A MODIFIED PAIRED COMBINATORIAL LOGIT ROUTE CHOICE MODEL WITH PROBIT-BASED EQUIVALENT IMPEDANCE

Reporter:

LAI Xinjun

Research Center of Intelligent Transportation System

Sun Yat-sen University

Guangzhou, China

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GUANGZHOU CITY

Guangzhou city

Evening peak of BRT

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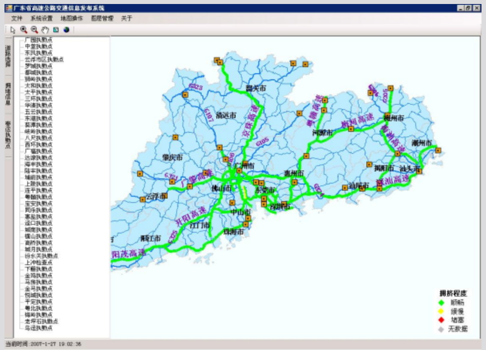
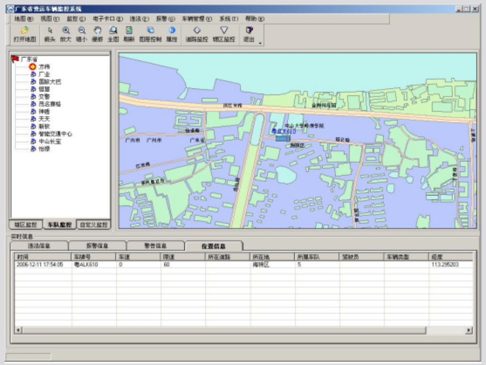


Guangdong province freeway traffic info issuing system



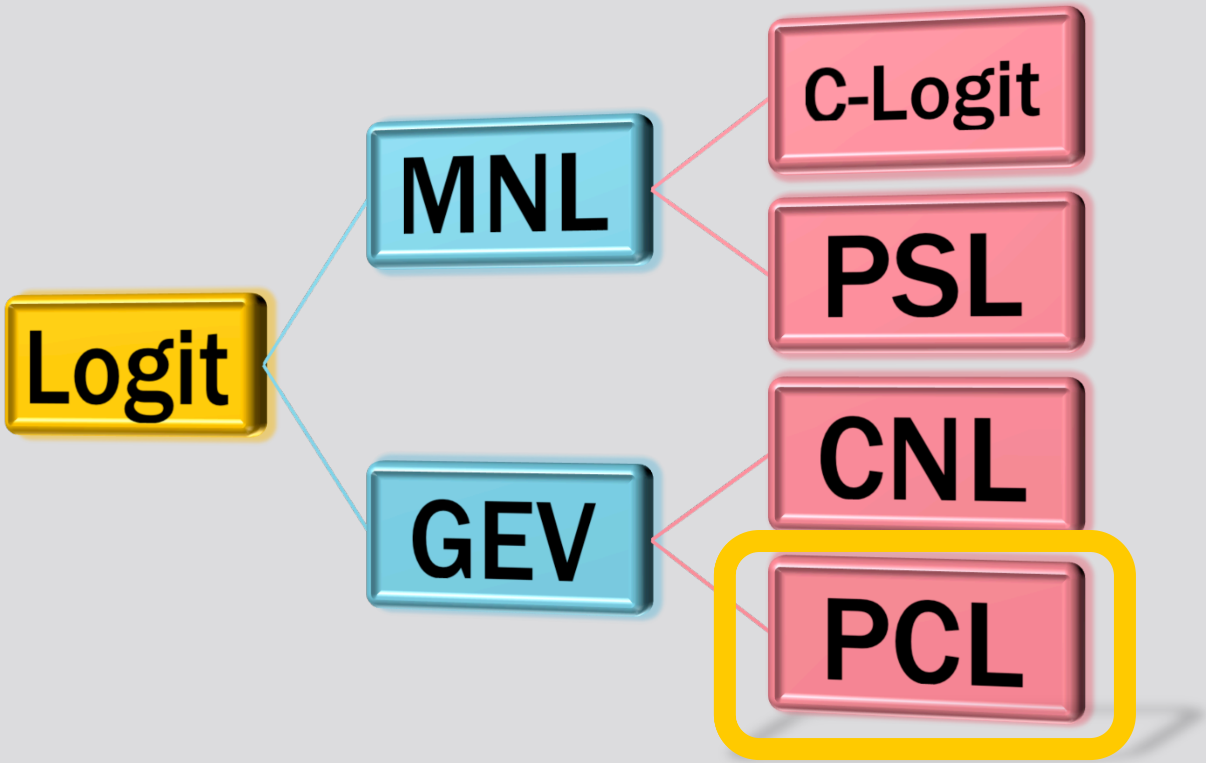
Guangdong province commercial vehicle manage system

