TRANSP-OR

Introduction
à l'optimisation
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Session 13: Conditions d'optimalité

Question 1:

a) The gradient of the function can be written as:

$$\begin{cases} \frac{\partial f}{\partial x_1} & 4x_1(x_1^2 - 4) \\ \frac{\partial f}{\partial x_2} & 2x_2 \end{cases}$$

By solving the above equations we obtain $(-2,0), (0,0), (2,0)$ as stationary points. Therefore for each point we check the hessian matrix. For $(-2,0)$ the hessian matrix is positive therefore it is a local minimum. with the same reasoning $(0,0)$ is a saddle point and $(2,0)$ is a local minimum.

b) The same procedure as (a), in this case, $(0,0)$ is a saddle point.

Question 2:

a)

$$\nabla f(x,y) = \begin{pmatrix} 4x^3 - 4x \\ 3y^2 - 3 \end{pmatrix}$$

$$\nabla f^2(x,y) = \begin{pmatrix} 12x^2 - 4x & 0 \\ 0 & 6y \end{pmatrix}$$

$(2,2)$ is not a minimum, $(-1,1)$ is minimum and $(0,-1)$ is maximum.

b) $x_0 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$$x_1 = x0 - (\nabla^2 f(2,2))^{-1} \nabla f(2,2) = \begin{pmatrix} 16/11 \\ 5/4 \end{pmatrix}$$

$$x_2 = x_1 - (\nabla^2 f(x_1))^{-1} \nabla f(x_1) = \begin{pmatrix} 1.151 \\ 1.025 \end{pmatrix}$$

Question 3: we calculate the gradient

$$\nabla f(x_k) = \begin{pmatrix} 2x_1 + 2x_2 \\ 2x_1 + 4x_2 \end{pmatrix}$$

the Hessian :

$$H(x_k) = \begin{pmatrix} 2 & 2 \\ 2 & 4 \end{pmatrix}$$

and its inverse:

$$H^{-1}(x_k) = \begin{pmatrix} 1 & -1/2 \\ -1/2 & 1/2 \end{pmatrix}$$

We know that we have

$$H^{-1}(x_k)\nabla f(x_k) = \begin{pmatrix} 1 & -1/2 \\ -1/2 & 1/2 \end{pmatrix} \begin{pmatrix} 2x_1 + 2x_2 \\ 2x_1 + 4x2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_k$$

Also, for $x_k \in \mathbb{R}^2$ we have:

$$x_{k+1} = x_k - H^{-1}(x_k)\nabla f(x_k) = x_k - x_k = (0, 0)$$

Therefore, for one iteration and independent of intial point (x_k) Newton method converge toward minimum of the function.