DYNAMIC PROGRAMMING APPROACHES FOR ESTIMATING AND APPLYING LARGE-SCALE DISCRETE CHOICE MODELS

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» Recursive models (correlated random terms)
» Results and comparisons
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INTRODUCTION

- This presentation gives an overview of Tien Mai’s dissertation (seven papers)
- Many transport related choice problems (e.g. location, activity, route, mode, departure time) share some characteristics
  - Network-based
  - Large number of alternatives
  - Dynamic
  - Alternatives are similar (can, in a RUM framework, be translated to correlated random terms)
- In this presentation we focus on route choice modelling
INTRODUCTION – ROUTE CHOICE

- Given an origin and destination in a transport network, which route does a traveller choose?
- Shortest path and/or recommended route
- Analyst has imperfect knowledge of travellers’ generalized cost and perception of network
- Discrete choice models estimated based on RP data are used to define choice probability distributions over alternatives
INTRODUCTION – ROUTE CHOICE

- Objectives
  - Models that can be consistently estimated using maximum likelihood
  - Models that produce accurate predictions in short computational time

- Main challenges
  - Definition of choice sets
  - Modelling correlation
INTRODUCTION

Choice sets

- Consideration choice sets
- Universal choice sets

- Choice sets are generated and treated as actual ones
- Alternative paths are not observed
- Observed paths may not be generated
- Parameter estimates differ depending on choice sets

- Sampling of alternatives
- Recursive logit (RL)

- Consistent estimation
- Straightforward for prediction
- No choice sets needed
- Prediction
RECURSIVE LOGIT

- Proposed by Fosgerau, Frejinger and Karlstrom (2013)
- Shortest path problems are typically solved by dynamic programming (DP)
  - Deterministic problem: labelling correction methods and associated heuristics such as A* 
  - Stochastic problem: find an optimal stationary policy in an infinite horizon formulation with absorbing state
- How to formulate a discrete choice model defining path choice probabilities using the DP framework (i.e. network-based approach)? Optimal policy is utility maximization and utilities are link-additive
RECURSIVE LOGIT

- Simple case: deterministic attributes and link choice model is logit which yields a logit model over all paths (no correlation)
- The recursive logit is based on results from Rust (1987)
RECURSIVE LOGIT

- Link additive instantaneous utilities $u(a|k) = v(a|k) + \mu \varepsilon(a)$
- $v(a|k) = v(x_{a|k}; \beta) < 0$, $v(d|k) = 0$
- $E[\varepsilon(a)] = 0$
RECURSIVE LOGIT

- Bellman’s equation: \( V^d(k) = E \left[ \max_{a \in A(k)} (v(a|k) + \mu \epsilon(a) + V^d(a)) \right] \)
- Logsum

\[
V^d(k) = \mu \ln \sum_{a \in A(k)} e^{\frac{1}{\mu} (v(a|k) + V^d(a))}
\]

- System of linear equations

\[
z = Mz + b \Leftrightarrow (I - M)z = b
\]

\( z \ (|\tilde{A}| \times 1) \), \( z_k = e^{\frac{1}{\mu} V(k)} \), \( b \ (|\tilde{A}| \times 1) \), \( b_k = 0 \ \forall k \in A \), \( b_k = 1, k = d \)

\( M \ (|\tilde{A}| \times |\tilde{A}|) \)

\[
M_{ka} = \begin{cases} 
\delta(a|k)e^{\frac{1}{\mu} v(a|k)} & \forall a \in \tilde{A}, \forall k \in A \\
0 & \text{otherwise}
\end{cases}
\]
RECURSIVE LOGIT

- Probability of choosing link $a$ given state $k$

\[ P(a|k) = \frac{e^{1/\mu (v(a|k)+V(a))}}{\sum_{a' \in A(k)} e^{1/\mu (v(a'|k)+V(a'))}} \]

- Path $\sigma = \{k_i\}_{i=0}^I$, $k_0$ is the origin and $k_I = d$, $P(\sigma) = \prod_{i=0}^{I-1} P(k_{i+1}|k_i)$

\[ P(\sigma) = \prod_{i=0}^{I-1} e^{1/\mu (v(k_{i+1}|k_i)+V(k_{i+1})-V(k_i))} \]

\[ = e^{-1/\mu V(k_0)} \prod_{i=0}^{I-1} e^{1/\mu v(k_{i+1}|k_i)} \]
CONTRIBUTIONS

Route choice models and estimation methods

- No path choice sets are needed
- Consistent estimation
- Straightforward for prediction
- Allow path utilities to be correlated (IIA is relaxed)
  - Nested logit
  - General MEV
  - Mixed logit
  - Random regret decision rule