ITS lab

0. CONTENS

1、 INTRODUCTION

1. INTRODUCTION

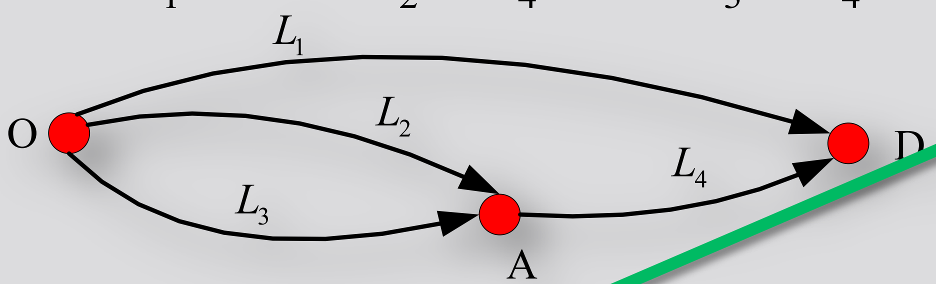


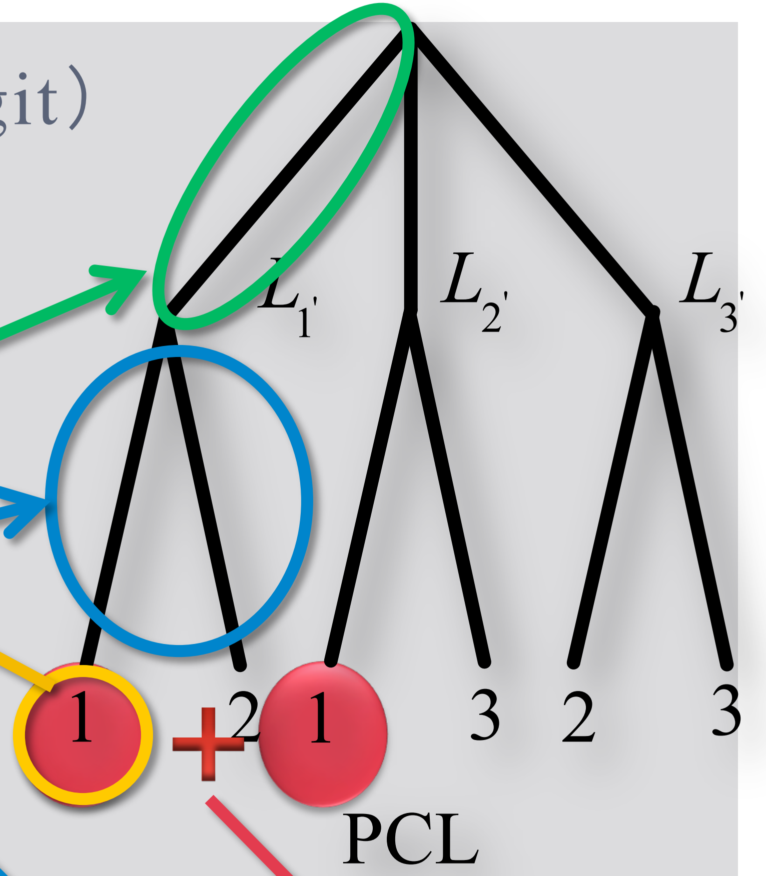
MNL: Multinomial Logit
C-Logit: *C* for *Commonality Factor*
PSL: Path Size Logit
GEV: Generalized Extreme Value

CNL: or called *GNL*, Generalized Nested Logit
PCL: Paired Combination Logit

1. INTRODUCTION

- PCL（Paired Combination Logit）
  - $1:\ L_1 \quad 2:\ L_2 + L_4 \quad 3:\ L_3 + L_4$





$$P(L_{1'})P(1 \mid L_{1'})$$

$$\frac{\left[\exp\left(-\frac{V_1}{1-\sigma_{12}}\right)+\exp\left(-\frac{V_2}{1-\sigma_{12}}\right)\right]^{1-\sigma_{12}}}{\sum_{k=1}^{3}\sum_{l=k+1}^{3}\left[\exp\left(-\frac{V_1}{1-\sigma_{12}}\right)+\exp\left(-\frac{V_2}{1-\sigma_{12}}\right)\right]^{1-\sigma_{12}}}$$

Similarity index

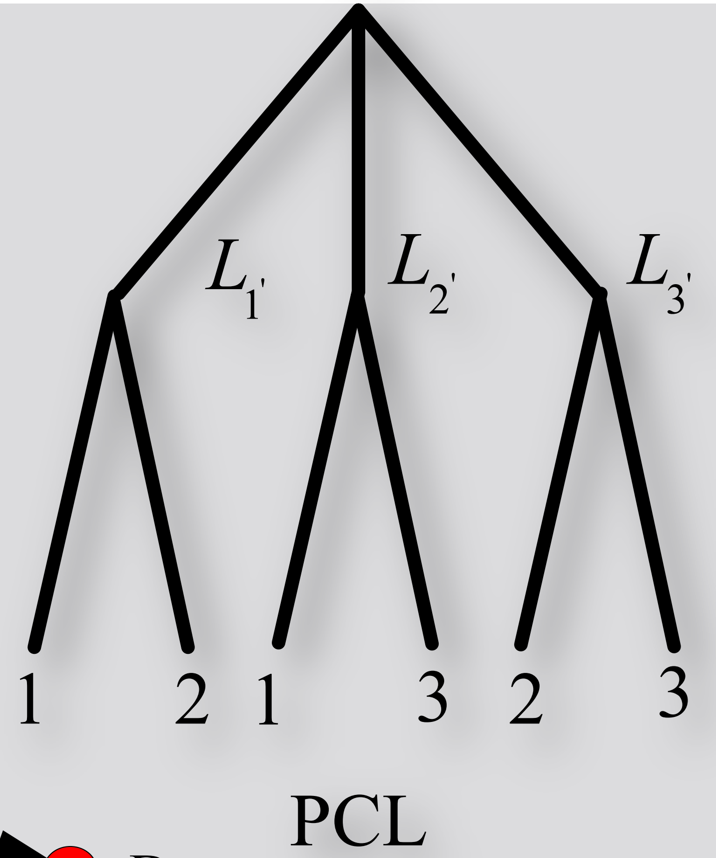
Binary logit

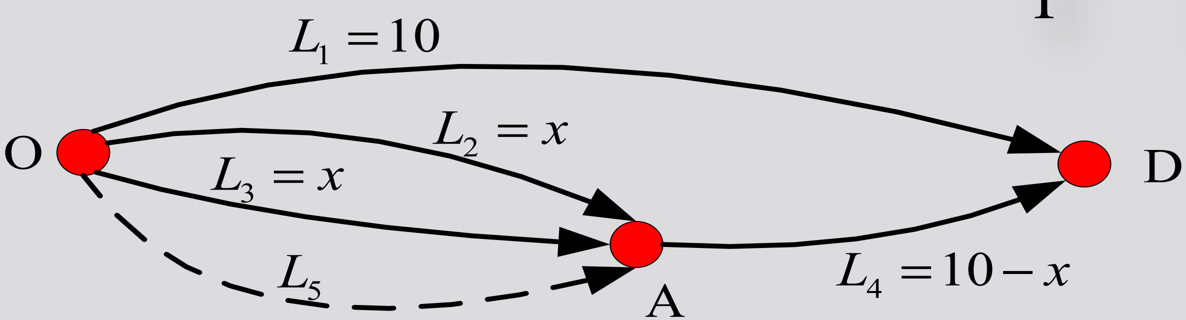
Probability of choosing route 1

1. INTRODUCTION

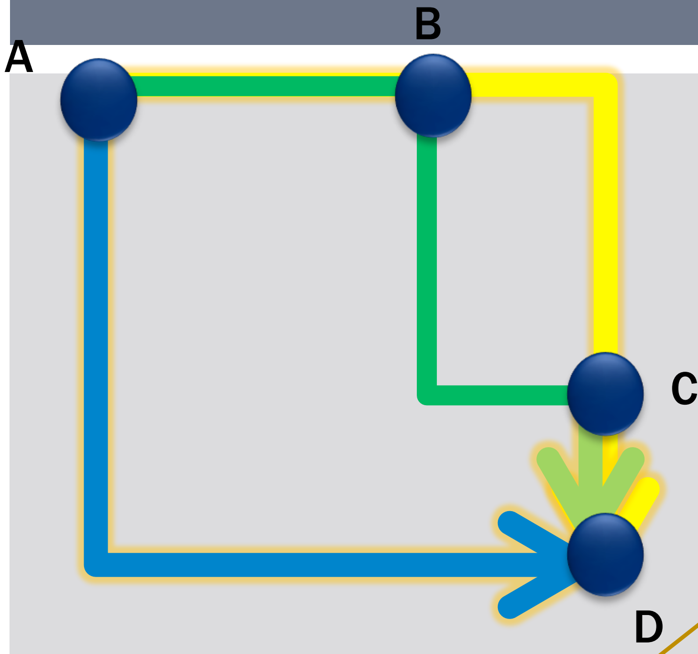
- How to fix the similarity index?
  - experience
- equivalent impedance

