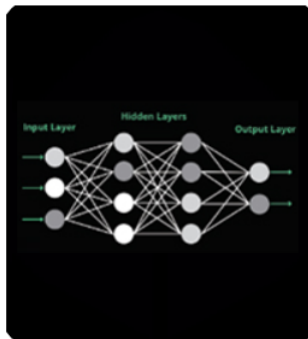
Very good prediction accuracy



# Machine learning



# Machine learning



# Enhancing Discrete Choice Models with Neural Networks



# General idea

Bringing the **predictive strength** of Neural Networks, a powerful machine learning-based technique, to the field of Discrete Choice Models without compromising the **interpretability** of these models



# Multinomial Logit as Convolution NN

## MNL

Choice set  $C_n = \{1, \dots, J_n\}$

Parameters  $\beta_1, \dots, \beta_p$

Variables  $x_{ikn}$

## CNN

Labels ( $1 \times J_n$ ) vector

Kernel weights ( $p \times 1$ )

Features:  $X$  ( $p \times J_n$ ) tensor

# Multinomial Logit as Convolution NN

Activation Function: Softmax

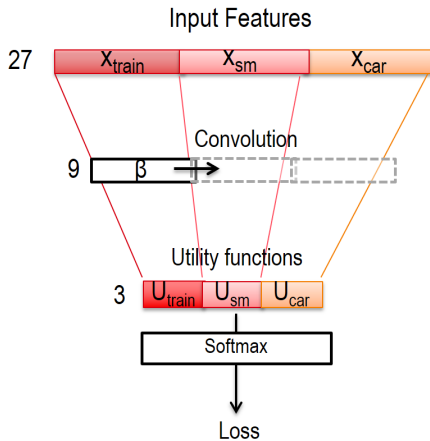
$$(\sigma(\mathbf{V}_n))_i = \frac{\exp^{V_{in}}}{\sum_{j=1}^p \exp^{V_{jn}}}$$

Loss: Categorical Cross-Entropy

$$H_n(\sigma, \mathbf{y}_n) = - \sum_{i=1}^P y_{in} \log [\sigma(\mathbf{V}_n)]_i$$

# Multinomial Logit as Convolution NN

A single convolution gives us the utility functions



# Model and Validation on Swissmetro

🤖 Stated Preference Survey

🤖 Simple utilities for validation purposes:

$$V_{car} = ASC_{car} + \beta_{cost} \cdot Cost_{car} + \beta_{time} \cdot Time_{car}$$

$$V_{train} = ASC_{train} + \beta_{cost} \cdot Cost_{train} + \beta_{time} \cdot Time_{train}$$

$$V_{SM} = ASC_{SM} + \beta_{cost} \cdot Cost_{SM} + \beta_{time} \cdot Time_{SM}$$

## Biogeme betas

Name	Value	Std err	t-test	p-value
ASC_CAR	-0.993	0.0385	-25.78	0.00
ASC_TRAIN	-1.49	0.0515	-28.86	0.00
B_COST	-0.663	0.0473	-14.03	0.00
B_TIME	-0.153	0.0373	-4.11	0.00

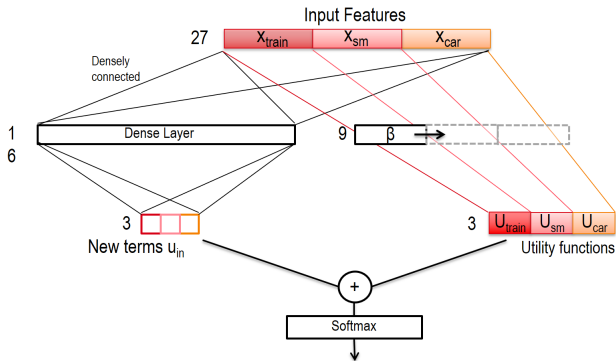
## Convolution Kernel weights

### Convolution Kernel weights

- ASC\_Car: -0.99298888
- ASC\_Train: -1.48712599
- B\_COST: -0.66329724
- B\_TIME: -0.15334089

