CHAIRE CN INTERMODALITÉ DES TRANSPORTS

DYNAMIC PROGRAMMING APPROACHES FOR ESTIMATING AND APPLYING LARGE-SCALE DISCRETE CHOICE MODELS

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CONTENU

- Introduction
- Recursive logit
- Summary of contributions
- Recursive models (correlated random terms)
- Results and comparisons
- Ongoing work
- Conclusions

INTRODUCTION

- This presentation gives an overview of Tien Mai's dissertation (seven papers)
- Many transport related choice problems (e.g. location, activity, route, mode, departure time) share some characteristics
  - Network-based
  - Large number of alternatives
  - Dynamic
  - Alternatives are similar (can, in a RUM framework, be translated to correlated random terms)
- In this presentation we focus on route choice modelling

INTRODUCTION – ROUTE CHOICE

- Given an origin and destination in a transport network, which route does a traveller choose?
- Shortest path and/or recommended route
- Analyst has imperfect knowledge of travellers' generalized cost and perception of network
- Discrete choice models estimated based on RP data are used to define choice probability distributions over alternatives

INTRODUCTION – ROUTE CHOICE

- Objectives
  - Models that can be consistently estimated using maximum likelihood
  - Models that produce accurate predictions in short computational time
- Main challenges
  - Definition of choice sets
  - Modelling correlation

INTRODUCTION

Choice sets

Consideration choice sets

- Choice sets are generated and treated as actual ones
- Alternative paths are not observed
- Observed paths may not be generated
- Parameter estimates differ depending on choice sets

Universal choice sets

Sampling of alternatives

- Consistent estimation
- Prediction

Recursive logit (RL)

- Consistent estimation
- Straightforward for prediction
- No choice sets needed

RECURSIVE LOGIT

- Proposed by Fosgerau, Frejinger and Karlstrom (2013)
- Shortest path problems are typically solved by dynamic programming (DP)
  - Deterministic problem: labelling correction methods and associated heuristics such as A^*
  - Stochastic problem: find an optimal stationary policy in an infinite horizon formulation with absorbing state
- How to formulate a discrete choice model defining path choice probabilities using the DP framework (i.e. network-based approach)? Optimal policy is utility maximization and utilities are link-additive

RECURSIVE LOGIT

- Simple case: deterministic attributes and link choice model is logit which yields a logit model over all paths (no correlation)
- The recursive logit is based on results from Rust (1987)

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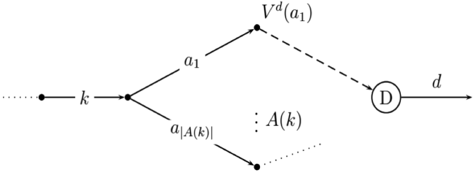
RECURSIVE LOGIT



- Link additive instantaneous utilities $u(a|k) = v(a|k) + \mu\varepsilon(a)$
- $v(a|k) = v(x_{a|k};\beta) < 0,\ v(d|k) = 0$
- $E[\varepsilon(a)] = 0$

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RECURSIVE LOGIT

- Bellman's equation: $V^d(k) = E\left[\max_{a \in A(k)}(v(a|k) + \mu\varepsilon(a) + V^d(a))\right]$
- Logsum

$$V^d(k) = \mu \ln \sum_{a \in A(k)} e^{\frac{1}{\mu}(v(a|k)+V^d(a))}$$

- System of linear equations

$$\mathbf{z} = \mathbf{M}\mathbf{z} + \mathbf{b} \Leftrightarrow (\mathbf{I} - \mathbf{M})\mathbf{z} = \mathbf{b}$$

- $\mathbf{z}\ (|\widetilde{A}| \times 1),\ z_k = e^{\frac{1}{\mu}V(k)},\ \mathbf{b}\ (|\widetilde{A}| \times 1),\ b_k = 0\ \forall k \in A,\ b_k = 1, k = d$
- $\mathbf{M}\ (|\widetilde{A}| \times |\widetilde{A}|)$

$$M_{ka} = \begin{cases} \delta(a|k)e^{\frac{1}{\mu}v(a|k)} & \forall a \in \widetilde{A},\ \forall k \in A \\ 0 & \text{otherwise} \end{cases}$$

RECURSIVE LOGIT

- Probability of choosing link a given state k

$$P(a|k) = \frac{e^{\frac{1}{\mu}(v(a|k)+V(a))}}{\sum_{a' \in A(k)} e^{\frac{1}{\mu}(v(a'|k)+V(a'))}}$$

- Path $\sigma = \{k_i\}_{i=0}^{I}$, k_0 is the origin and $k_I = d$, $P(\sigma) = \prod_{i=0}^{I-1} P(k_{i+1}|k_i)$

$$\begin{aligned} P(\sigma) &= \prod_{i=0}^{I-1} e^{\frac{1}{\mu}(v(k_{i+1}|k_i)+V(k_{i+1})-V(k_i))} \\ &= e^{-\frac{1}{\mu}V(k_0)} \prod_{i=0}^{I-1} e^{\frac{1}{\mu}v(k_{i+1}|k_i)} \end{aligned}$$

CONTRIBUTIONS

Route choice models and estimation methods

- No path choice sets are needed
- Consistent estimation
- Straightforward for prediction
- Allow path utilities to be correlated (IIA is relaxed)
  - ✓ Nested logit
  - ✓ General MEV
  - ✓ Mixed logit
  - ✓ Random regret decision rule

Methods for other related problems

- A model misspecification test
- Estimation of large-scale MEV models
- Optimization algorithms for maximum likelihood estimation (MLE)