LI Jun and OUYANG Jun. A MODIFIED PAIRED COMBINATORIAL LOGIT ROUTE CHOICE MODEL WITH UNIFIED PARAMETERLPCL (LPCL)

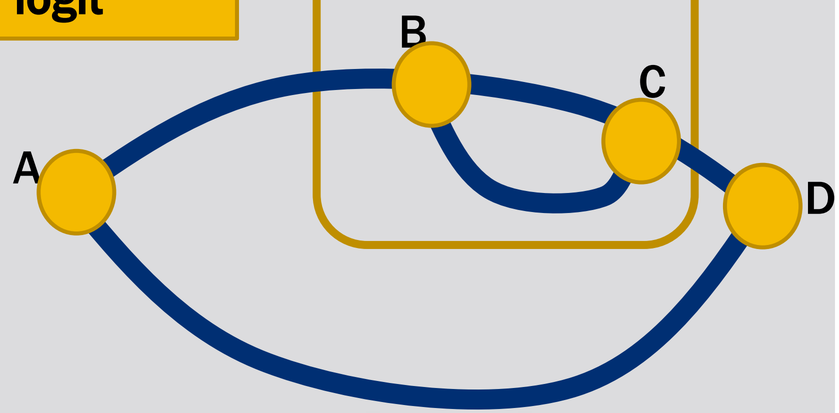




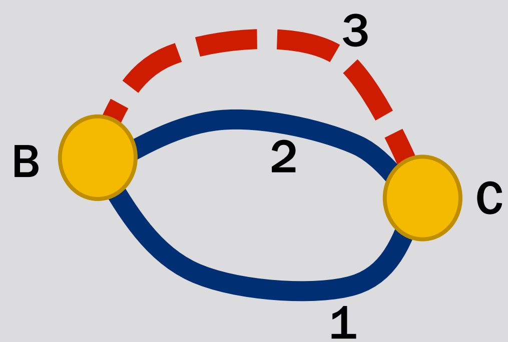
1. INTRODUCTION



The overlapping link
IIA of logit



Equivalent impedance



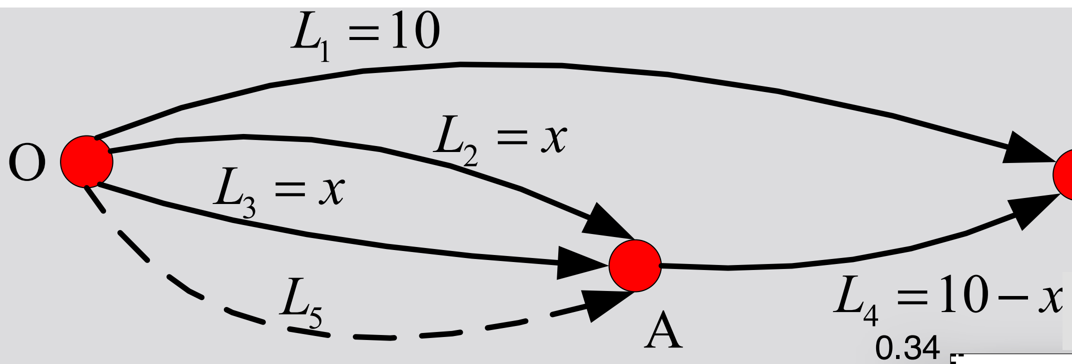
$$\frac{exp(-\theta T_3)}{\sum[exp(-\theta T_3)+exp(-\theta T_1)+exp(-\theta T_2)]} = \frac{exp(-\theta T_2)}{\sum[exp(-\theta T_3)+exp(-\theta T_1)+exp(-\theta T_2)]} + \frac{exp(-\theta T_1)}{\sum[exp(-\theta T_3)+exp(-\theta T_1)+exp(-\theta T_2)]}$$

1. INTRODUCTION

- $\exp(-\theta T_3) / \sum [\exp(-\theta T_3) + \exp(-\theta T_1) + \exp(-\theta T_2)] = \exp(-\theta T_2) / \sum [\exp(-\theta T_3) + \exp(-\theta T_1) + \exp(-\theta T_2)] = \exp(-\theta T_1) / \sum [\exp(-\theta T_3) + \exp(-\theta T_1) + \exp(-\theta T_2)]$

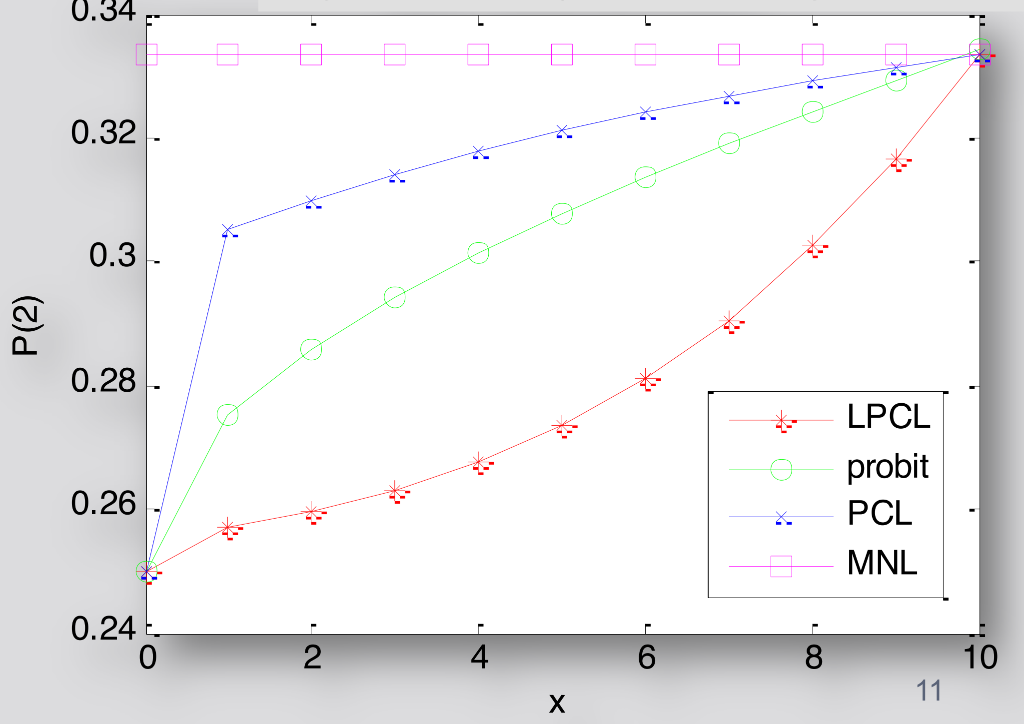
T_1 = *length of L1/length of the shortest route*

