Conceptual model of the shippers' choice between sea, rail and road transport

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Background: Transfer of goods from road to sea or rail

- According to the National Transport Plan of Norway, growth in long distance freight transport is to be taken by **rail** or **sea** to the largest possible extent.
  - *Several attempts have been made to achieve this transition, but the **mechanisms** behind each individual shipper's choice between road, sea and rail **are not well understood***

- Two elements considered to be of importance:
  - *Uncertainty*
  - *Economies of scale*

Background: The Norwegian commodity flow survey

- Collected by Statistics Norway. Meant to give a complete picture of all freight flows in Norway during one year (2014).
- All shipments from the **8 largest transport operators** in Norway registered
- As well as a survey among **4224 firms**
    - *Varying sizes and industries*
    - *All shipments in 2014 were registered …*
    - ***… Including mode of transport***
    - *91 % of the firms had answered by 1/12-2015*
- By now, data regarding 70,000,000 shipments is collected
- Data will (should) be ready some time during May

➢ We have very good data on freight flows and chosen transport modes – but **not on actual transport costs**

  ➢ *How to utilize this data in the best possible way?*

Project idea

- Focus on interesting commodity groups and origin-destination pairs; choose some **case studies**
- For each shipper, **calculate logistics costs** associated with each (potential) mode based on generic cost parameters
- Use these calculations **as input in a discrete mode choice model**
- Aggregating over shippers to **predict the size of each mode specific commodity flow**
- This **affects the freight rate** for each shipper due to economies of scale (think large container ship)
- To assess the potential for transferring goods from road to sea or rail: **Simulate a policy change and iterate until convergence**

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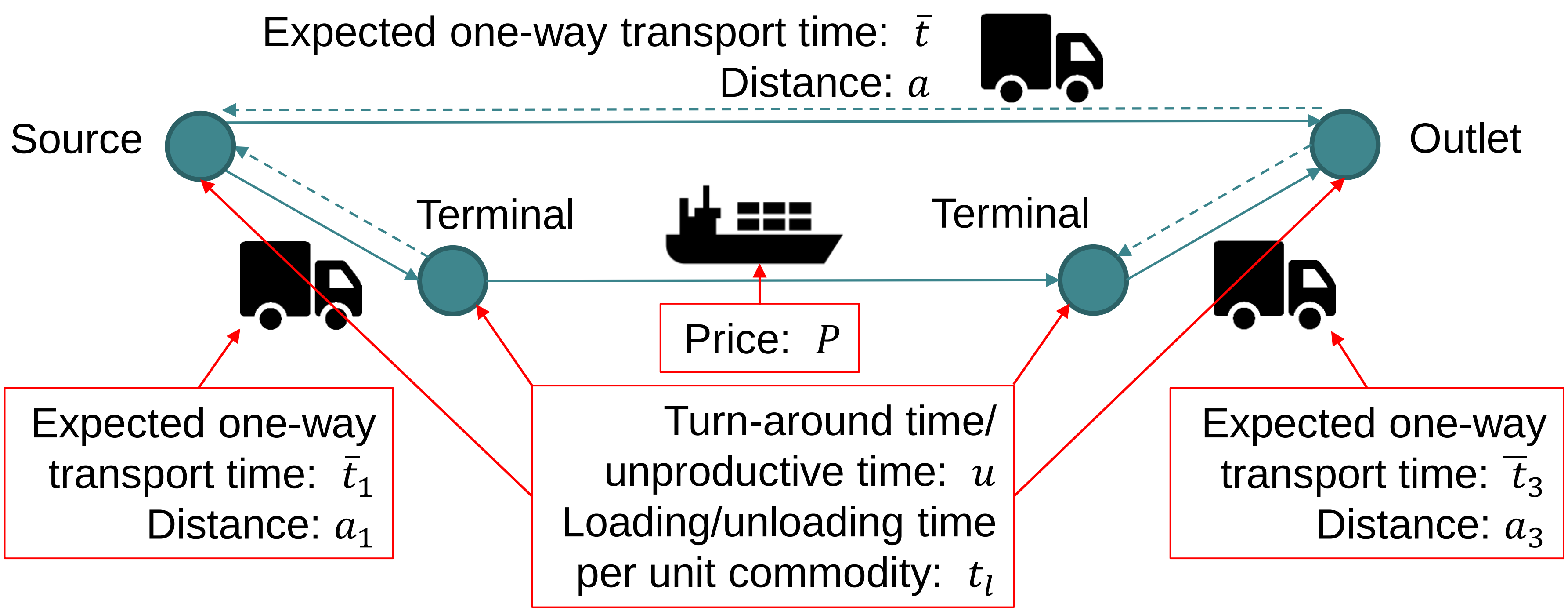
The supply chain

