Decision-aid Methodologies in Transportation
A Brief Introduction to VRP Heuristics

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Overview

Introduction

- What is VRP?
  - Proposed by Dantzig and Ramser (1959), the vehicle routing problem, or VRP, is a combinatorial optimization and integer programming problem that seeks to find the most efficient utilization and routing of a vehicle fleet to service a set of customers subject to an array of constraints.
  - It is one of the most practically applicable and widely studied problems in Operations Research.
- Types of VRP:
  - VRP with time windows
  - Open VRP
  - Site-dependent VRP
  - Multi-depot VRP
  - Pickup and delivery problem
  - Dial-a-ride problem
  - Location-routing problem
  - Heterogeneity of fleets, loads, maximum travel time, etc...

- Using standalone models for complex real-life problems is impossibly slow.
- Even state-of-the-art exact methods will struggle on larger instances.
- Consider the following situation:
  - It takes you on average 1 day to solve a complex real-world instance. Not so bad, right?
  - But what if you have to solve it every day?
- Short planning horizons mean that we need to get solutions fast, even at the expense of sacrificing optimality.
- Nevertheless, well-implemented heuristics can consistently reach results within a very small margin of optimality, e.g. 5%, in relatively short computation times.
- However, they don't have the ability to prove whether an obtained solution is optimal or how far it is from optimality.

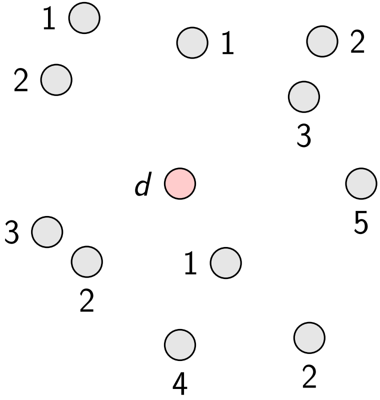
- What are the choices we have when we go about solving complex real-life problems?
- Rules of thumb:
  - Rules that have historically proved to work well in a given environment.
  - For example, don't add a customer to an existing route if its addition would result in a deviation of more than X km.
- Heuristics
  - Wikipedia: a technique designed for solving a problem more quickly when classic methods are too slow, [...] trading optimality, completeness, accuracy, or precision for speed.
- Metaheuristics
  - Wikipedia: a higher-level procedure or heuristic designed to find, generate, or select a lower-level procedure or heuristic [...] that may provide a sufficiently good solution to an optimization problem.
- Hybrid techniques - a mixture of exact and heuristic methods
  - For example, use a heuristic to assign customers to vehicles.
  - Then use an exact method to solve a TSP for each vehicle and only the customers assigned to it.

Construction Heuristics

- Starting from a pool of unassigned customers and a pool of available vehicles, construct each vehicle's route. Many different approaches exists, such as:
  - Sequential constriction
  - Parallel construction
  - Semi-parallel construction
- Complications:
  - Time windows
  - Precedence constraints
  - Incompatibility constraints
  - Accessibility constraints
  - Maximum travel time constraints
  - Maximum time spent on vehicle constraints
  - ...
- Examples:
  - Greedy construction
  - Savings algorithm (Clarke and Wright, 1964)
  - Sweep algorithm (Gillett and Miller, 1974)
  - and many many more...including customized and problem specific approaches

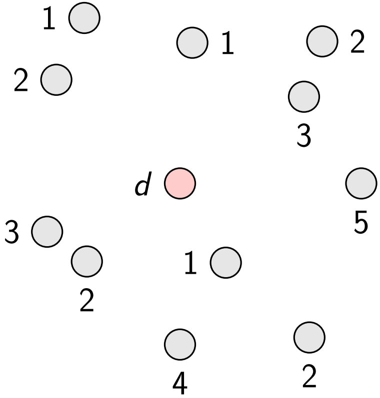
- Consider the following problem:
  - A single depot
  - A set of customers with known demands (node labels)
  - An unlimited homogeneous fleet, where each vehicle has a capacity of 10 units
  - The distances between all pairs of customers or customer-depot are known in advance.
  - Construct the minimum-length tours to service all customers.

Greedy Construction



- A greedy heuristic is one that always takes the next best step and never looks back.
- In the tour construction context, we did the following:
  1. Start from the depot.
  2. Always go to the next closest customer
  3. When demand of the next closest will exceed capacity, go back to the depot and start another tour.
- It is easy and straightforward to implement in most contexts.
- However, it usually leads to poor results, see for example the tour that crosses itself twice.
- We obtained a total tour length of **493** km.

Sweep Construction



- To construct the tours, we rotated counterclockwise an imaginary ray starting from the depot.
- Each new revealed point is connected to the previous one, going through the depot if capacity would be exceeded otherwise.
- We assumed that we have the geographic coordinates of the customers, not just distances between them.
- We also assumed that the depot is situated more or less in the center of the customer nodes.
- Solutions may be very different depending on the initial position of the ray. We started at 0°
- ...and obtained a total tour length of **460** km.