1. INTRODUCTION



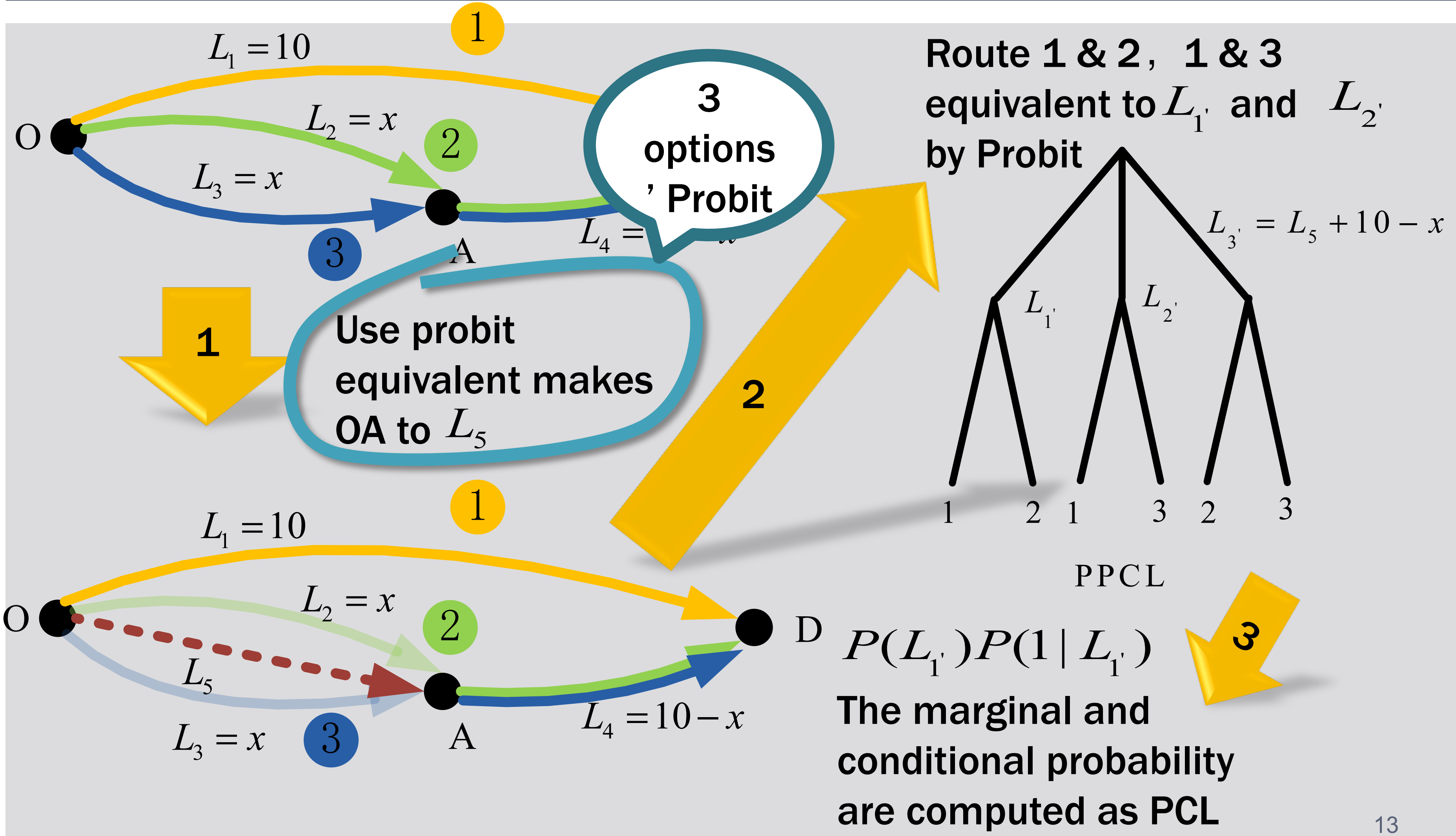
LPCL makes "equivalent impedance + PCL" workable

Fig. probability of choosing route 2



2 ： THE PROPOSED MODEL:PPCL （PROBIT PCL）

2 THE PROPOSED MODEL: PPCL



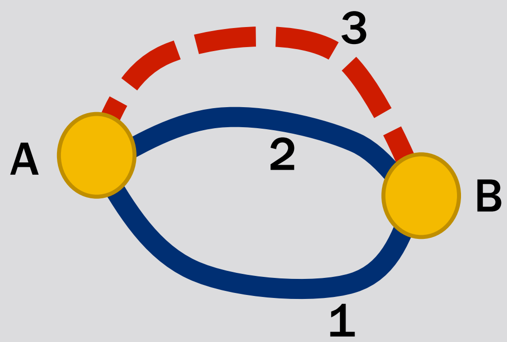
2 THE PROPOSED MODEL:PPCL

$$\max(U_1,U_2) \sim N(\mu_{12},\varpi_{12}-\mu_{12})$$

- Approximation for 3 options' Probit

$$P_3 = \Pr ob[U_3 > \boxed{\max(U_1,U_2)}] = \Phi\left(\frac{V_3 - E[\max(U_1,U_2)]}{\sqrt{\operatorname{var}(U_3 + E[\max(U_1,U_2)])}}\right)$$

$$= \Phi\left(\frac{V_3 - \mu_{12}}{\sqrt{\sigma_3^2 + \sigma_m^2 - 2\rho_3^{'}\sigma_3\sigma_m}}\right)$$



$$P_1 + P_2 + P_3 = 1$$

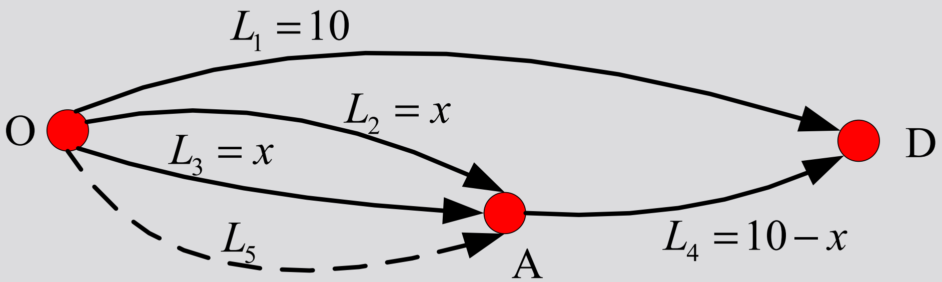
$$0$$

$$P_3 = P_2 + P_1 = \Phi\left(\boxed{\frac{V_3 - \mu_{12}}{\sqrt{\sigma_3^2 + \sigma_m^2 - 2\rho_3^{'}\sigma_3\sigma_m}}}\right) = 0.5$$

