Modelling competition in demand-based optimization models

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Outline

Competition in transportation

- Competition is often present in the form of oligopolies (regulations, limited capacity of the infrastructure, barriers to entry, etc.).
- Deregulation often led to oligopolistic markets.
  - Airlines
  - Railways
  - Buses
  - Multi-modal networks

Trending topic

TRANSPORTS FLIXBUS ET EUROBUS S'ALLIENT POUR DESSERVIR LA SUISSE

Les deux compagnies de bus Flixbus et Eurobus se sont mises d'accord pour démarrer le cabotage en Suisse à partir du 10 juin. C'est une concurrence accrue pour les CFF.

PAR PASCAL SCHMUCK
ZURICH
05.06.2018

9

ARTICLES EN RELATION
- Gros succès de Flixbus en Suisse
- Flixbus épinglé pour cabotage

Flixbus s'implante en Suisse. A partir du 10 juin, la compagnie allemande de bus desservira les trajets St-Gall-Aéroport de Genève, Coire-Sion, Coire-Aéroport de Zurich et Bâle EuroAirport-Lugano. Elle s'associe avec Eurobus, la plus grande entreprise de bus en Suisse, révèle le *Blick*.

Réforme de la **SNCF** : à quoi va ressembler la suite après le vote du ...
LCI - 5 hours ago
Après son vote par l'Assemblée en avril, puis par le Sénat le 5 juin, le projet de loi de **réforme** de la **SNCF** doit faire l'objet d'une commission ...

Réforme de la **SNCF**. Faut-il vous préparer à une poursuite de la ...
Ouest-France - 6 Jun 2018

Contre la **réforme** ferroviaire, les cheminots envahissent le siège de la ...
Le HuffPost - 5 Jun 2018

SNCF : le Sénat a voté le projet de **réforme** ferroviaire
Franceinfo - 6 Jun 2018

Vote au Sénat de la **réforme** de la **SNCF** : la grève n'est pas finie
In-Depth - La Tribune.fr - 6 Jun 2018

Le Sénat vote la **réforme** de la **SNCF**
In-Depth - Le Figaro - 5 Jun 2018

How to study competitive transport markets?

- Modelling demand
- Modelling supply
- Modelling competition

Demand

- Each customer chooses the alternative that maximizes his/her utility.
- Customers have different tastes and socioeconomic characteristics that influence their choice.

Supply

- Operators take decisions that optimize their objective function (e.g. revenue maximization).
- Decisions can be related to pricing, capacity, frequency, availability ...
- Decisions are influenced by:
  - The preferences of the customers
  - The decisions of the competitors

Competition

- We consider non-cooperative games.
- We aim at understanding the Nash equilibrium solutions of such games, i.e. stationary states of the system in which no competitor has an incentive to change its decisions.

Modelling the problem

Starting point:
MILP for the demand-based optimization problem for one operator (Pacheco et al. (2017)).

The goal:
MILP that models the non-cooperative multi-leader-follower game played by operators and customers.

The framework

Three elements to be modelled: customers, operators and market.

1. **Customers**: discrete choice models take into account preference heterogeneity and model individual decisions. These can be integrated in a MILP by relying on simulation to draw from the distribution of the error term of the utility function.

The framework

Three elements to be modelled: customers, operators and market.

1. **Customers**: discrete choice models take into account preference heterogeneity and model individual decisions. These can be integrated in a MILP by relying on simulation to draw from the distribution of the error term of the utility function.

2. **Operators**: a mixed integer linear program can maximize any relevant objective function.

The framework

Three elements to be modelled: customers, operators and market.

1. **Customers**: discrete choice models take into account preference heterogeneity and model individual decisions. These can be integrated in a MILP by relying on simulation to draw from the distribution of the error term of the utility function.
2. **Operators**: a mixed integer linear program can maximize any relevant objective function.
3. **Market**: Nash equilibrium solutions are found by enforcing best response constraints.