- We are considering supply chains where goods are regularly **supplied from a single source** and **sold at a single outlet**. (At least) two possible transport chains:



Transport costs

- The shipper can always choose the most appropriate truck size C from a continuous interval $[C_{min}, C_{max}]$
- We assume economies of scale in vehicle size:
  - *A linear relationship between vehicle size and kilometer cost:* $k = k_0 + k_1 C$
  - *And between vehicle size and hourly vehicle capital cost:* $i = i_0 + i_1 C$
- Shipment size Q can never be larger than C (but smaller)
- Let x be annual demand, Y be annual transport capacity and assume no unnecessary trips are made:
  - *If* $Q \in [C_{min}, C_{max}]$*, then* $Y = x$ *(and consequently* $C = Q$*)*
  - *If* $Q \leq C_{min}$ *then* $Y = x * \frac{C_{min}}{Q}$ $(> x)$

Transport costs

- This allows us to write the annual transport costs of the shipper (K_T) as a function of two choice variables: shipment size (Q) and annual transport capacity (Y)

$$
\begin{aligned}
&K_T \\
&= \{2k_0(a_1 + a_3) + (w + i_0)[2(\bar{t}_1 + \bar{t}_3) + (u_1 + u_3)]\}\frac{x}{Q} \\
&+ \{i_l t_l\}QY + \{(w + i_0 + w_l)t_l + P\}x \\
&+ \{2k_1(a_1 + a_3) + i_1[2(\bar{t}_1 + \bar{t}_3) + (u_1 + u_3)]\}Y
\end{aligned}
$$

Ordering costs and inventory holding costs, no uncertainty

- The ordering cost: b per shipment
- The expected annual cost of the stationary inventory:

$$\frac{1}{2}H(1+\varepsilon)Q$$

Inventory holding cost per year and unit of stationary inventory

- The inventory holding cost for units tied up in transport:

$$\frac{J}{\eta}(\bar{t}+t_l Q)x$$

Inventory holding cost per year and unit of mobile inventory

Uncertainty – demand during lead time

- Assume commodities are demanded one at the time; demand is generated by a stationary stochastic process; and demand per business hour is $\sim N(\mu_D, \sigma_D^2)$, where $\mu_D = x/\eta$
- Lead time: from a shipment is ordered until the commodity is on the shelf (including transport time). Assume lead time $\sim N(\mu_T, \sigma_T^2)$
- Then, demand during lead time is $\sim N(\mu_L, \sigma_L^2)$, where (Hadley and Whitin, 1963):

$$\mu_L = \mu_D \mu_T$$
$$\sigma_L^2 = \mu_T \sigma_D^2 + \mu_D^2 \sigma_T^2$$

Uncertainty – demand during lead time

- Stock-outs are allowed to occur, but at a cost.
  - *Assume **stock-outs are backordered** with a cost per instance plus a cost depending on the time until delivery takes place*
- The firms trade off **stock-out costs** and **inventory holding costs** by **fixing a reorder point $\boldsymbol{R}$**
  - *Whenever the inventory position reaches R, a new shipment is ordered*
- Adding R as an instrument, we say that **the firm follows a $(\boldsymbol{Q}, \boldsymbol{Y}, \boldsymbol{R})$ policy**
  - Q *and* Y *must be strictly positive, but* R *might be negative*