- Φ is standard normal distribution, ϕ is standard density function of normal distribution

2 THE PROPOSED MODEL:PPCL

$$V_3 = \mu_{12} = V_1\Phi(\frac{V_1 - V_2}{\sqrt{\sigma_1^2 + \sigma_2^2}}) + V_2\Phi(\frac{V_2 - V_1}{\sqrt{\sigma_1^2 + \sigma_2^2}}) + \sqrt{\sigma_1^2 + \sigma_2^2}\,\phi(\frac{V_1 - V_2}{\sqrt{\sigma_1^2 + \sigma_2^2}})$$



$$V_5 = -L_5 = -L_2\Phi(\frac{-L_2 + L_3}{\sqrt{\sigma_2^2 + \sigma_3^2}}) - L_3\Phi(\frac{-L_3 + L_2}{\sqrt{\sigma_2^2 + \sigma_3^2}}) + \sqrt{\sigma_2^2 + \sigma_3^2}\,\phi(\frac{-L_2 + L_3}{\sqrt{\sigma_2^2 + \sigma_3^2}})$$

$$L_5 = x\Phi(0) + x\Phi(0) - \sqrt{2\beta x}\,\phi(0) = x - \sqrt{\frac{\beta x}{\pi}}$$

compared to LPCL $L_5 = \dfrac{\theta x}{\log 2 + \theta}$

2 THE PROPOSED MODEL:PPCL

- where

$$\mu_{12} = E[\max(U_1, U_2)] = V_1\Phi(\alpha_{12}) + V_2\Phi(-\alpha_{12}) + \sqrt{\mathrm{var}(U_1 + U_2)}\phi(\alpha_{12})$$

$$\alpha_{12} = (V_1 - V_2)\Big/\sqrt{\mathrm{var}(U_1 + U_2)}$$

$$\mathrm{var}(U_1 + U_2) = \sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2$$

$$\rho_3^{'} = \rho[U_3, \max(U_1 + U_2)] = \frac{\sigma_1\rho_{13}\Phi(\alpha_{12}) + \sigma_2\rho_{13}\Phi(-\alpha_{12})}{\sqrt{\varpi_{12} - \mu_{12}^2}}$$

$$\varpi_{12} = E[\max(U_1 + U_2)] = (V_1^2 + \sigma_1^2)\Phi(\alpha_{12})$$

$$+ (V_2^2 + \sigma_2^2)\Phi(-\alpha_{12}) + (V_1^2 + V_2^2)\sqrt{\mathrm{var}(U_1 + U_2)}\phi(\alpha_{12})$$

$$\sigma_m^2 = \mathrm{var}[\max(U_1, U_2)] = \varpi_{12} - \mu_{12}^2$$

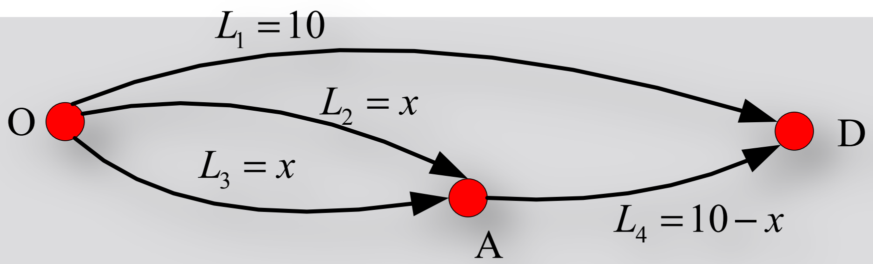
3 NUMERICAL EXAMPLES

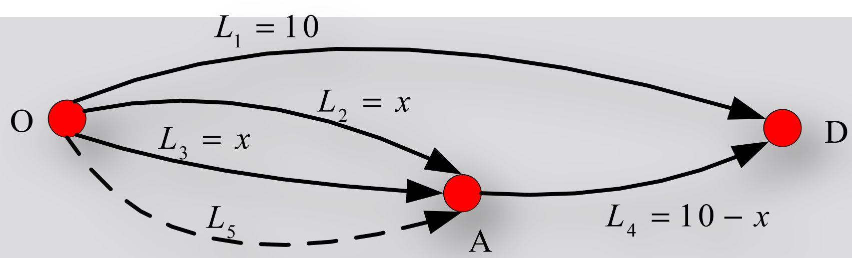
3 NUMERICAL EXAMPLES

- Probit obeys the normal distribution of $(E, \beta x)$
- Logit obeys the gumbel distribution of $(E, \frac{\pi^2}{6\theta^2})$
- In order to compare the models ,assume that the variances of 2 models are equal, so