The framework: customer level

- For all customers $n \in N$ and all alternatives $i \in I$, R draws are extracted from the error term distribution, each corresponding to a different behavioral scenario. For each $r \in R$ we have:

$$U_{inr} = \beta_{in} p_{in} + q_{in} + \xi_{inr}$$

  where p_{in} is a variable endogenous to the optimization model, β_{in} is the corresponding parameter, q_{in} is the exogenous term and ξ_{inr} is the error term.

- In each scenario, customers choose the alternative with the highest utility:

$$w_{inr} = 1 \text{ if } U_{inr} = \max_{j \in I} U_{jnr}, \text{ and } w_{inr} = 0 \text{ otherwise}$$

- Over multiple scenarios, the probability of $n \in N$ choosing $i \in I$ is given by:

$$P_{in} = \frac{\sum_{r \in R} w_{inr}}{R}$$

The framework: operators level

- We assume that an operator $k \in K$ can decide on price p_{in} and availability y_{in} of each alternative $i \in C_k$ for all customers $n \in N$.
- Stackelberg game: the operator (the leader) knows the best response of the customers ("collective" follower) to all strategies.
- Objective function to be maximized by operator k:

$$V_k = \frac{1}{R} \sum_{i \in C_k} \sum_{n \in N} \sum_{r \in R} p_{in} w_{inr}$$

Optimization model for the single operator

$$\max \quad \frac{1}{R} \sum_{i \in I \setminus \{0\}} \sum_{n \in N} \sum_{r \in R} \alpha_{inr} \tag{1}$$

$$s.t. \quad \sum_{i \in I} w_{inr} = 1 \qquad \forall n \in N, \forall r \in R \tag{2}$$

$$w_{inr} \leq y_{inr} \qquad \forall i \in I, \forall n \in N, \forall r \in R \tag{3}$$

$$y_{inr} \leq y_{in} \qquad \forall i \in I, \forall n \in N, \forall r \in R \tag{4}$$

$$y_{in} = 0 \qquad \forall i \in I, \forall n \in N : i \notin I_n \tag{5}$$

$$\sum_{n \in N} w_{inr} \leq C_i \qquad \forall i \in I \setminus \{0\}, \forall r \in R \tag{6}$$

$$C_i(y_{in} - y_{inr}) \leq \sum_{m \in N : L_{im} < L_{in}} w_{imr} \qquad \forall i \in I \setminus \{0\}, \forall n \in N, \forall r \in R \tag{7}$$

$$\sum_{m \in N : L_{im} < L_{in}} w_{imr} \leq (C_i - 1)y_{inr} + (n-1)(1 - y_{inr}) \qquad \forall i \in I \setminus \{0\}, \forall n \in N, \forall r \in R \tag{8}$$

$$U_{inr} = \beta_{in} p_{in} + q_{in} + \xi_{inr} \qquad \forall i \in I, \forall n \in N, \forall r \in R \tag{9}$$

$$lbU_{nr} \leq z_{inr} \leq lbU_{nr} + M_{U_{nr}} y_{inr} \qquad \forall i \in I, \forall n \in N, \forall r \in R \tag{10}$$

$$U_{inr} - M_{U_{nr}}(1 - y_{inr}) \leq z_{inr} \leq U_{inr} \qquad \forall i \in I, \forall n \in N, \forall r \in R \tag{11}$$

$$z_{inr} \leq U_{nr} \qquad \forall i \in I, \forall n \in N, \forall r \in R \tag{12}$$

$$U_{nr} \leq z_{inr} + M_{U_{nr}}(1 - w_{inr}) \qquad \forall i \in I, \forall n \in N, \forall r \in R \tag{13}$$

$$lbP_{in} \leq p_{in} \leq ubP_{in} \qquad \forall i \in I, \forall n \in N, \forall r \in R \tag{14}$$

$$lbP_{in} w_{inr} \leq \alpha_{inr} \leq ubP_{in} w_{inr} \qquad \forall i \in I, \forall n \in N, \forall r \in R \tag{15}$$

$$p_{in} - (1 - w_{inr}) ubP_{in} \leq \alpha_{inr} \leq p_{in} \qquad \forall i \in I, \forall n \in N, \forall r \in R \tag{16}$$