# Neural Network enhanced DCM

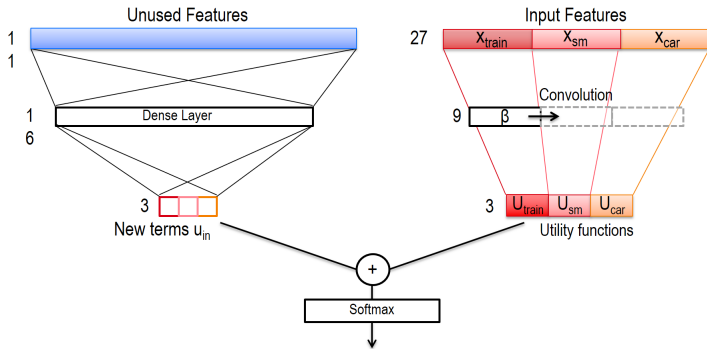
## Case 1: same inputs



The new term from the NN overruns the MNL linear parameters

# Neural Network enhanced DCM

## Case 2: different inputs



New term in the utility specification coming from a NN using all unused features

# Neural Network enhanced DCM

## Case 2: different inputs

$$U_{in} = ASC_i + \beta_{cost} \cdot Cost_{in} + \beta_{time} \cdot Time_{in} + \beta_{NN} \cdot NN_{in} + \varepsilon_{in}$$

🤖 Interpretation:  $NN_{in}$  = uncaptured information of MNL model

- ✓ MNL estimates keep their significance
- ✓ Increase in log-likelihood



# Swissmetro case

## Utility Functions

Variable		Alternative		
		Car	Train	Swissmetro
ASC	Constant	Car-Const		SM-Const
TT	Travel Time	B-Time	B-Time	B-Time
Cost	Travel Cost	B-Cost	B-Cost	B-Cost
Freq	Frequency		B-Freq	B-Freq
GA	Annual Pass		B-GA	B-GA
Age	Age in classes		B-Age	
Luggage	Pieces of luggage	B-Luggage		
Seats	Airline seating			B-Seats

# Swissmetro case

## Unused Features

- Travel purpose: Discrete value between 1 to 9 (Business, leisure, travel,... )
- First class: 0 for no or 1 for yes if passenger is a first class traveler in public transport
- Ticket: Discrete value between 0 to 10 for the ticket type (One-way, half-day, ...)
- Who: Discrete value between 0 to 3 for who pays the travel (self, employer, ...)
- Male: Traveler's gender, 0 for female and 1 for male
- Income: Discrete value between 0 to 4 concerning the traveler's income per year
- Origin: Discrete value defining the canton in which the travel begins
- Dest: Discrete value defining the canton in which the travel ends

# Results

## Benchmark - standard MNL

Parameter number	Description	Coeff. estimate	Robust Asympt. std. error	<i>t</i> -stat	<i>p</i> -value
1	$ASC_{Car}$	1.20	0.183	6.58	0.00
2	$ASC_{SM}$	1.19	0.182	6.53	0.00
3	$\beta_{age}$	0.175	0.0512	3.41	0.00
4	$\beta_{cost}$	-0.00690	0.000577	-11.97	0.00
5	$\beta_{freq}$	-0.00704	0.00116	-6.09	0.00
6	$\beta_{GA}$	1.54	0.168	9.17	0.00
7	$\beta_{luggage}$	-0.113	0.0479	-2.36	0.02
8	$\beta_{seats}$	0.432	0.115	3.76	0.00
9	$\beta_{time}$	-0.0129	0.000842	-15.34	0.00

