Choice-based routing problem in the context of flexible mobility on demand

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Motivation
Motivation

Silverstein, 2014, calculates the fares for a sample trip of 5 miles in 10 minutes under car speed of 30MPH with no waiting time.

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<thead>
<tr>
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Motivation

Source: http://bruegel.org/2014/09/the-economics-of-uber/
Motivation

Conventional public transportation services are not personalized. Fixed route, Fixed schedule, Low frequency etc.

Most people cannot afford using taxi service on a daily basis.

Personalized transportation services using mobile apps are emerging Uber, Lyft, GrabTaxi.
Problem definition

How to increase operator profit and user satisfaction?

Flexibility to demand fluctuations is necessary. Currently, due to lack of the flexibility:

Off-peak:
  => Drivers cannot find passengers

On-peak:
  => Passengers cannot find drivers.
  Some passengers may give up taking public transportation.
  => Operator lose revenue opportunity.
What is FMOD?

Flexible Mobility on Demand
Real-time system
Flexibility to demand fluctuations

Concepts
Dynamic allocation of vehicle to service modes
Optimized travel menus are offered to the customer
Dynamic allocation of vehicle to service modes
Same vehicle is dynamically reassigned to different service modes according to the evolving demand.
FMOD
FMOD

Supply Demand

FMOD Server

optimization

maximizing profit/welfare

request

offer

Customer

Request:
Origin: A, Destination: B
Preferred Departure Time: 8:00 – 8:30
Preferred Arrival Time: 8:45 – 9:00

Assortment:
taxi: DT: 8:25/AT: 8:45, $20
shared-taxi: DT: 8:27/AT: 8:57, $10
mini-bus: DT: 8:14/AT: 8:59, $5

Acceptance (or rejection):
service: shared-taxi
DT: 8:27/AT: 8:57, $10
Modes of transport

- Serves a single passenger at a time
- Provides door-to-door service
- No fix location for pickup and delivery
- Fastest alternative
- Highest fare.

Taxi
Modes of transport

- Multiple passengers in the same vehicle
- Provides door-to-door service
- Arbitrary locations for pickup and delivery
- Travel time may increase due to the pick-up and drop-off of other passengers.
Modes of transport

- Fixed routes
- Pick-up / drop-off locations are predetermined
- Adapted schedule for passengers similar to the shared-taxi
FMOD app
Integrated choice-based optimization framework

- Choice Modeling
- Vehicle Routing
- Assortment Optimization
- Dynamic Pricing
Simulation (Sequential framework)

**Product** $p_{n,m,l}$
A service on a vehicle departing at a certain time period

**Feasible product** $p_{n,m,l} \in F$
A product that satisfies the capacity and scheduling constraints
- Vehicle capacity
- No conflict with existing schedules
- Deviation from preferred time window

**Assortment**
A list of feasible products on the travel menu

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**Phase 1. Feasible product set generation**
Feasible products set are generated taking into account:
- Capacity constraints
- Scheduling constraints

**Phase 2. Assortment optimization**
Assortment to be offered to the customer is optimized
- Maximize operator’s profit and/or consumer surplus based on a choice model

Simulation (model)

\( N \): set of vehicles,
\( M \): set of service modes
\( L \): set of time periods

\[ X = \{ x_{n,m,l} \mid x_{n,m,l} \in \{0,1\} \} \]

Decide which feasible products are included in the assortment

\[ x_{n,m,l} = 0 \quad \forall p_{n,m,l} \notin F \]

Only feasible products are included

\[ \max R_{current}(X) = \sum_{n \in N} \sum_{m \in M} \sum_{l \in L} r_{n,m,l} \cdot \text{Prob}_{n,m,l}(X) \]

Expected profit from each product

\[ \text{s.t. } \sum_{n \in N} \sum_{l \in L} x_{n,m,l} = 1 \quad \forall m \in M \]

One product is offered for each service

\[ r_{n,m,l} \quad \text{Profit associated with } p_{n,m,l} \]

\[ \text{Prob}_{n,m,l} \quad \text{Choice probability } p_{n,m,l} \]

Integrated choice-based optimization framework

- Assortment Optimization
- Choice Modeling
- Real time
- Vehicle Routing
- Dynamic Pricing
Assumptions

• Integrating vehicle routing, assortment optimization and dynamic pricing.
• Flexible service providing
• Homogeneous fleet with the same capacity of 8 persons.
• Dynamic change of role among taxi and shared-taxi
• Information of the ride request:
  - Origin and destination of the requested trip
  - Preferred departure time interval or preferred arrival time
  - Number of passengers

The passenger could accept or reject the proposed option.
The server may reject the request:
  (1) there is no vehicle available to serve the customer
  (2) the associated profit to the offered choices is negative
Assumptions

**Arrival / departure time**
A time window (in minutes) is received $\delta^P(+/–)15$

**Fare**
Base fare charged once
Price per kilometer (shortest path between O-D)
Three levels of price

**Utility of taxi and shared taxis**

$u_{\text{taxi}} = \beta_0 P_{\text{Base}} + \beta_1 P_{\text{km}} D + \beta_2 (TTime) + \beta_3 (SD) + \epsilon$

$u_{\text{shared taxi}} = \beta_0 P_{\text{Base}} + \beta_1 P_{\text{km}} D + \beta_2 (MaxRideTime) + \beta_3 (SD) + \epsilon$