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CONTRIBUTIONS

Link attributes	Deterministic	Stochastic
Static	• Uni-modal network (car) • Revealed preferences data	
Dynamic	• State is defined by time and location • Ramos et al. (2012) and ongoing work	• State is time, location and perceived real-time information (e.g. day to day travel time variability) • Challenges: a large number of states, and complicated dynamic programming problems • Ongoing work

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RECURSIVE MODELS

- Mai T., Fosgereau M., Frejinger E. (2015). A nested recursive logit for route choice analysis, Transportation Research Part B, 75(1), p.100-112.
- Mai T., Recursive network MEV model for route choice analysis, submitted to Transportation Research Part B.
- Mai, T., Bastin, F., and Frejinger, E. A decomposition method for estimating recursive logit based route choice models, under revision EURO Journal on Transportation and Logistics.
- Mai, T., Bastin, F., and Frejinger, E. Comparing regret minimization and utility maximization for route choice using the recursive logit model, under revision Journal of Choice Modeling

RECURSIVE MODELS – NESTED

- Scale parameters of the random terms are assumed to be link specific
- Logit at each choice stage but IIA property does not hold
- Bellman's equation:
  $V^d(k) = E\left[\max_{a \in A(k)}(v(a|k) + \mu_k \varepsilon(a) + V^d(a))\right]$, $\varepsilon(a)$ are i.i.d. EV

$$z_k^d = \sum_{a \in A(k)} M_{ka} (z_a^d)^{\frac{\mu_a}{\mu_k}} + b_k$$

- Large system of non-linear equations, can be solved by value iteration (we propose "dynamic accuracy")
  Fixed point solution exits if $\sum_{a \in A(k)} M_{ka} < 1 \quad \forall k$

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RECURSIVE MODELS – GENERALIZED

- Model at each choice stage can be any network MEV model (Daly and Bierlaire, 2006)
- Bellman's equation $V^d(k) = E\left[\max_{a \in A(k)}(v(a|k) + \varepsilon(a|k) + V^d(a))\right]$ where $\varepsilon(a|k) \ \ \forall a \in A(k)$ follow a MEV distribution with CPGF G_k
- Challenge: compute G_k and $\partial G_k \ \forall k$
- Trick: change the graph to include correlation structure at each stage (state augmentation), then use the same way to compute value function as the nested recursive model

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RECURSIVE MODELS – MIXED

- Error component model combined with subnetwork (Frejinger and Bierlaire, 2007)
- Challenge: solve a very large number of linear systems
- Decomposition method that allows to solve *one* system of linear equations to obtain the value functions for all observations (useful also for RL model)

RECURSIVE MODELS – RANDOM REGRET

- Random regret minimization instead of random utility maximization
- Three different link regret functions (GRRM, ERRM, ARRM)

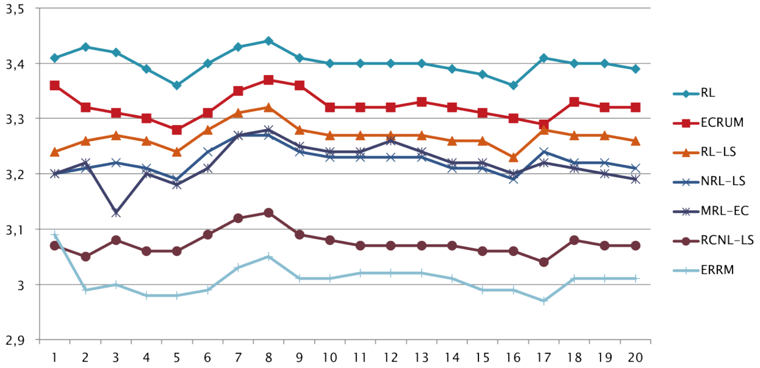
$$r^{ERRM}(a|k) = \sum_{a' \in A(k)} \sum_t \ln\left(\lambda_t + e^{\beta_t \left(x(a'|k)_t - x(a|k)_t\right) + \delta_t x(a'|k)_t}\right)$$

- "Competitive RUM" models

RESULTS – COMPARISON

- Borlänge data (some of the models have also been estimated and applied to Delft and Eugene networks but they are not presented here)
- Static and deterministic network
- 1832 observations, 466 destinations
- 5 attributes and all parameters have expected signs, are significant and have plausible relative magnitudes
- In-sample model fit cannot be compared across all models

RESULTS – COMPARISON



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RESULTS – COMPARISON

- Non parallelized MATLAB code running under an Intel(R) 3.20GHz machine with a x64-based processor
- Estimation cost
  RL: 4 minutes (with the DeC), 2 hours (without the DeC method)
  RL-LS : 8 hours
  NRL-LS: 30 hours
  RCNL-LS: 3 days
  MRL-LS (500 draws): 5-7 days
  RRM, CRUM models: 10 hours (with the DeC method)
- For all the recursive models
  Less than 1 minute to solve Bellman's equation
  Few seconds to compute link flows, simulate a path

ONGOING WORK

Link attributes	Deterministic	Stochastic
Static	1. NFXP for recursive models 2. Discount factors in recursive models 3. Bike route choice modeling	
Dynamic	4. Recursive models for dynamic and deterministic networks	5. A recursive routing policy choice model for stochastic time-dependent networks

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CONCLUSIONS

- Different ways to analyze route choices (estimation and prediction) using discrete choice models
  - No generation of choice sets of paths
  - Correlated utilities
  - Prediction
- MATLAB code distributed on GitHub
  `https://github.com/maitien86/RecursiveLogitCode`

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