$$\beta = \frac{\pi^2}{6\theta^2 x}$$

3 NUMERICAL EXAMPLES: NETWORK 1





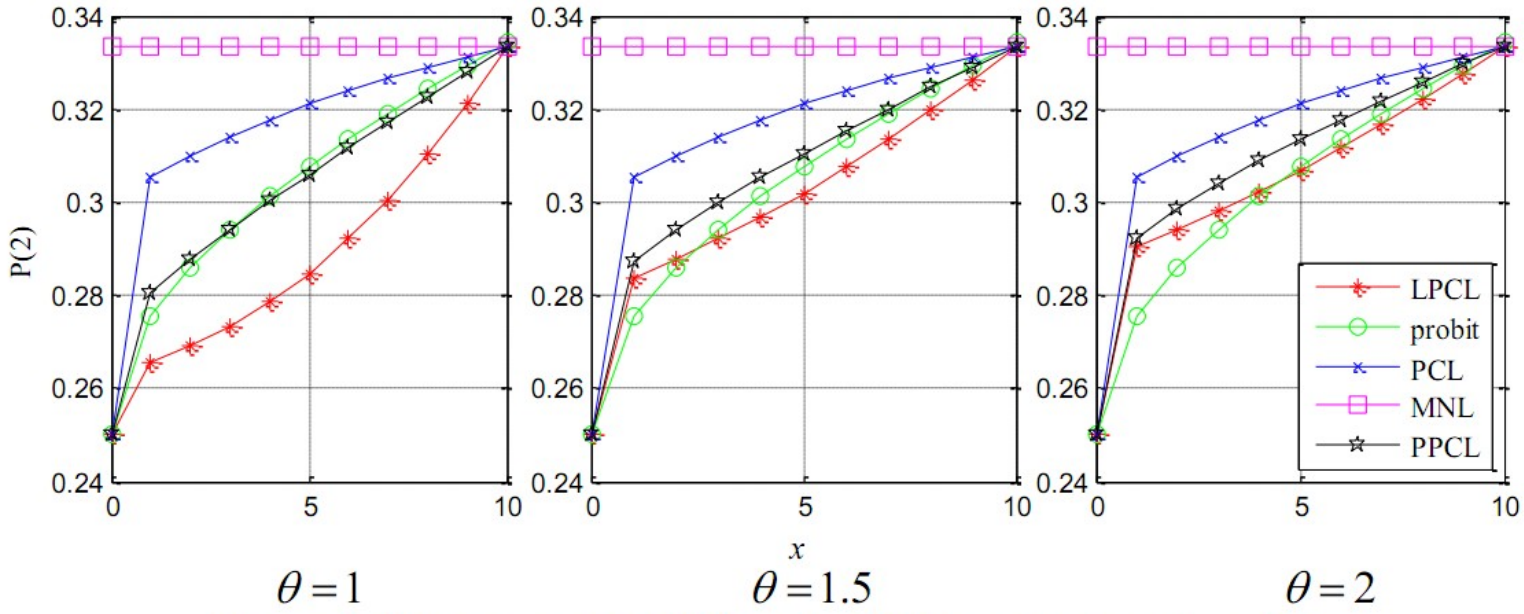
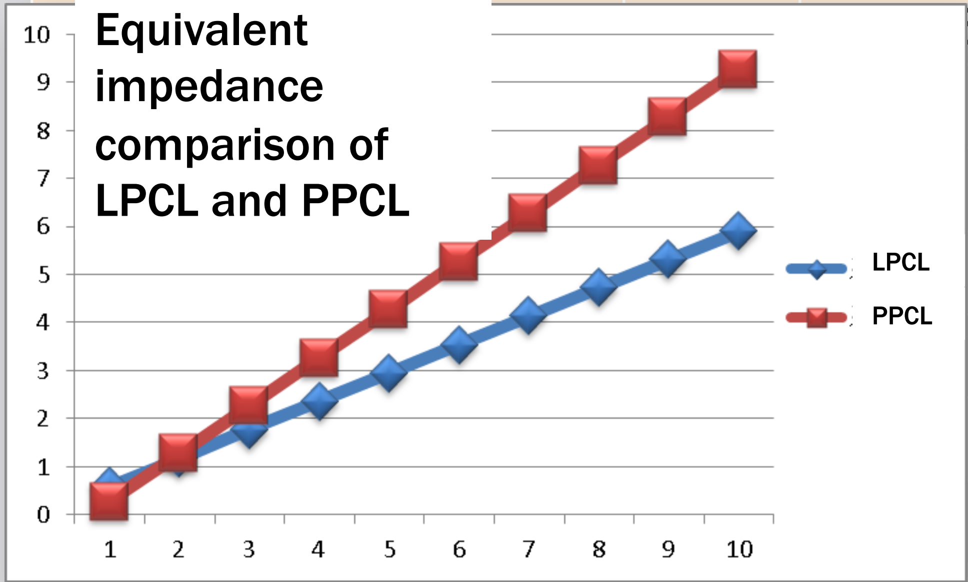


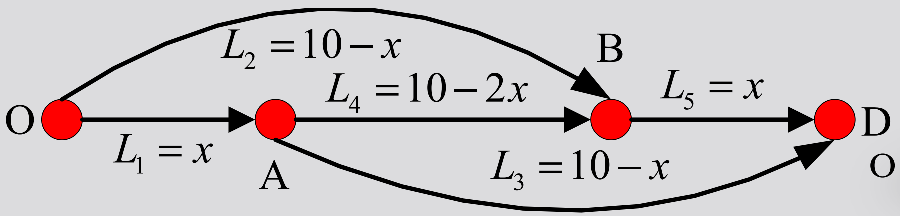
Figure 2. Choice probabilities of middle route in network 1

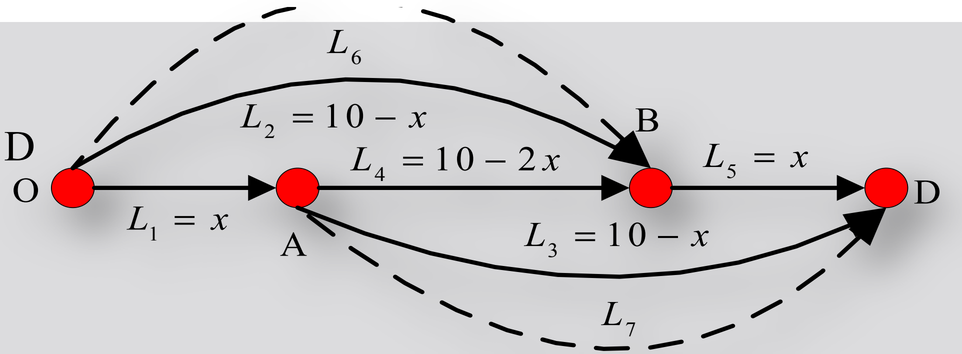
3 NUMERICAL EXAMPLES: NETWORK 1

	Equivalent impedance			Relative error（compared to Probit，%）		
x	LPCL	PPCL	x	LPCL	PCL	PPCL
0	0	NaN	0	0	0	NaN
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]636	10.8729	0.5917
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]564	8.3616	2.2591
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]996	6.7034	3.2757
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]518	5.4039	3.4398
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]572	4.2910	3.1735
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]413	3.2864	2.6721
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]711	2.3499	2.0355
8	4.7249	7.2764	8	9.0065	1.4588	1.3191
9	5.3155	8.2764	9	5.1701	0.5997	0.5554
10	5.9062	9.2764	10	0.2357	0.2357	0.2357
$\theta = 1$			mean	8.95	4.35	0.24