Number of observations = 7234

$$\mathcal{L}(\hat{\beta}) = -5766.705$$

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# Results

## Hybrid model

Parameter number	Description	Coeff. estimate	Robust Asympt. std. error	$t$ -stat	$p$ -value
1	$ASC_{Car}$	0.0652	0.179	0.37	0.71
2	$ASC_{SM}$	0.327	0.171	1.92	0.06
3	$\beta_{age}$	0.376	0.0464	8.12	0.00
4	$\beta_{cost}$	-0.0141	0.000595	-23.63	0.00
5	$\beta_{freq}$	-0.00807	0.00123	-6.55	0.00
6	$\beta_{GA}$	0.130	0.181	0.72	0.47
7	$\beta_{luggage}$	0.0153	0.0505	0.30	0.76
8	$\beta_{seats}$	0.207	0.106	1.95	0.05
9	$\beta_{time}$	-0.0157	0.000952	-16.53	0.00
10	$\beta_{NN}$	1.24	0.0524	23.74	0.00

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++ Increased likelihood

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++ Keeps important parameters significant

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– Some parameters loose significance



# Results

## Simple hybrid model - only key parameters

Parameter number	Description	Coeff. estimate	Robust Asympt. std. error	<i>t</i> -stat	<i>p</i> -value
1	$ASC_{Car}$	0.966	0.0977	9.89	0.00
2	$ASC_{SM}$	1.13	0.0941	11.97	0.00
3	$\beta_{cost}$	-0.0165	0.000666	-24.71	0.00
4	$\beta_{freq}$	-0.00820	0.00129	-6.38	0.00
5	$\beta_{time}$	-0.0171	0.000853	-20.05	0.00
6	$\beta_{NN}$	1.25	0.0854	14.65	0.00

Number of observations = 7234

$$\mathcal{L}(\hat{\beta}) = -4894.539$$

# Results

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Increased likelihood + significant parameters

# Results

## Comparison

Parameter	MNL	Hybrid	Simple Hybrid
$\beta_{cost}$	100.0%	204.3%	239.1%
$\beta_{freq}$	100.0%	114.6%	116.5%
$\beta_{time}$	100.0%	121.7%	132.5%
Value of Time	0.54	0.89	0.96
Value of Frequency	0.98	1.75	2.01
Final Log-Likelihood	-5766.71	-5009.00	-4894.54
Number of parameters	9	10	6

# Conclusions



Enhancing DCM greatly increases likelihood



Same input or correlated input

- Breaks original parameter significance



Independent Input

- Best performances for likelihood
- All parameters have good sign and significance

# Future work

## Interpretability: what is really happening?

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Value of Frequency	0.98	1.75	2.01
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Number of parameters	9	10	6

What do you think?



# Automatic Utility Specification Using Machine Learning Techniques

# Credit



Nicola Ortelli



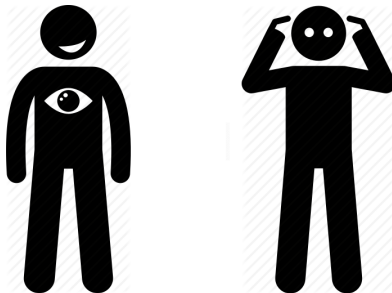
# Motivation

Determining the appropriate utility specification for a particular application is a difficult task



# Motivation

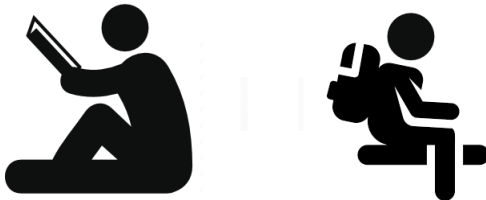
Determining the appropriate utility specification for a particular application is a difficult task



Expertise + Intuition

# Motivation

Determining the appropriate utility specification for a particular application is a difficult task



Inspiration + Experience

# Motivation

Determining the appropriate utility specification for a particular application is a difficult task



Trial-and-error

# Motivation

Determining the appropriate utility specification for a particular application is a difficult task