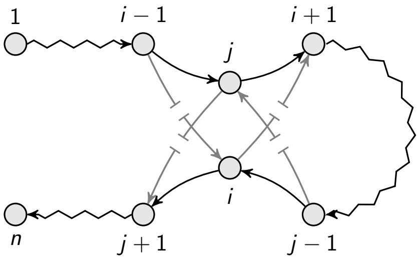
Improvement Heuristics

- As good as your construction heuristics may be, the tours will rarely be close to optimal.
- To improve them, you apply what are often called improvement heuristics.
- We can divide those into two very broad classes:
  - Genetic and evolutionary algorithms
  - Local search algorithms
- Most of the research on VRP has focused on local search algorithms, although recently genetic algorithms have produced remarkable results, see e.g. the work of Thibaut Vidal `http://w1.cirrelt.ca/~vidalt/en/home-thibaut-vidal.html`

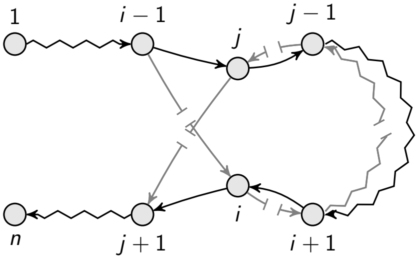
- What is local search?
  - An exploration of the immediate neighbors of an incumbent solution.
  - A neighbor is a solution that can be reached by a relatively simple well-defined manipulation, known as a move or an operator.
- Examples of local search algorithms include:
  - Iterated local search
  - Variable neighborhood search
  - Tabu search
  - ...
- So let's see some local search operators!

Single-tour Improvement

Single-tour 1-1 exchange

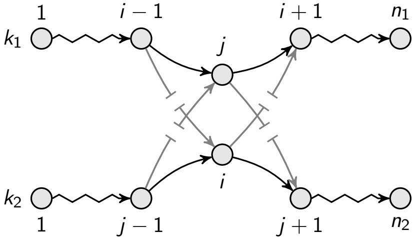


Single-tour 2-opt

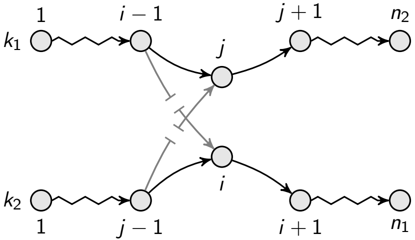


Multi-tour Improvement

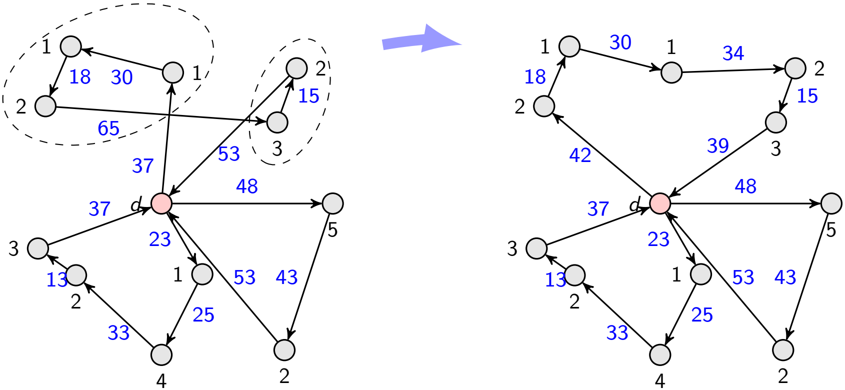
Multi-tour 1-1 exchange



Multi-tour 2-opt



- Take the example of the tours produced by the greedy heuristic.
- There is a tour that would definitely benefit from the application of a 2-opt operator. . . twice!
- We have reduced the total tour length by **40** km, from **493** km down to **453** km.



- Implementations of these algorithms run for many iterations until some stopping criterion/criteria are met, for example a maximum number of iterations.
- The search is guided by rules such as:
  - Maximum number of iterations for each operator
  - Maximum number of non-improving iterations for each operator
  - When to change from one operator to another
  - How to balance between diversification and intensification
  - ...
- The search may be designed to produce feasible neighbors only or to go through infeasible intermediate solutions with the goal of recovering feasibility with some techniques.
- **In the end, the goal is to try to avoid getting stuck at local optima!**
- **. . . and of course to be efficient and fast!**

Try it yourself

- HeuristicLab `http://dev.heuristiclab.com/trac/hl/core`
  - A framework for heuristic and evolutionary algorithms developed by members of the Heuristic and Evolutionary Algorithms Laboratory, Upper Austria University of Applied Sciences.
  - Written in C#
- OsmSharp `http://www.osmsharp.com`
  - A framework for map rendering, map data processing, routing and logistics optimization developed by Ben Abelshausen.
  - Written in C#
- Concorde TSP solver `http://www.math.uwaterloo.ca/tsp/concorde`
- Others? `http://wiki.openstreetmap.org/wiki/Routing`

Thank you!

Clarke, G. and Wright, J. W. (1964). Scheduling of vehicles from a central depot to a number of delivery points. *Operations Research*, 12:568–581.

Dantzig, G. and Ramser, R. (1959). The truck dispatching problem. *Management Science*, 6:80–91.

Gillett, B. E. and Miller, L. R. (1974). A heuristic algorithm for the vehicle-dispatch problem. *Operations Research*, 22(2):340–349.