The framework: market level

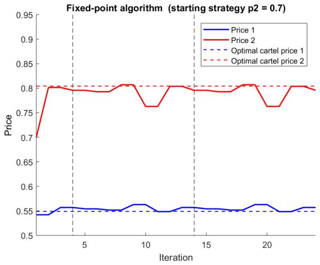
- The payoff of an operator also depends on the strategies of the competitors
- Let's define as X_k the set of strategies that can be played by operator $k \in K$
- Condition for Nash equilibrium (best response constraints):

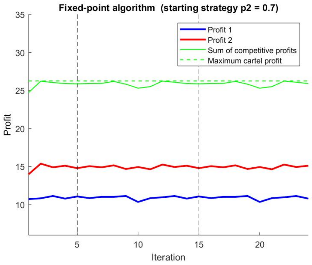
$$V_k = V_k^* = \max_{x_k \in X_k} V_k(x_k, x_{K \setminus \{k\}}) \qquad \forall k \in K$$

- Nash (1951) proves that every finite game has at least one mixed strategy equilibrium solution

A fixed-point iteration method

- Sequential algorithm to find Nash equilibrium solutions of a two-player game:
  - Initialization: one player selects an initial feasible strategy.
  - Iterative phase: operators take turns and each plays its best response pure strategy to the last strategy played by the competitor.
  - Termination criterion: either a Nash equilibrium or a cyclic equilibrium is reached.





MILP formulations

Pure strategies:

- Each operator $k \in K$ chooses a pure strategy from a finite set S_k.
- Number of pure strategy solutions of the game: $|S| = \prod_{k \in K} S_k$.
- For each solution $s \in S$ we can derive a payoff function V_{ks} for each operator $k \in K$.
- If $s \in S$ includes only best response strategies for all operators, then it is a pure strategy Nash equilibrium for the finite game.

Mixed strategies:

- Operator k chooses a mixed strategy from the finite set S_k, i.e. a vector of probabilities p_{s_k} associated to all pure strategies s_k in S_k, such that $\sum_{s_k \in S_k} p_{s_k} = 1$.

Operator and market level (Pure strategies)

Find $s \in S$ such that $e_s = 1$

$s.t.$

Equilibrium constraints:

$$e_s \geq \sum_{k \in K} x_{ks} - (|K| - 1) \qquad \forall s \in S \qquad (17)$$

$$e_s \leq x_{ks} \qquad \forall k \in K, \forall s \in S \qquad (18)$$

Operator constraints:

$$V_{ks} = \frac{1}{R} \sum_{i \in C_k} \sum_{n \in N} \sum_{r \in R} p_{ins} w_{inrs} \qquad \forall k \in K, \forall s \in S \qquad (19)$$

$$V_{ks} \leq V_{kt}^{max} \qquad \forall k \in K, \forall s \in S_k, \forall t \in S_k^C \qquad (20)$$

$$V_{kt}^{max} \leq V_{ks} + M_r(1 - x_{ks}) \qquad \forall k \in K, \forall s \in S_k, \forall t \in S_k^C \qquad (21)$$

$$\sum_{s \in S} x_{ks} = \left| S_k^C \right| \qquad \forall k \in K \qquad (22)$$

Operator and market level (Mixed strategies)

Find $p_{s_k}, b_{s_k}, r_{s_k}, V_{s_k}, V_k$ such that... or $\max \sum_{k \in K} V_k$ or...