Time consuming

# Objective

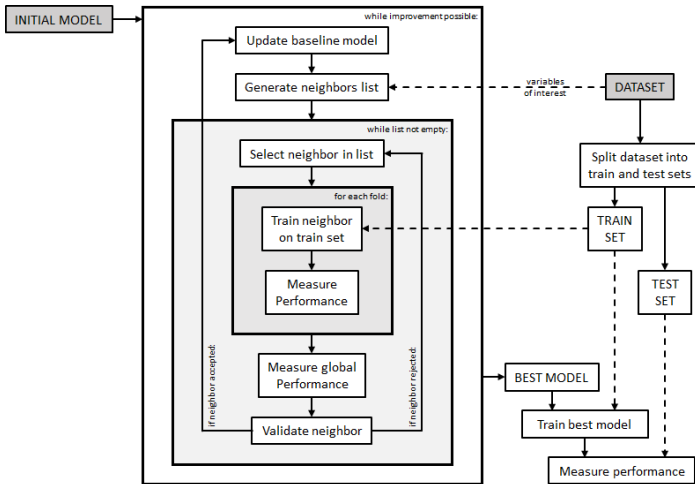


Build a procedure that automatically finds a *good* utility specification based on the data

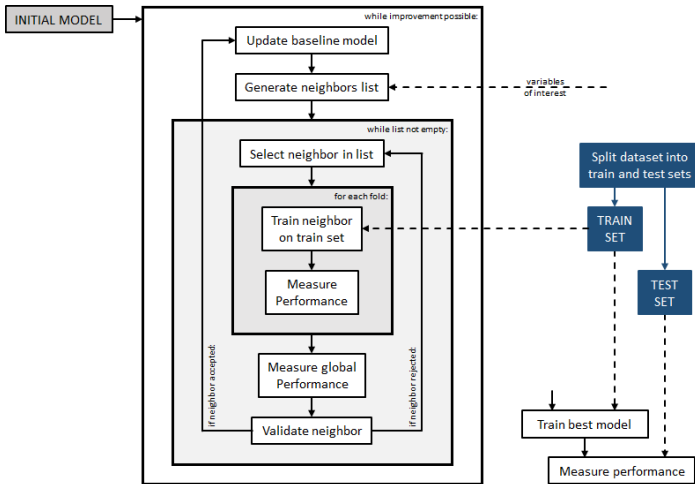


Define neighborhood relations between specifications and use classical local search algorithms

# Automatic utility specification framework



# Data partition

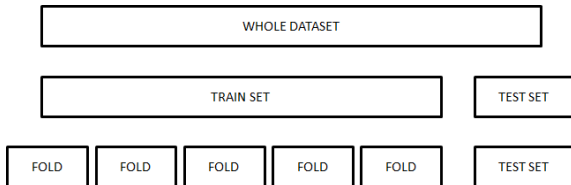




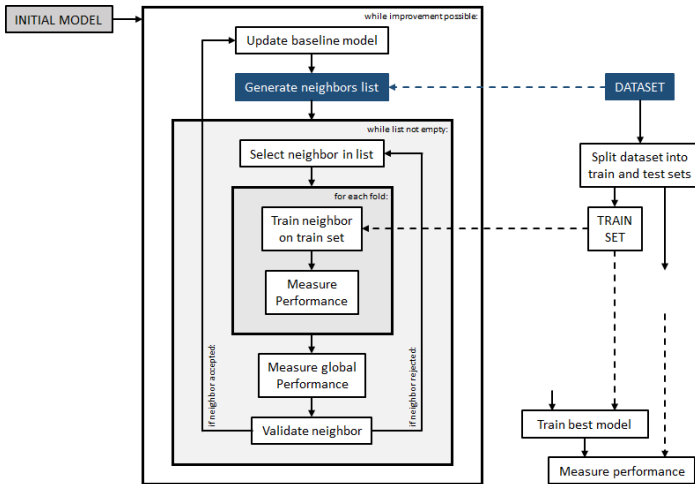
# Data partition

🚗 Data set separated into train set and test set

🚗 Train set separated into K folds



# Specifications and neighborhood structure



# Specifications and neighborhood structure

## Current assumptions:

- 🐞 Only continuous and binary variables
- 🐞 Each continuous variable is included either on its own or in interaction with one binary variable
- 🐞 All parameters are alternative specific
- 🐞 Example:

$$V = \dots + \beta_x \cdot x + \dots$$

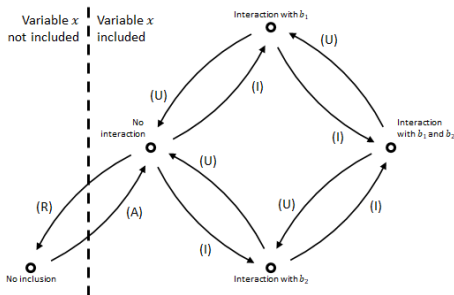
$$V = \dots + \beta_{x,b_1=0} \cdot x \cdot (1 - b_1) + \beta_{x,b_1=1} \cdot x \cdot b_1 + \dots$$

$$V = \dots + \beta_{x,b_2=0} \cdot x \cdot (1 - b_2) + \beta_{x,b_2=1} \cdot x \cdot b_2 + \dots$$

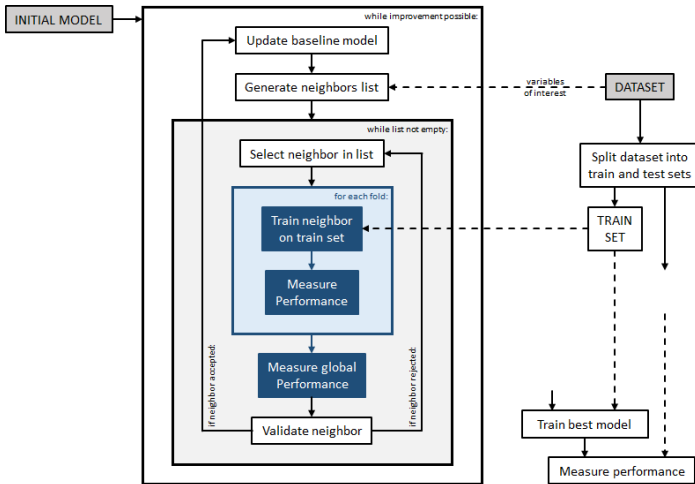
$$V = \dots + \beta_{x,b_1=0,b_2=0} \cdot x \cdot (1 - b_1) \cdot (1 - b_2) + \beta_{x,b_1=1,b_2=0} \cdot x \cdot b_1 \cdot (1 - b_2) \\ + \beta_{x,b_1=0,b_2=1} \cdot x \cdot (1 - b_1) \cdot b_2 + \beta_{x,b_1=1,b_2=1} \cdot x \cdot b_1 \cdot b_2 + \dots$$

# Specifications and neighborhood structure

- Neighbors of a particular specification = all specifications that are a single change away from it
- Four possible changes: add, remove, interact, un-interact



