Optimisation linéaire : première phase

Question 1:

$$\begin{aligned} \min \quad & -x_1 - x_2 \\ & x_1 + 2x_2 + e_1 = 3 \\ & x_1 + x_2 - e_2 = 1 \\ & x_1, x_2 \geq 0 \\ & e_1, e2 \geq 0 \end{aligned} \tag{1}$$

La solution initiale:
$e_1 = 3, e_2 = -1$ and $x_1 = 0, x_2 = 0$
C'est une solution non-réalisable. Donc, on utlise la méthode de 2 phases.

	0	0	0	0	1	
	x_1	x_2	e_1	e_2	a_2	Valeur
e_1	1	2	1	0	0	3
a_2	1	1	0	−1	1	1
c_j	−1	−1	0	1	0	

	0	0	0	0	1	
	x_1	x_2	e_1	e_2	a_2	Valeur
e_1	0	1	1	1	−1	2
x_1	1	1	0	−1	1	1
c_j	0	0	0	0	1	

$e_1 = 2, e_2 = 0$ and $x_1 = 1, x_2 = 0$
C'est la solution initiale de deusième phase.

$$\begin{aligned} \min \quad & -x_1 - x_2 \\ & x_1 + 2x_2 + e_1 = 3 \\ & x_1 + x_2 - e_2 = 1 \\ & x_1, x_2 \geq 0 \\ & e_1, e2 \geq 0 \end{aligned} \tag{2}$$

	-1	-1	0	0	
	x_1	x_2	e_1	e_2	Valeur
e_1	0	1	1	1	2
x_1	1	1	0	-1	1
c_j	0	-1	0	1	

	-1	-1	0	0	
	x_1	x_2	e_1	e_2	Valeur
e_2	0	1	1	1	2
x_1	1	2	1	0	3
c_j	0	1	1	0	

$e_1 = 0, e_2 = 2$ and $x_1 = 3, x_2 = 0$

Question 2:

(a) (Uniqueness) $\frac{1}{a} \neq \frac{c}{1} \Rightarrow ac \neq 1$ and $c \neq 0$
(Optimality condition) $a \geq 1$ and $ac \leq 1$.
(Degeneracy) $a = 1$

(b) This happens when the objective function is parallel with one of the constraints, therefore, the optimal point will be all the points $1/a = c/1 \rightarrow ac = 1$ and $a > 0$ (parallel with the first constraint)
$c = 0$ and $a < 1$ if it is parallel with the second constraint.

Question 3:

2 phase algorithm will use to solve the problem. for the optimal solution we have $x_1 = 26/8$, $x_2 = 118/16$ and $Z = -58.75$

Question 3

phase 1 :

min $-9x_1 - 4x_2$

$5x_1 + 2x_2 \leq 31$

$-3x_1 + 2x_2 \leq 5$

$-2x_1 - 3x_2 \leq -1$

$x_i \geq 0$

min $-9x_1 - 4x_2$

$5x_1 + 2x_2 + e_1 = 31$

$-3x_1 + 2x_2 + e_2 = 5$

$-2x_1 - 3x_2 + e_3 = -1$

$+2x_1 + 3x_2 - e_3 = 1$

initial sol: $(0, 0, 31, 5, -1)$

X

infeasible sol.

$\Rightarrow$ min a_3

$5x_1 + 2x_2 + e_1 = 31$

$-3x_1 + 2x_2 + e_2 = 5$

$2x_1 + 3x_2 - e_3 + a_3 = 1$

		0	0	0	0	0	1		
		x_1	x_2	e_1	e_2	e_3	a_3		
0	e_1	5	2	1	0	0	0	31	31/2
0	e_2	-3	2	0	1	0	0	5	5/2
1	a_3	2	(3)	0	0	-1	1	1	1/3
		2	3	0	0	-1	1		
		-2	-3 ↑	0	0	1	0		

		0	0	0	0	0	1	
		x_1	x_2	e_1	e_2	e_3	a_3	
0	e_1	11/3	0	1	0	2/3	-2/3	91/3
0	e_2	-13/3	0	0	1	2/3	-2/3	13/3
0	x_2	2/3	1	0	0	-1/3	1/3	1/3
		0	0	0	0	0	0	
		0	0	0	0	0	1	

$\Rightarrow e_1 = 91/3$

$e_2 = 13/3$

$x_2 = 1/3$

$x_1 = 0$

$a_3 = 0$

$x_3 = 0$

→ end of phase 1

$\Rightarrow$ Solve the 2nd phase : optimal sol

$x_1^* = 26/8 \quad x_2^* = 118/16 \quad Z = -58.$