**Sets**

- $a \in A$: Set of nodes generated for a new request (Each node represents a product)
- $s \in S$: Set of services \{Taxi, Shared Taxi\}
- $P^S$: Set existing pickup nodes for service S
- $D^S$: Set of existing delivery nodes for service S
- $k \in K$: set of vehicles which can be used either as a taxi or as a shared-taxi
- $V$: Set of nodes in the graph $P^S \cup D^S \cup A \cup Depot$
- $V_i^+$: Set of exiting arcs from the node $i$
- $V_i^-$: Set of entering arc to the node $i$
Notations

Parameters

- $C_n$: Total routing cost (excluding cost associated with products) for request $n$
- $c_{ij}$: Cost of traveling from node $i$ to node $j$
- $t_{ij}$: Travel time between node $i$ and $j$
- $[e_i, l_i]$: Service time window at node $i$
- $Q$: Vehicle capacity
- $q_i$: Load at node $i$, positive value for pickup and negative value for delivery
  it is of quantity 1 for shared-taxi, and $Q$ for taxi
- $T^{\text{max}}$: Maximum ride time for shared taxi
- $M$: Large constants
- $u_0$: Utility of no-purchase (reject) option
- $f_a$: Charging fare associated with alternative $a$
- $v_a$: Utility weight of alternative $a$

Variables

- $p_a$: The probability of selecting alternative $a$ by customer
- $x^k_{ij}$: Binary variable, 1 if vehicle $k$ travels from node $i$ to node $j$
- $w_i$: Arrival time of vehicle at node $i$
- $l_i^k$: Load of vehicle $k$ at node $i$

Max $\sum_{a \in A} f_a p_a - (\sum_{i \in V} \sum_{j \in V} \sum_{k \in K} c_{ij} x^k_{ij})$
\[ \sum_{j \in V'} \sum_{k \in K} x_{ij}^k = 1 \quad (\forall i \in P \setminus \{p', p''\}) \]

\[ \sum_{j \in V} x_{ij}^k - \sum_{j \in V} x_{n+i,j}^k = 0 \quad (\forall i \in P, k \in K) \]

\[ \sum_{i \in V} x_{ij}^k = \sum_{i \in V} x_{ji}^k \quad (\forall j \in P \cup D, k \in K) \]

\[ w_j^k \geq w_i^k + t_{ij} - M \mathbf{1}_{i,j} \cdot (1 - x_{ij}^k) \quad (\forall i, j \in V, k \in K) \]

\[ e_i \leq w_i^k \leq l_i \quad (\forall i \in V) \]

\[ t_{i,n+i} \leq w_{n+i}^k - w_i^k \leq T_{\text{max}} \quad (\forall i \in P, k \in K) \]

\[ w_{n+i}^k \geq w_i^k \quad (\forall i \in P, k \in K) \]

\[ v_j^k \geq v_i^k + q_i - M \mathbf{1}_{i,j} \cdot (1 - x_{ij}^k) \quad (\forall i, j \in V, k \in K) \]

\[ \max\{0, q_i\} \leq v_i^k \leq \min\{Q, Q + q_i\} \quad (\forall i \in V, k \in K) \]

\[ x_{ij}^k \in \{0, 1\} \quad (\forall i, j \in V, \forall k \in K) \]

\[ \sum_{l \in L} \sum_{b \in B} \sum_{s \in S} \pi_{a,l,b,s} \leq \sum_{j \in V} x_{ij}^k \quad (\forall a \in A, k \in K | a = k, i \in \{p', p''\}) \]

\[ u_{a,l,b,s} = \beta_{P,a,l}^{p} + \beta_{P,a,b}^{d} + \beta_{s}^{t}(tt_{i,s}^k) \quad (\forall a \in A, l \in L, b \in B, s \in S, k \in K | a = k, i \in \{p', p''\}) \]

\[ \sum_{a \in A} \sum_{l \in L} \sum_{b \in B} \sum_{s \in S} \pi_{a,l,b,s} = 1 - \pi_0 \]

\[ \sum_{a \in A} \sum_{l \in L} \sum_{b \in B} \sum_{s \in S} \frac{\pi_{a,l,b,s}}{\exp(u_{a,l,b,s})} \leq \frac{\pi_0}{\exp(u_0)} \quad \forall a \in A \]

\[ 0 \leq \frac{\pi_{a,l,b,s}}{\exp(u_{a,l,b,s})} \leq \frac{\pi_0}{\exp(u_0)} \quad (\forall a \in A, l \in L, b \in B, s \in S) \]
Computational results

Alternatives:

- Two type of scheduled delay
- Two price levels
- Two types of vehicles

<table>
<thead>
<tr>
<th>Customer</th>
<th>Offer</th>
<th>Profit</th>
<th>Time</th>
<th>Selection</th>
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Individual based opt out included

Initial cost and per kilometer cost is different from taxi to shared taxi.

Profit is defined for the assortment.
Computational results
Conclusion

- Integrated framework for FMOD
- Adding minibus in the system
- More sophisticated pricing planning
- Intelligent heuristic for large size network
Thank You
Conclusion

\[
\text{Max} \sum_{a \in A} \sum_{l \in L} \sum_{b \in B} \sum_{s \in S} \pi_{a,l,b,s}(p_{a,l}^0 + p_{a}d_{a,b}) - \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} c_{ij} x_{ij}^k
\]