# Measure of performance



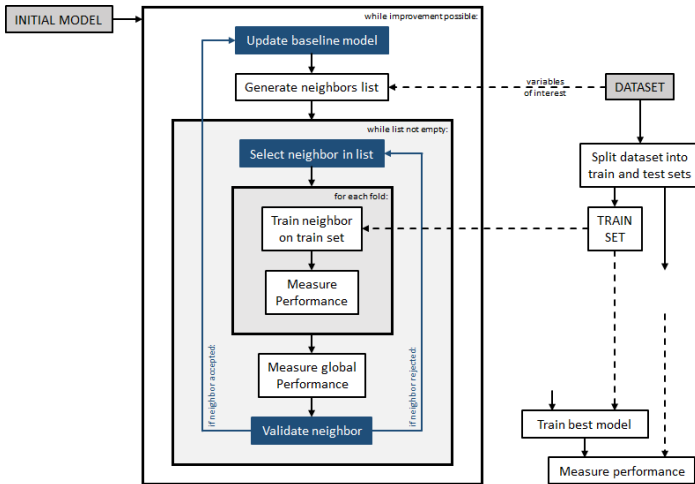
# Measure of performance

## Current assumptions:

- Performance of a specification is measured as the log-likelihood it yields on new data
- Cross-validation is used to avoid overfitting or favorizing models with a large number of parameters
- The global performance  $P_m$  of model  $m$  is defined as the sum of the log-likelihoods obtained on each fold after estimation on the  $K-1$  others:

$$P_m = \sum_{k=1}^K \mathbb{L}_{mk}$$

# Neighbor validation



# Neighbor validation

## Conditions of acceptance of a model $m$ :

- 🤖 A neighbor that performs better than the baseline is always accepted

$$P(\text{accept } m) = 1 \text{ if } P_m \geq P_{m_b}$$

- 🤖 One that performs worse might still get accepted, depending on the difference of performance  $P_m - P_{m_b}$  and the current iteration number

$$P(\text{accept } m) = \frac{\exp(P_m - P_{m_b})}{T_t} \text{ if } P_m < P_{m_b}$$

- 🤖 The algorithm ends when all neighbors of a model are rejected



## Case study 1: Optima

# Case study: Optima

## Revealed preference survey conducted in Switzerland for CarPostal:

- 🚗 Three alternatives: public transports, car and soft modes
- 🚗 1'814 observations left after discarding incomplete ones
- 🚗 Two binary and six continuous variables are considered
- 🚗 250'000 different specifications can be obtained

Variable	Description
Gender	<i>Traveler's gender. 0 if female, 1 if male.</i>
UrbRur	<i>Area where the traveler lives. 1 rural, 0 if urban.</i>
TimePT	<i>Duration of a loop performed in public transport [minutes].</i>
MarginalCostPT	<i>Public transport cost, taking into account travel cards ownership [CHF].</i>
TimeCar	<i>Duration of a loop performed using the car [minutes].</i>
CostCar	<i>Total gas cost of a loop performed with the car [CHF].</i>
distance_km	<i>Total distance performed for a loop [km].</i>
NbTrajects	<i>Number of trips in a loop.</i>

# Best specification

Parameter	Final value	<i>t</i> -stat
$ASC_{PT}$	1.16	3.84
$B_{\text{MarginalCostPT}}$	-0.0651	-7.75
$B_{\text{TimePT}}$	-0.0106	-5.69
$B_{\text{PT-distance\_km}}$	0.231	10.1
$B_{\text{PT-NbTrajects}}$	-0.759	-4.69
$ASC_{CAR}$	1.76	6.21
$B_{\text{TimeCar}}$	-0.0424	-6.19
$B_{\text{Car-distance\_km}}$	0.233	9.74
$B_{\text{Car-NbTrajects}}$	-0.66	-4.47
$ASC_{SM}$	0	Fixed
$B_{\text{SM-distance\_km}}$	0	Fixed
$B_{\text{SM-NbTrajects}}$	0	Fixed

# Comparison with an existing model

Data	Statistic	Best encountered	Benchmark model
Train set	Log-likelihood	-952.51	-843.29
	Accuracy	61.2%	66.2%
	Correct guesses	71.8%	76.5%
Test set	Log-likelihood	-216.54	-208.47
	Accuracy	60.3%	65.1%
	Correct guesses	69.0%	74.7%
Number of estimated parameters		9	18
Number of considered variables		5	13

## Case study 2: SwissMetro

# Case study: SwissMetro

## Stated choice survey to analyze the impact of the Swissmetro:

- 🚆 Three alternatives: train, Swissmetro and car
- 🚆 Nine different situations for each of the 1'192 respondents
- 🚆 Two binary and eight continuous variables are considered
- 🚆 6'250'000 different specifications can be obtained