$s.t.$

MILP mixed-strategy Nash:

$$\sum_{s_k \in S_k} p_{s_k} = 1 \qquad \forall k \in K \qquad (23)$$

$$V_{s_k} = \sum_{s_k^C \in S_k^C} p_{s_k^C} V_k(s_k, s_k^C) \qquad \forall k \in K, \forall s_k \in S_k \qquad (24)$$

$$V_k \geq V_{s_k} \qquad \forall k \in K, \forall s_k \in S_k \qquad (25)$$

$$r_{s_k} = V_k - V_{s_k} \qquad \forall k \in K, \forall s_k \in S_k \qquad (26)$$

$$p_{s_k} \leq 1 - b_{s_k} \qquad \forall k \in K, \forall s_k \in S_k \qquad (27)$$

$$r_{s_k} \leq M b_{s_k} \qquad \forall k \in K, \forall s_k \in S_k \qquad (28)$$

Pure strategy payoffs:

$$V_k(s_k, s_k^C) = \frac{1}{R} \sum_{i \in C_k} \sum_{n \in N} \sum_{r \in R} p_{ins} w_{inrs} \qquad \forall k \in K, \forall (s_k, s_k^C) \in S \qquad (29)$$

Numerical example: pure strategy equilibria

Payoff matrix of player 1

S1 \ S2	*0,70*	*0,73*	*0,76*	*0,79*	*0,82*	*0,85*
0,50	10,00	10,00	10,00	10,00	10,00	10,00
0,53	10,49	**10,60**	**10,60**	10,60	10,60	10,60
0,56	**10,53**	10,42	10,53	**10,86**	**11,20**	11,20
0,59	10,27	10,03	9,80	9,91	10,62	**11,45**
0,62	10,04	9,80	9,42	9,42	9,42	9,92
0,65	9,62	9,36	8,84	8,45	8,71	8,58

Payoff matrix of player 2

S1 \ S2	*0,70*	*0,73*	*0,76*	*0,79*	*0,82*	*0,85*
0,50	14,00	14,45	14,74	14,69	**14,76**	14,62
0,53	14,00	14,45	14,74	**15,01**	14,60	14,45
0,56	14,00	14,60	14,74	**14,85**	14,76	14,28
0,59	14,00	14,60	15,05	**15,48**	15,09	14,45
0,62	14,00	14,60	15,20	15,48	**15,91**	15,81
0,65	14,00	14,60	15,20	15,80	15,91	**16,32**

(a) Game with 1 pure strategy Nash equilibrium

Payoff matrix of player 1

S1 \ S2	*0,75*	*0,77*	*0,79*	*0,81*	*0,83*	*0,85*
0,50	10,00	10,00	10,00	10,00	10,00	10,00
0,52	10,40	10,40	10,40	10,40	10,40	10,40
0,54	**10,80**	**10,80**	10,80	10,80	10,80	10,80
0,56	10,42	10,53	**10,86**	**11,09**	**11,20**	11,20
0,58	9,74	9,86	10,09	10,44	10,67	**11,37**
0,60	9,60	9,60	9,72	10,08	10,44	10,68

Payoff matrix of player 2

S1 \ S2	*0,75*	*0,77*	*0,79*	*0,81*	*0,83*	*0,85*
0,50	14,70	**14,78**	14,69	14,74	14,28	14,62
0,52	14,70	**15,09**	14,85	14,58	14,61	14,45
0,54	14,85	14,94	**15,17**	14,74	14,44	14,45
0,56	14,85	**14,94**	14,85	14,90	14,61	14,28
0,58	15,00	15,09	**15,17**	15,07	15,11	14,45
0,60	15,00	15,25	**15,48**	15,39	15,27	14,30

(b) Game with no pure strategy Nash equilibrium

Payoff matrices for two games with different support strategies. Best response payoffs are in bold. Equilibrium payoffs are in blue.

Numerical example: mixed strategy equilibria

Payoff matrices for player 1 and player 2

S1 \ S2	*0,75*	*0,77*	*0,79*	*0,81*	*0,83*	*0,85*	p_1	V_1
0,50	10,00	10,00	10,00	10,00	10,00	10,00	*0*	*10,00*
0,52	10,40	10,40	10,40	10,40	10,40	10,40	*0*	*10,40*
0,54	10,80	10,80	10,80	10,80	10,80	10,80	*0.27*	*10,80*
0,56	10,42	10,53	10,86	11,09	11,20	11,20	*0.73*	*10,80*
0,58	9,74	9,86	10,09	10,44	10,67	11,37	*0*	*10,05*
0,60	9,60	9,60	9,72	10,08	10,44	10,68	*0*	*9,70*