3 NUMERICAL EXAMPLES: NETWORK 2





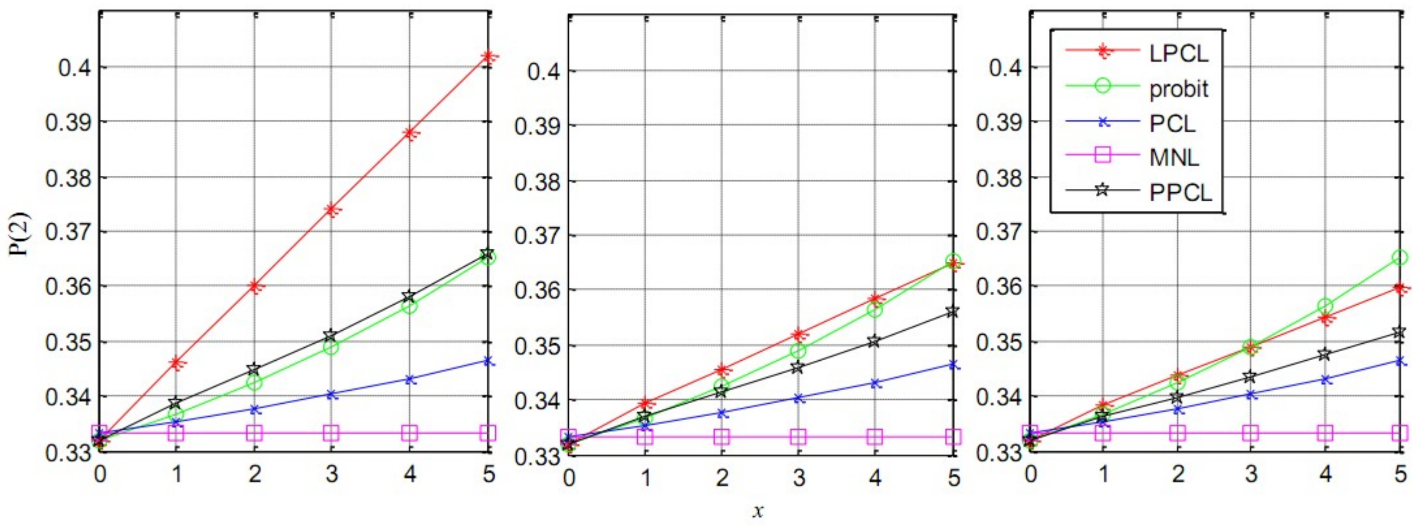
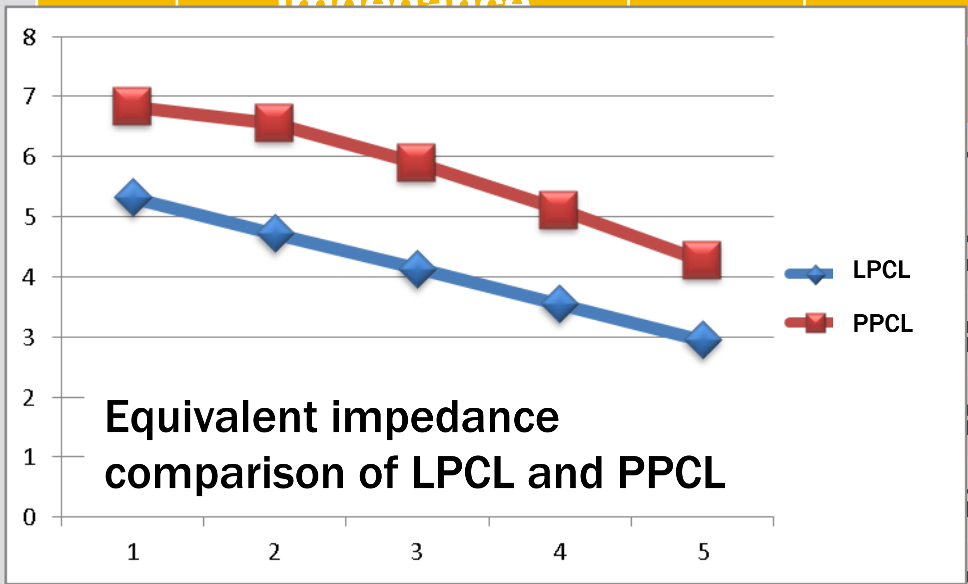


Figure 4. Choice probabilities of upper route in network 2

3 NUMERICAL EXAMPLES: NETWORK 2



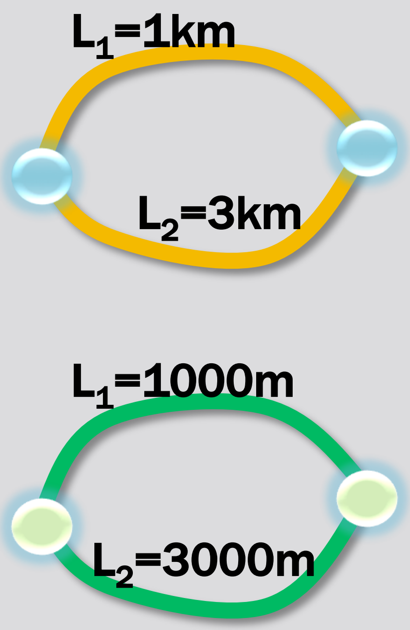
	Equivalent imdedance			Relative error （compared to Probit， %）		
				CL	PCL	PPCL
				747	0.4747	NaN
				289	0.4074	0.9607
				126	1.3814	0.1991
				531	2.4668	0.9024
				584	3.6910	2.2889
5	2.9531	4.2764	5	22.8518	5.0938	3.9708
	$\theta = 1$		mean	15.66	2.60	1.66

4. CONCLUSION

- PPCL PROS
  - 1. Retains the advantage of the PCL model to resolve the overlapping route problem
  - 2. Better represent the random error term in the route choice decision process
  - 3. Clear theoretical background while original PCL may require different parameter for each OD pair
  - 4. demonstrate more improvements on the IIA issue of original Logit route choice model

5 RELATIVE IMPEDANCE & DISCUSSION

5.1 RELATIVE IMPEDANCE



$$P_1 = 82\%$$

$$P_1 = 100\%$$

5.1 RELATIVE IMPEDANCE

- Why ?
- Basic assumption of Logit

$$\text{variance} \quad \sigma^2 = \frac{\pi^2}{6\theta^2}$$

Perception error regardless of impedance

- Modification to the variance

Min impedance （shortest route $V_{\min}$

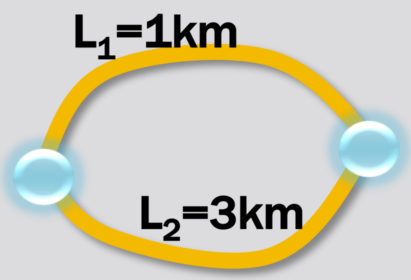
$$\text{modified variance} \quad \sigma^2 = \frac{\pi^2}{6\theta'^2} V^2 = \frac{\pi^2}{6\left(\frac{\theta'}{V}\right)}$$

$$\theta = \frac{\theta'}{V_{\min}}$$

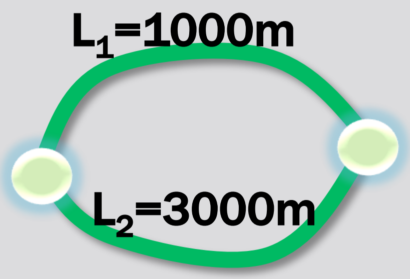
$$P_i = \frac{\exp(-\theta' \frac{V_i}{V_{\min}})}{\sum_j \exp(-\theta' \frac{V_j}{V_{\min}})} = \frac{1}{1 + \sum_{j \neq i} \exp\left[-\theta'\left(\frac{V_i - V_j}{V_{\min}}\right)\right]}$$