Variable	Description
GENDER	<i>Traveler's gender. 0 if female, 1 if male.</i>
GA	<i>GA travel card ownership. 1 if the traveler owns one, 0 otherwise.</i>
TRAIN <sub>TT</sub>	<i>Train travel time [minutes]. Door-to-door, based on the car distance.</i>
TRAIN <sub>CO</sub>	<i>Train cost [CHF]. If the traveler owns a GA, equal to its annual price.</i>
TRAIN <sub>HE</sub>	<i>Train headway [minutes].</i>
SM <sub>TT</sub>	<i>Swissmetro travel time [minutes]. A speed of 500 km/h is considered.</i>
SM <sub>CO</sub>	<i>Swissmetro cost [CHF]. Equal to the rail fare multiplied by a fixed factor.</i>
SM <sub>HE</sub>	<i>Swissmetro headway [minutes].</i>
CAR <sub>TT</sub>	<i>Car travel time [minutes].</i>
CAR <sub>CO</sub>	<i>Car cost [CHF]. A fixed average cost per kilometer is considered.</i>

# Best specification

Parameter	Final value	t-stat
$ASC_{TRAIN,GA=0}$	0.472	3.23
$ASC_{TRAIN,GA=1}$	4.31	9.29
$B_{TRAIN-TT,GA=0,GENDER=0}$	-0.00815	-8.36
$B_{TRAIN-TT,GA=0,GENDER=1}$	-0.0213	-21.9
$B_{TRAIN-TT,GA=1,GENDER=0}$	0.000281	0.122
$B_{TRAIN-TT,GA=1,GENDER=1}$	-0.0000779	-0.0516
$B_{TRAIN-CO,GA=0}$	-0.00746	-8.76
$B_{TRAIN-CO,GA=1}$	-0.00102	-8.72
$B_{TRAIN-HE}$	-0.00719	-7.23
$ASC_{SM,GA=0,GENDER=0}$	1.4	8.83
$ASC_{SM,GA=0,GENDER=1}$	0.286	2.81
$ASC_{SM,GA=1,GENDER=0}$	5.45	9.3
$ASC_{SM,GA=1,GENDER=1}$	4.7	9.9
$B_{SM-TT}$	-0.0137	-19.4
$B_{SM-CO,GA=0,GENDER=0}$	-0.0058	-7.75
$B_{SM-CO,GA=0,GENDER=1}$	-0.0078	-16.1
$B_{SM-CO,GA=1,GENDER=0}$	-0.00103	-7.6
$B_{SM-CO,GA=1,GENDER=1}$	-0.000589	-6.74
$B_{SM-HE}$	-0.0071	-2.46
$ASC_{CAR}$	0	Fixed
$B_{CAR-TT,GENDER=0}$	-0.00383	-4.07
$B_{CAR-TT,GENDER=1}$	-0.0125	-17.8
$B_{CAR-CO}$	-0.00672	-7.58

# Comparison with an existing model

Data	Statistic	Best encountered	Bierlaire <i>et al.</i> (2001)
Train set	Log-likelihood	-6'431.72	-6'759.69
	Accuracy	54.1%	51.8%
	Correct guesses	67.2%	62.8%
Test set	Log-likelihood	-1'551.94	-1'695.30
	Accuracy	54.9%	51.5%
	Correct guesses	68.3%	62.1%
Number of estimated parameters		22	10
Number of considered variables		10	12



## Conclusions

# Conclusions



Despite a certain number of restrictions, results show that this topic is worth further investigation



The procedure reaches very good specifications in a matter of minutes



In some cases, the obtained model outperforms the benchmark with less variables under consideration

# Future work



VNS  $\rightarrow$  neighborhood structures + relax assumptions



What is a good utility specification ?

Predictability

Parameters significance

Behavioral interpretation

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Thank you!

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*The fact is I've always loved both bands :-)*