Enseignant: M. Bierlaire
Assistante: Sh. Sharif Azadeh

Introduction
à l'optimisation
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Question 1:

A company has contracted for five jobs. These jobs can be performed in six of its manufacturing plants. Because of the size of the jobs, it is not feasible to assign more than one job to a particular manufacturing facility. The estimated cost (in thousand dollars) of performing the jobs in different manufacturing plants are summarized in the following table:

	PLANT					
Job	1	2	3	4	5	6
1	50	55	42	57	48	52
2	66	70	64	68	75	63
3	81	78	72	80	85	78
4	40	42	38	45	46	42
5	62	55	58	60	56	65

a) Formulate this problem of assigning jobs to the plants in a way that the total cost is minimized. Use *linprog* function of MATLAB to solve the problem.

b) The company faces some environmental restrictions on assigning jobs to plants. These restrictions are:

   (i) Job 1 should be either assigned to plant 1 or 3

   (ii) Regardless of the previous part, now assume that jobs 4 and 5 should be assigned to one of these groups of plants: (1,2,3) or (4,5,6).
   Add these constraints to your model and solve it again.

Question 2:

Consider the following Knapsack problem:

$$\max \quad 12.95x_1 + 7.08x_2 + 7.08x_3 + 5.9x_4 + 6x_5 + 4.72x_6 + 1.18x_7 + 23.6x_8 \tag{1}$$
$$11x_1 + 6x_2 + 6x_3 + 5x_4 + 5x_5 + 4x_6 + x_7 + 20x_8 \leq 19 \tag{2}$$
$$x_i \in \{0,1\} \quad i = 1,2,..,8 \tag{3}$$

Answer to the following questions. For each question you are asked to fill the table.

a) Solve the linear programming problem (relaxation model).

b) By analysing the constraint, identify the value of the variable(s) that you can fix before solving the relaxation. Then, solve the relaxation model again.

c) The linear programming model can be strengthened by adding logical constraints to the problem when they do not eliminate the integer feasible solutions. For example, the following constraint:

$$x_3 + x_4 + x_5 + x_6 \leq 3 \tag{4}$$

can be added to the original problem (i.e.,between x_3, x_4, x_5, x_6 at most three variables could be selected). Sort the coefficients then add valid inequalities to avoid overpassing the capacity. Pay attention to choose minimum size combinations of variables $(x_1, \ldots, x_8)$. You should add around 15 constraints in the model.

d) Some of the constraints added before can be further improved. For example, constraint $x_3 + x_4 + x_5 + x_6 \leq 3$ can be extended to $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \leq 3$. Further we may increase the value of coeffficient for variable x_1 and x_2. This can be done by having additional logical analysis on the original constraint. Suppose that $x_2 = 0$ and a is the coefficient of x_1, we want to know the maximum value of a that satisfies the constraint. If $x_1 = 1$ then the remaining capacity is 19-11=8. In this case, at most one of the variables of x_2, x_3, x_4, x_5 could have the value euqal to one and the rest will be equal to zero. That means if $x_1 = 1$ then $x_2 + x_3 + x_4 + x_5 \leq 1$ therefore, we can find an upper bound value of a. (i.e., $a + 1 \leq 3 \rightarrow a = 2$).
The constraint becomes: $2x_1 + bx_2 + x_3 + x_4 + x_5 \leq 3$. Repeat the same procedure in order to find the value of b. add this constraint to the model and solve it again.

e) This problem can also be solved via a greedy algorithm. To do that sort the coefficients of the original constraint in a descending order and use a greedy procedure to find a solution

f) Do the previous step, however, this time sort the items based on their weighted constribution to the objective function.

	Obj	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
a									
b									
c									
d									
e									
f									