Methods for other related problems

- A model misspecification test
- Estimation of large-scale MEV models
- Optimization algorithms for maximum likelihood estimation (MLE)
## CONTRIBUTIONS

<table>
<thead>
<tr>
<th>Link attributes</th>
<th>Deterministic</th>
<th>Stochastic</th>
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</table>
| Static          | • Uni-modal network (car)  
                 • Revealed preferences data |           |
| Dynamic         | • State is defined by time and location  
                 • Ramos et al. (2012) and ongoing work | • State is time, location and perceived real-time information (e.g. day to day travel time variability)  
                 • Challenges: a large number of states, and complicated dynamic programming problems  
                 • Ongoing work |
RECURSIVE MODELS

- Mai T., Recursive network MEV model for route choice analysis, submitted to Transportation Research Part B.
RECURSIVE MODELS – NESTED

- Scale parameters of the random terms are assumed to be link specific
- Logit at each choice stage but IIA property does not hold
- Bellman’s equation:
  \[
  V^d(k) = E \left[ \max_{a \in A(k)} (v(a|k) + \mu_k \epsilon(a) + V^d(a)) \right], \epsilon(a) \text{ are i.i.d. EV}
  \]

  \[
  z^d_k = \sum_{a \in A(k)} M_{ka} (z^d_a) \frac{\mu_a}{\mu_k} + b_k
  \]

- Large system of non-linear equations, can be solved by value iteration (we propose “dynamic accuracy”)
  Fixed point solution exits if \( \sum_{a \in A(k)} M_{ka} < 1 \quad \forall k \)
RECURSIVE MODELS – GENERALIZED

- Model at each choice stage can be any network MEV model (Daly and Bierlaire, 2006)

- Bellman’s equation \( V^d(k) = E \left[ \max_{a \in A(k)} (v(a|k) + \epsilon(a|k) + V^d(a)) \right] \)
  where \( \epsilon(a|k) \) \( \forall a \in A(k) \) follow a MEV distribution with CPGF \( G_k \)

- Challenge: compute \( G_k \) and \( \partial G_k \) \( \forall k \)

- Trick: change the graph to include correlation structure at each stage (state augmentation), then use the same way to compute value function as the nested recursive model
RECURSIVE MODELS – MIXED

- Error component model combined with subnetwork (Frejinger and Bierlaire, 2007)
- Challenge: solve a very large number of linear systems
- Decomposition method that allows to solve one system of linear equations to obtain the value functions for all observations (useful also for RL model)
RECURSIVE MODELS – RANDOM REGRET

- Random regret minimization instead of random utility maximization
- Three different link regret functions (GRRM, ERRM, ARRM)

\[ r^{ERRM}(a|k) = \sum_{a' \in A(k)} \sum_t \ln \left( \lambda_t + e^{\beta_t (x(a'|k)_t - x(a|k)_t) + \delta_t x(a'|k)_t} \right) \]

- “Competitive RUM” models
RESULTS – COMPARISON

- Borlänge data (some of the models have also been estimated and applied to Delft and Eugene networks but they are not presented here)
- Static and deterministic network
- 1832 observations, 466 destinations
- 5 attributes and all parameters have expected signs, are significant and have plausible relative magnitudes
- In-sample model fit cannot be compared across all models
RESULTS – COMPARISON
RESULTS – COMPARISON

- Non parallelized MATLAB code running under an Intel(R) 3.20GHz machine with a x64-based processor

- Estimation cost
  - RL: 4 minutes (with the DeC), 2 hours (without the DeC method)
  - RL-LS: 8 hours
  - NRL-LS: 30 hours
  - RCNL-LS: 3 days
  - MRL-LS (500 draws): 5-7 days
  - RRM, CRUM models: 10 hours (with the DeC method)

- For all the recursive models
  - Less than 1 minute to solve Bellman’s equation
  - Few seconds to compute link flows, simulate a path
## ONGOING WORK

<table>
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<tr>
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| **Static**      | 1. NFXP for recursive models  
2. Discount factors in recursive models  
3. Bike route choice modeling | |
| **Dynamic**     | 4. Recursive models for dynamic and deterministic networks | 5. A recursive routing policy choice model for stochastic time-dependent networks |
CONCLUSIONS

- Different ways to analyze route choices (estimation and prediction) using discrete choice models
  - No generation of choice sets of paths
  - Correlated utilities
  - Prediction
- MATLAB code distributed on GitHub
  https://github.com/maitien86/RecursiveLogitCode