
Exercise session 4
Routing a fleet of heterogeneous trucks

1 Introduction

During the last two lab sessions, we discussed how the use of electric public transportation can help curb CO₂ in urban areas. However, optimization can also be used to improve the efficiency of freight transportation. According to Eurostat (2015) inland freight transportation in the EU was close to 2'100 billion tonne-kilometers for 2012, of which 75.1% were transported by road. Routing improvements in road freight transportation can thus lead to lower CO₂ emissions, cost savings, and less congestion on the road infrastructure.

2 Problem statement

A small distributor of sporting equipment owns the 3 trucks whose characteristics are described in table 1. The trucks' capacities are expressed in terms of number of standard boxes that the equipment is packaged in. The trucks have different capacities, and also different average speeds. On a given day, the trucks have to deliver sporting equipment to 20 stores located in different cities and towns in the canton of Vaud. The equipment is heavy, and a crane is used to unload the boxes. Because a crane is not available at all stores, some of them require the truck that delivers the equipment to have it. Also, the parking space at some stores is limited, and therefore they require that the truck should not have a trailer.

In this context, table 1 lists the above truck attributes as follows:

- solo: 1 if the truck does not have a trailer, 0 if it has a trailer
- crane: 1 if the truck has a crane, 0 if it does not

Table 1: Truck characteristics

| truck | capacity | speed | solo | crane |
|---------|-----------|---------|------|-------|
| truck 1 | 150 boxes | 70 kmph | 0 | 1 |
| truck 2 | 100 boxes | 70 kmph | 1 | 1 |
| truck 3 | 80 boxes | 90 kmph | 1 | 0 |

The stores have also imposed the following service constraints:

- Orders are indivisible, i.e. they must be fully delivered in one single visit.

- The stores have to plan ahead for the deliveries and commit additional personnel. Therefore, they have specified delivery intervals which have to be respected. These are referred to as time windows. A time window for a store i is defined by its lower bound λ_i and its upper bound μ_i in terms of hours since midnight. If the truck arrives to the store before λ_i it has to wait. Also, the delivery operation cannot continue after μ_i . To give an example, suppose store i specifies a time window [13:00, 15:00], and it is estimated that the service duration is 30 min. Then, the delivery can start between 13:00 and 14:30.

The distributor's depot is in Lausanne, where all trucks are initially located. Trucks can start their tours at 8:00 at the earliest and are required to be back at the depot at 18:00 at the latest. Moreover, every trucks' schedule is limited to 8 hours per day, including waiting time.

The company has mandated you to develop a decision support tool that minimizes the operational cost of the vehicle routing problem. Two costs are considered - 4 CHF per kilometer traveled and 80 CHF per hour worked.

3 Tasks

1. Read carefully the description of the problem and propose a schematic representation of it. Determine the decision variables and the objective function (descriptive).
2. Based on the defined framework, provide the description of the constraints that have to be respected.
3. Beside the provided information, determine what type of additional information is required in order to present the problem mathematically.
4. Provide the mathematical description of the problem in the following order:
 - (a) Define sets and parameters
 - (b) Define variables
 - (c) Objective function
 - (d) Constraints
5. Implement your mathematical model in CPLEX with the provided data.
6. You will find that your model is taking too long to solve. We can try to improve the solution time using the following techniques:
 - Providing better bounds: better lower bounds lead to faster elimination of branches. What are some bounds that we can provide as constraints in the formulation?
 - Variable fixing: some variables are certain to take specific values. By fixing them from the start, we can eliminate the evaluation of many combinations.
7. The company would like to know how much waiting time accrues for each truck. You are provided with a very simple post-processing script that prints a detailed solution. You are asked to extend it to report waiting time as well.