Safety stock and stock-out costs

- Let
  - *E: The average number of backorders per year*
  - *B: The average number of backorders at any point in time*
- Hadley and Whitin (1963) show that:

$$E = E(Q,R) = \frac{x}{Q}\alpha(R)$$

$$B = B(Q,R) = \frac{1}{Q}\beta(R)$$

- Where:

$$\alpha(R) = \sigma_L\left[\left(1+\left(\frac{R-\mu_L}{\sigma_L}\right)^2\right)\left(1-F\left(\frac{R-\mu_L}{\sigma_L}\right)\right)-\frac{R-\mu_L}{\sigma_L}f\left(\frac{R-\mu_L}{\sigma_L}\right)\right]$$

$$\beta(R) = \frac{1}{2}\sigma_L^2\left[\left(1+\left(\frac{R-\mu_L}{\sigma_L}\right)^2\right)\left(1-F\left(\frac{R-\mu_L}{\sigma_L}\right)\right)-\frac{R-\mu_L}{\sigma_L}f\left(\frac{R-\mu_L}{\sigma_L}\right)\right]$$

Safety stock and stock-out costs

- Then, the stock-out costs will be:

$$\pi E(Q,R) + \hat{\pi} B(Q,R)$$

Fixed unit cost per back-order

Cost per year of a back-order

- The average excess inventory compared to the deterministic case will be:

$$R - \mu_L + B(Q,R)$$

The logistics cost function

Adding all the cost elements gives:

$$K = [\gamma_1 + \psi(R)]\frac{x}{Q} + \gamma_2 Qx + \gamma_3 QY + \gamma_4 Q + \gamma_5 x + \gamma_6 Y + pH(R - \mu_L)$$

Where:

$\gamma_1 = 2k_0(a_1 + a_3) + (w + i_0)[2(\bar{t}_1 + \bar{t}_3) + (u_1 + u_3)] + b$

$\gamma_2 = 2J\eta^{-1}t_l$

$\gamma_3 = i_1 t_l$

$\gamma_4 = {}^1\!/_2\, H(1 + \varepsilon)$

$\gamma_5 = (w + i_0 + w_l)t_l + P + J\eta^{-1}(\bar{t}_1 + \bar{t}_2 + \bar{t}_3)$

$\gamma_6 = 2k_1(a_1 + a_3) + i_1[2(\bar{t}_1 + \bar{t}_3) + (u_1 + u_3)]$

$\psi(R) = \pi\alpha(R) + (H + \hat{\pi})x^{-1}\beta(R)$

The logistics cost function

- The decision maker's problem is to minimize logistics costs subject to $C_{min} \leq C \leq C_{max}$ and $Y \geq x$. This can be written as:

$$\begin{array}{l} \mathrm{Max}_{Q,Y,R} \; -K \\ s.t. \\ \quad -QY \leq -C_{min}x \quad (\lambda_1) \\ \quad QY \leq C_{max}x \quad (\lambda_2) \\ \quad -Y \leq -x \quad (\lambda_3) \end{array}$$

Given the Lagrangian:

$$L = -K - \lambda_1(C_{min}x - QY) - \lambda_2(QY - C_{max}x) - \lambda_3(x - Y)$$

The Kuhn-Tucker conditions for an optimum are:

$$\begin{array}{l} \frac{\partial L}{\partial Q} = \frac{\partial L}{\partial Y} = \frac{\partial L}{\partial R} = 0 \\ \lambda_1 \geq 0 \;\; (= 0 \; if \; QY > C_{min}x) \\ \lambda_2 \geq 0 \;\; (= 0 \; if \; QY < C_{max}x) \\ \lambda_3 \geq 0 \;\; (= 0 \; if \;\; Y > x \;) \end{array}$$

8 potential cases

$$L = -K - \lambda_1(C_{min}x - QY) - \lambda_2(QY - C_{max}x) - \lambda_3(x - Y)$$

