# Decision-Aid Methodologies in Transportation

## Price Optimization

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# Introduction

- Choice model captures demand
- Demand is elastic to price
- Predicted demand varies with price, if it is a variable of the model
- In principle, the probability to use/purchase an alternative decreases if the price increases.
- The revenue per user increases if the price increases.
- Question: what is the optimal price to optimize revenue?

In short:

- Price$\uparrow \Rightarrow$ profit/passenger$\uparrow$ and number of passengers $\downarrow$
- Price$\downarrow \Rightarrow$ profit/passenger$\downarrow$ and number of passengers $\uparrow$
- What is the best trade-off?

# Revenue calculation

Number of persons choosing alternative $i$ in the population

$$\hat{N}(i) = \sum_{s=1}^{S} N_s P(i|x_s, p_{is})$$

where

- $p_s$ is the price of item $i$ in segment $s$
- $x_s$ gathers all other variables corresponding to segment $s$
- the population is segmented into $S$ homogeneous strata
- $P(i|x_s, p_{is})$ is the choice model
- $N_s$ is the number of individuals in segment $s$

# Revenue calculation

The total revenue from $i$ is therefore:

$$R_i = \sum_{s=1}^{S} N_s P(i|x_s, p_{is}) p_{is}$$

If the price is constant across segments, we have

$$R_i = p_i \sum_{s=1}^{S} N_s P(i|x_s, p_i)$$

# Price optimization

Optimizing the price of product $i$ is solving the problem

$$\max_{p_i} p_i \sum_{s=1}^{S} N_s P(i|x_s, p_i)$$

Notes:

- It assumes that everything else is equal
- In practice, it is likely that the competition will also adjust the prices

# Illustrative example

A binary logit model with

$$\begin{aligned} V_1 &= \beta_p p_1 - 0.5 \\ V_2 &= \beta_p p_2 \end{aligned}$$

so that

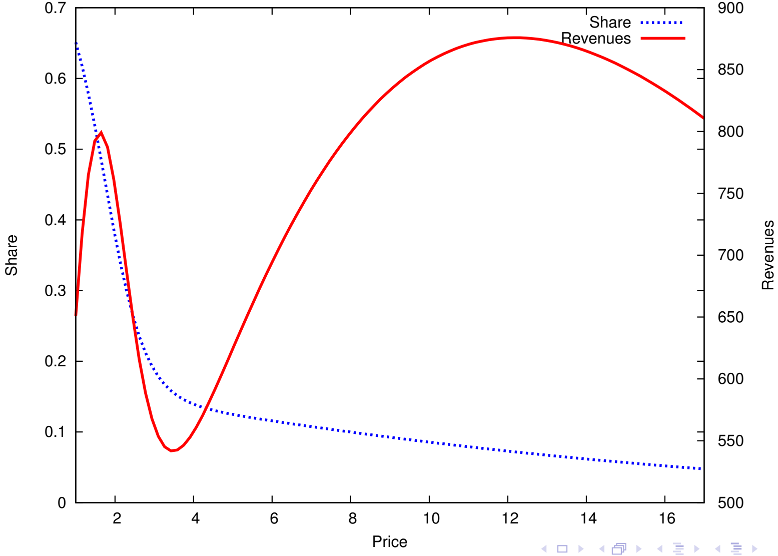
$$P(1|p) = \frac{e^{\beta_p p_1 - 0.5}}{e^{\beta_p p_1 - 0.5} + e^{\beta_p p_2}}$$

Two groups in the population:

- Group 1: $\beta_p = -2$, $N_s = 600$
- Group 2: $\beta_p = -0.1$, $N_s = 400$

Assume that $p_2 = 2$.

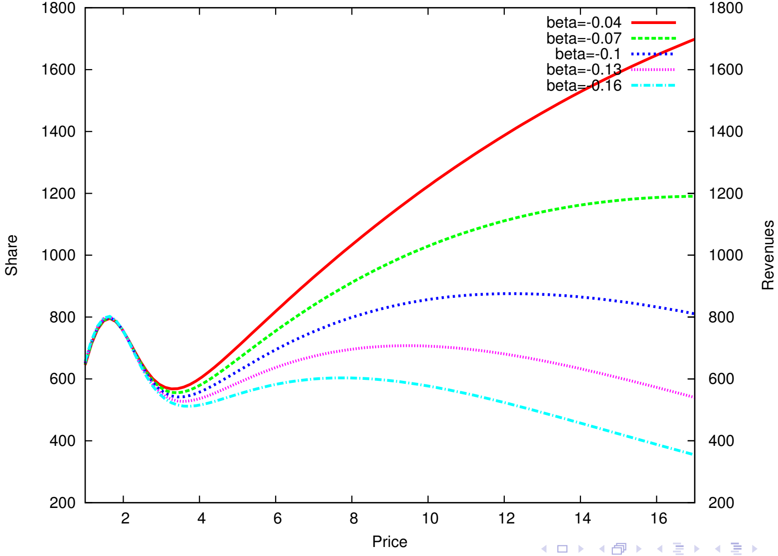
# Illustrative example



# Sensitivity analysis

- Parameters are estimated, we do not know the real value
- 95% confidence interval: $[\widehat{\beta}_p - 1.96\sigma, \widehat{\beta}_p + 1.96\sigma]$
- Perform a sensitivity analysis for $\beta_p$ in group 2

# Sensitivity analysis



# Summary

### Comments

- Typical non concavity of the revenue function due to taste heterogeneity.
- In general, decision making is more complex than optimizing revenues.
- Applying the model with values of $x$ very different from estimation data may be highly unreliable.
- accounting for market organization and type of competition strongly affects the problem to model