5.1 RELATIVE IMPEDANCE

$$P_i = \frac{\exp(-\theta' \frac{V_i}{V_{\min}})}{\sum_j \exp(-\theta' \frac{V_j}{V_{\min}})} = \frac{1}{1 + \sum_{j \neq i} \exp\left[-\theta' \left(\frac{V_i - V_j}{V_{\min}}\right)\right]}$$



$$P_1 = 82\%$$



$$P_1 = 82\%$$

Variance modification

Relative impedance

5.2 DISCUSSION

- **Relationship between impedance and perception error ?**

$$\sigma_{\text{logit}}^2 = \sigma_{\text{probit}}^2 \rightarrow \begin{cases} \sigma_{\text{logit}}^2 = \dfrac{\pi^2}{6\theta^2}V^2 \\ \sigma_{\text{probit}}^2 = \beta V \end{cases}$$

  - Assumption 1: shortest route x, other routes are proportionate to x

x

cx (c is constance)

$$D(cx) = c^2 D(x) \Rightarrow \sigma^2 = \frac{\pi^2}{6\theta^2}V^2$$

  - Assumption 2: independent perception for each link (reasonable)

$a+b$

$$D(a+b) = D(a) + D(b) \Rightarrow \sigma^2 = \beta V$$

$$\sigma_{\text{probit}}^2 = \beta V^2 \Leftarrow \sigma_{\text{logit}}^2 = \frac{\pi^2}{6\theta^2}V^2$$

$$\Uparrow$$

$$P_i = \frac{1}{1 + \sum_{j \neq i} \exp\left[-\theta'\left(\dfrac{V_i - V_j}{V}\right)\right]}$$

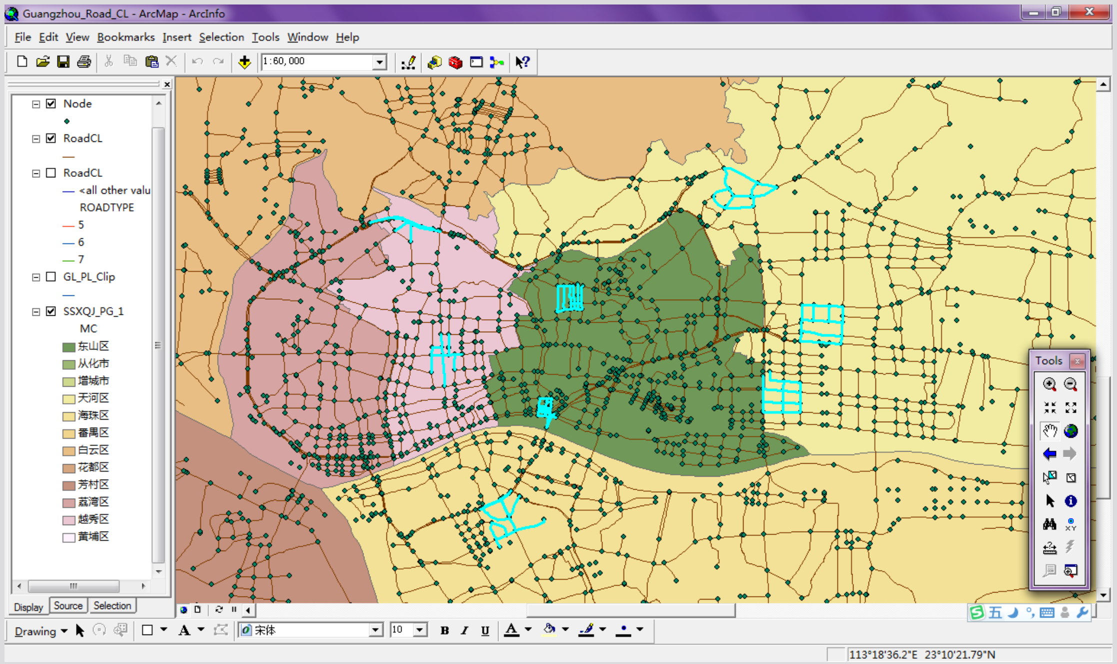
$$\sigma_{\text{probit}}^2 = \beta V \Rightarrow \sigma_{\text{logit}}^2 = \frac{\pi^2}{6\theta^2}V$$

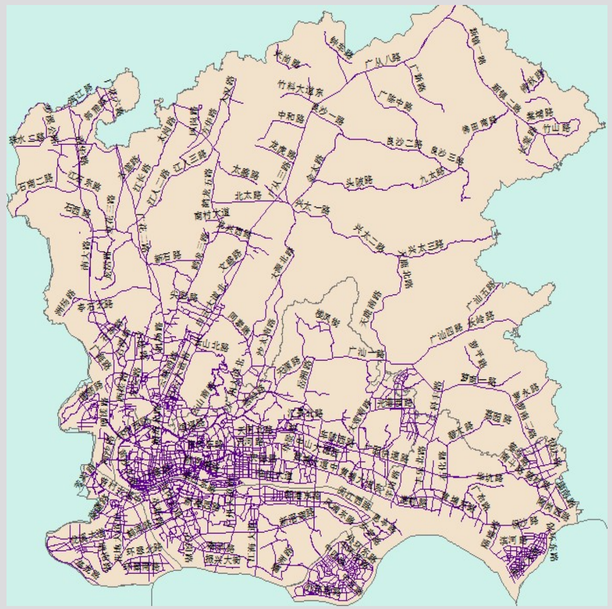
$$\Downarrow$$

$$P_i = \frac{1}{1 + \sum_{j \neq i} \exp\left[-\theta'\left(\dfrac{V_i - V_j}{\sqrt{V}}\right)\right]}$$

5.3 FUTURE WORK

- Parameter calibration
- Test the model
  - Source: GPS taxi data of Guangzhou city





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