S1 \ S2	*0,75*	*0,77*	*0,79*	*0,81*	*0,83*	*0,85*
0,50	14,70	14,78	14,69	14,74	14,28	14,62
0,52	14,70	15,09	14,85	14,58	14,61	14,45
0,54	14,85	14,94	15,17	14,74	14,44	14,45
0,56	14,85	14,94	14,85	14,90	14,61	14,28
0,58	15,00	15,09	15,17	15,07	15,11	14,45
0,60	15,00	15,25	15,48	15,39	15,27	14,30
p_2	*0*	*0.19*	*0.81*	*0*	*0*	*0*
V_2	*14.85*	*14.94*	*14.94*	*14.86*	*14.56*	*14.33*

Figure: Game with mixed strategy Nash equilibrium

Discussion

- The model requires finite strategy sets (enumeration), therefore the problem is solvable with small solution spaces only.
- The assumption of a finite game requires price discretization.
- Formulation 1: all pure strategy Nash equilibria of the game can be found, if they exist.
- Formulation 2: among the mixed strategy Nash equilibria, it is possible to select one by choosing a relevant objective function, e.g. total welfare maximization.

A MILP model for the fixed-point problem

- What if we can write a MILP model to minimize the distance between two consecutive fixed-point iterations?

- A solution for a two-operator problem: (x_1^b, x_2^b)

- Optimization problems for the operators:

$$x_1^* = \arg\max_{x_1 \in X_1} V_1(x_1, x_2^b)$$
$$x_2^* = \arg\max_{x_2 \in X_2} V_2(x_1^b, x_2)$$

- Fixed-point problem:

$$\min_{x_1, x_2, x_1^*, x_2^*} \|x_1^* - x_1^b\| + \|x_2^* - x_2^b\|$$

Initial configuration

- No optimization at operator level: any feasible strategy could be selected.
- Constraints:
  - Customer choice
  - Customer utility maximization
  - Capacity

Initial configuration

$$\sum_{i \in I} w^b_{inr} = 1 \qquad \forall n \in N, \forall r \in R \qquad (30)$$

$$w^b_{inr} \leq y^b_{inr} \qquad \forall i \in I, \forall n \in N, \forall r \in R \qquad (31)$$

$$y^b_{inr} \leq y^b_{in} \qquad \forall i \in I, \forall n \in N, \forall r \in R \qquad (32)$$

$$y^b_{in} = 0 \qquad \forall i \in I, \forall n \in N : i \notin I_n \qquad (33)$$

$$\sum_{n \in N} w^b_{inr} \leq C_i \qquad \forall i \in I \setminus \{0\}, \forall r \in R \qquad (34)$$

$$C_i(y^b_{in} - y^b_{inr}) \leq \sum_{m \in N : L_{im} < L_{in}} w^b_{imr} \qquad \forall i \in I \setminus \{0\}, \forall n \in N, \forall r \in R \qquad (35)$$

$$\sum_{m \in N : L_{im} < L_{in}} w^b_{imr} \leq (C_i - 1) y^b_{inr} + (n - 1)(1 - y^b_{inr}) \qquad \forall i \in I \setminus \{0\}, \forall n \in N, \forall r \in R \qquad (36)$$

$$U^b_{inr} = \beta_{in} p^b_{in} + q_{in} + \xi_{inr} \qquad \forall i \in I, \forall n \in N, \forall r \in R \qquad (37)$$

$$lbU_{nr} \leq z^b_{inr} \leq lbU_{nr} + M y^b_{inr} \qquad \forall i \in I, \forall n \in N, \forall r \in R \qquad (38)$$

$$U^b_{inr} - M(1 - y^b_{inr}) \leq z^b_{inr} \leq U^b_{inr} \qquad \forall i \in I, \forall n \in N, \forall r \in R \qquad (39)$$

$$z_{inr} \leq U_{nr} \qquad \forall i \in I, \forall n \in N, \forall r \in R \qquad (40)$$