Cases	1st constraint $C \geq C_{min}$	2nd constraint $C \leq C_{max}$	3rd constraint $Y \geq x$
1	$C = C_{min}$	$C = C_{max}$	$\mathrm{Y} = \mathrm{x}$
2	$C = C_{min}$	$C = C_{max}$	$Y > \mathrm{x}$
3	$C = C_{min}$	$C < C_{max}$	$\mathrm{Y} = \mathrm{x}$
4	$C = C_{min}$	$C < C_{max}$	$Y > \mathrm{x}$
5	$C > C_{min}$	$C = C_{max}$	$\mathrm{Y} = \mathrm{x}$
6	$C > C_{min}$	$C = C_{max}$	$Y > \mathrm{x}$
7	$C > C_{min}$	$C < C_{max}$	$\mathrm{Y} = \mathrm{x}$
8	$C > C_{min}$	$C < C_{max}$	$Y > \mathrm{x}$

4 possible cases

$$L = -K - \lambda_1(C_{min}x - QY) - \lambda_2(QY - C_{max}x) - \lambda_3(x - Y)$$

Cases	1st constraint $C \geq C_{min}$	2nd constraint $C \leq C_{max}$	3rd constraint $Y \geq x$	
1	$C = C_{min}$	$C = C_{max}$	$Y = x$	Contradiction
2	$C = C_{min}$	$C = C_{max}$	$Y > x$	Contradiction
3	$C = C_{min}$	$C < C_{max}$	$Y = x$	Possible case
4	$C = C_{min}$	$C < C_{max}$	$Y > x$	Possible case
5	$C > C_{min}$	$C = C_{max}$	$Y = x$	Possible case
6	$C > C_{min}$	$C = C_{max}$	$Y > x$	Contradiction
7	$C > C_{min}$	$C < C_{max}$	$Y = x$	Possible case
8	$C > C_{min}$	$C < C_{max}$	$Y > x$	Contradiction

4 possible cases - solutions

	Case 1	Case 2	Case 3	Case 4
$Q^*=$	$\sqrt{\frac{x(\gamma_1+\psi(R)+\gamma_6 C_{min})}{\gamma_2 x+\gamma_4}} < C_{min}$	C_{min}	$\sqrt{\frac{\gamma_1+\psi(R))x}{(\gamma_2+\gamma_3)x+\gamma_4}} \in [C_{min}, C_{max}]$	C_{max}
$Y^*=$	$C_{min} x \sqrt{\frac{\gamma_2 x+\gamma_4}{x(\gamma_1+\psi(R)+\gamma_6 C_{min}}} > x$	x	x	x
$R^*=$	from $\frac{\partial L}{\partial R}=0$	from $\frac{\partial L}{\partial R}=0$	from $\frac{\partial L}{\partial R}=0$	from $\frac{\partial L}{\partial R}=0$

- As the cost per shipment $\gamma_1+\psi(R^*)$ increase, the optimal solution moves from the first of the cases, to the second, third and fourth.

- Cases 2 and 4 are explicit solutions (solved analytically). Cases 1 and 3 are implicit solutions (they are simple to solve numerically).

Summary

- We have developed a conceptual framework for logistics cost minimization, where:
- Decisions are made on an annual basis
    - *Shipment size and number of trips are endogenous*
- Not all transport costs are proportional to number of tonnes
    - *Unlike conventional inventory theory models, transport costs will affect the choice variables*
- Uncertain demand and uncertain lead time
    - *Firms choose a reorder point depending on demand during lead time*
- Solution depends on a set of generic cost parameters
    - *Most of these parameters are already available for Norwegian conditions*
- Framework can be used to assess "value of reliability"
    - *Which can be problematic in a SP setting*
- Cost minimization can (almost) be solved analytically
    - *Short computation time*

Summary

- According to our judgement, the best way to utilize the commodity flow survey data for discrete choice model predictions
- Next steps:
  - *Collect some missing data regarding parameter values*
  - *Calculate minimized logistics costs for a set of firms chosen from the commodity flow survey*
  - *Estimate a discrete choice model for modal choice*
  - *Embed it in an equilibrium model with network externalities*
  - *Conduct policy simulations*

Thank you for your attention!

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