Optimisation linéaire : Dualié - Solution

Question 1:

a)

$$
\begin{aligned}
\max z = 7y_2 + 2y_3 \\
y_1 \in R \\
y_2 \leq 0 \\
y_3 \geq 0 \\
y_1 + y_2 + 2y_3 = -1 \\
-2y_1 + y_2 + y_3 \leq -1
\end{aligned}
$$

b)

Dual/Primal	Optimal	Unbounded	Infeasible
Optimal	Yes		
Unbounded			Yes
Infeasible		Yes	Yes

Question 2:

a) The dual of the problem is

$$
\begin{aligned}
\max 10y_1 + 16y_2 + 5y_3 \\
2y_1 + y_2 \leq 1 \\
y_1 + y_3 = -4 \\
4y_2 - y_3 \geq 2 \\
y_1 \geq 0 \\
y_2 \in R \\
y_3 \leq 0
\end{aligned}
$$

b)

$$
\begin{aligned}
\max 16y_1 + 31y_2 + 24y_3 + 68y_4 + 22y_5 \\
-5y_1 + y_2 + y_3 + 3y_4 + y_5 \leq -4 \\
4y_1 + 3y_2 + 2y_3 + 5y_4 + y_5 \leq -7 \\
y_1, y_2, y_3, y_4, y_5 \leq 0
\end{aligned}
$$

Optimisation linéaire : Dualité-Solution

Question 3:

It happens when $c = -b$ and matrix A has the property that it is equal to its own dual (skew-symmetric) then A is said to be self-dual.

For example:

$$\begin{array}{llll} \min & 3x_1 + 4x_2 & & \\ & 2x_2 & \geq & -3 \\ -2x_1 & & \geq & -4 \\ & x_1, \quad x_2 & \geq & 0 \end{array}$$

where A equals to

$$A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

The dual problem is:

$$\begin{array}{llll} \min & -3y_1 - 4y_2 & & \\ & -2y_2 & \leq & 3 \\ 2y_1 & & \leq & 4 \\ & y_1, \quad y_2 & \geq & 0 \end{array}$$