$$U^b_{nr} \leq z^b_{inr} + M(1 - w^b_{inr}) \qquad \forall i \in I, \forall n \in N, \forall r \in R \qquad (41)$$

$$lbP_{in} \leq p^b_{in} \leq ubP_{in} \qquad \forall i \in I, \forall n \in N, \forall r \in R \qquad (42)$$

$$lbP_{in} w^b_{inr} \leq \alpha^b_{inr} \leq ubP_{in} w^b_{inr} \qquad \forall i \in I, \forall n \in N, \forall r \in R \qquad (43)$$

$$p_{in} - (1 - w^b_{inr}) ubP_{in} \leq \alpha^b_{inr} \leq p^b_{in} \qquad \forall i \in I, \forall n \in N, \forall r \in R \qquad (44)$$

"Best response" configurations

For each operator, solve an optimization problem having:

- Customer choice constraints
- Customer utility maximization constraints
- Capacity constraints

- Strategies of the other operator(s) equal to those of the initial solution
- Best response strategy to the initial solution for the optimizing operator

"Best response" configurations

Best response constraints:

$$V_{ks} = \frac{1}{R} \sum_{i \in I_k} \sum_{n \in N} \sum_{r \in R} p_{ins} w_{inrs}^{aft} \qquad \forall k \in K, \forall s \in S_k \tag{45}$$

$$V_{ks} \leq V_{ks}^{max} \qquad \forall s \in S_k \tag{46}$$

$$V_{ks}^{max} \leq V_{ks} + M(1 - x_{ks}) \qquad \forall s \in S_k \tag{47}$$

$$\sum_{s \in S} x_{ks} = 1 \tag{48}$$

Capacity constraints

...

Customer choice constraints

...

Customer utility maximization constraints

...

Customer utilities:

$$U_{inrs}^{aft} = \beta_{in} p_{ins} + q_{in} + \xi_{inr} \qquad \forall i \in I_k, \forall n \in N, \forall r \in R, \forall s \in S_k \tag{49}$$

$$U_{inrs}^{aft} = U_{inr}^{b} \qquad \forall i \in I \setminus I_k, \forall n \in N, \forall r \in R, \forall s \in S_k \tag{50}$$

Objective function

- Minimization problem:

$$z^* = \min_{x_1, x_2, x_1^*, x_2^*} \|x_1^* - x_1^b\| + \|x_2^* - x_2^b\|$$

- If $z^* = 0$, we have an equilibrium. What can we say about this equilibrium?
  If $z^* > 0$, can we conclude something? Are we in an equilibrium region?

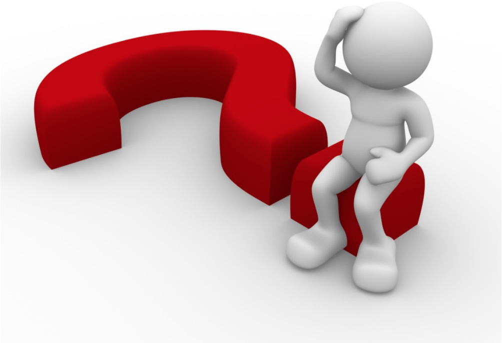
```
// solution (optimal) with objective 0.00094676129297977

BEFORE
Alternative 1 - Price = 0.56
Alternative 2 - Price = 0.769053239
AFTER
Alternative 1 - Price = 0.56
Alternative 2 - Price = 0.77

Revenue[1] = 11.2
Revenue[2] = 14.150579592
```

```
// solution (optimal) with objective 0.00743590825444868

BEFORE
Alternative 1 - Price = 0.172591923
Alternative 2 - Price = 0.279972169
AFTER
Alternative 1 - Price = 0.18
Alternative 2 - Price = 0.28

Revenue[1] = 3.658948769
Revenue[2] = 8.063198458
```

Upcoming work and ideas

- Further test the MILP model for the fixed-point problem.
- Increase the numbers of draws to better understand marginal changes in the objective function.
- Efficient search for equilibria in the solution space
- Insertion of the assortment problem in the model.
- Investigation of the concept of Nash equilibrium region for real-life applications.

Questions?



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