

---

# Choice theory

Michel Bierlaire

`michel.bierlaire@epfl.ch`

Transport and Mobility Laboratory

# Framework

---

Choice: outcome of a sequential decision-making process

- Definition of the choice problem: **How do I get to EPFL?**
- Generation of alternatives: **car as driver, car as passenger, train**
- Evaluation of the attributes of the alternatives: **price, time, flexibility, comfort**
- Choice: **decision rule**
- Implementation: **travel**

# Framework

---

A choice theory defines

1. decision maker
2. alternatives
3. attributes of alternatives
4. decision rule

# Framework

---

## Decision-maker :

- Individual or a group of persons
- If group of persons, we ignore internal interactions
- Important to capture difference in tastes and decision-making process
- Socio-economic characteristics: age, gender, income, education, etc.

# Framework

---

## Alternatives :

- Environment: *universal choice set* ( $\mathcal{U}$ )
- Individual  $n$ : *choice set* ( $\mathcal{C}_n$ )

## Choice set generation:

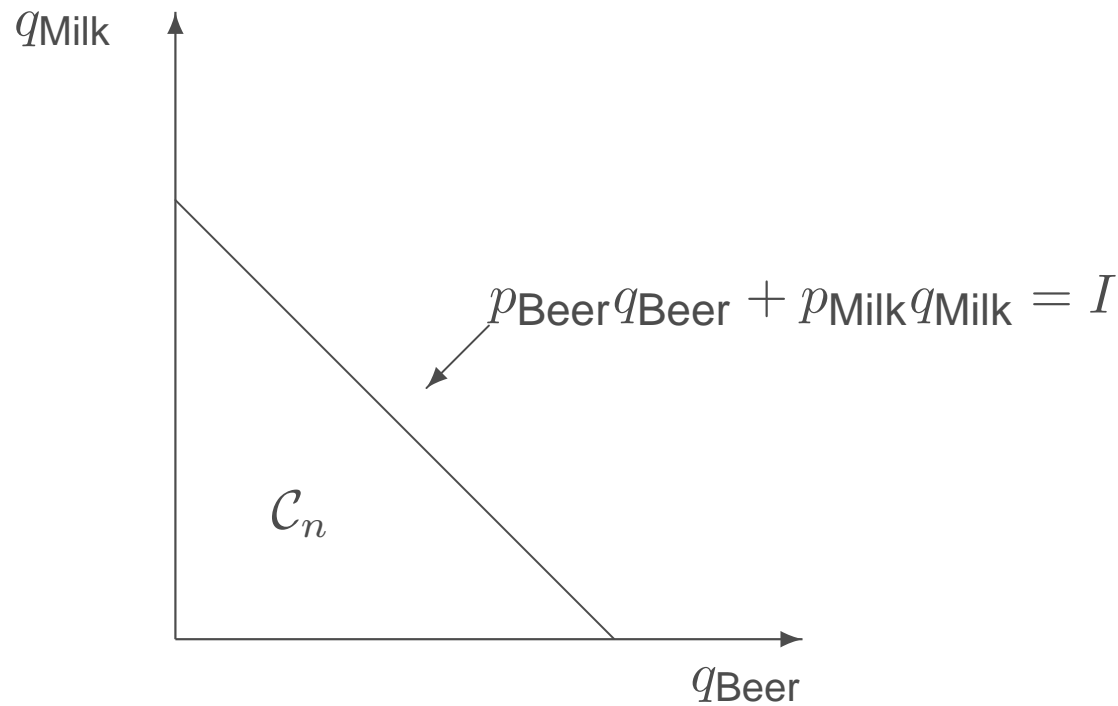
- Availability
- Awareness

**Swait, J. (1984)** *Probabilistic Choice Set Formation in Transportation Demand Models*  
Ph.D. dissertation, Department of Civil Engineering, MIT, Cambridge, Ma.

# Framework

## Continuous vs. discrete

Continuous choice set:



Discrete choice set:

$$C_n = \{ \text{Car, Bus, Bike} \}$$

# Framework

---

## Attributes

- cost
  - travel time
  - walking time
  - comfort
  - bus frequency
  - etc.
- ✓ Generic vs. specific
  - ✓ Quantitative vs. qualitative
  - ✓ Perception

# Framework

---

## Decision rules

Neoclassical economic theory

Preference-indifference operator  $\succsim$

(i) reflexivity

$$a \succsim a \quad \forall a \in \mathcal{C}_n$$

(ii) transitivity

$$a \succsim b \text{ and } b \succsim c \Rightarrow a \succsim c \quad \forall a, b, c \in \mathcal{C}_n$$

(iii) comparability

$$a \succsim b \text{ or } b \succsim a \quad \forall a, b \in \mathcal{C}_n$$



# Framework

---

## Decision rules

### Neoclassical economic theory (ctd)

#### 👉 Numerical function

$\exists U_n : \mathcal{C}_n \longrightarrow \mathbb{R} : a \rightsquigarrow U_n(a)$  such that

$$a \succeq b \Leftrightarrow U_n(a) \geq U_n(b) \quad \forall a, b \in \mathcal{C}_n$$

Utility

# Framework

---

## Decision rules

- Utility is a latent concept
- It cannot be directly observed

# Framework

---

## Continuous choice set

- $Q = \{q_1, \dots, q_L\}$  consumption bundle
- $q_i$  is the quantity of product  $i$  consumed
- Utility of the bundle:

$$U(q_1, \dots, q_L)$$

- $Q_a \succeq Q_b$  iff  $U(q_1^a, \dots, q_L^a) \geq U(q_1^b, \dots, q_L^b)$
- Budget constraint:

$$\sum_{i=1}^L p_i q_i \leq I.$$

# Framework

---

Decision-maker solves the optimization problem

$$\max_{q \in \mathbb{R}^L} U(q_1, \dots, q_L)$$

subject to

$$\sum_{i=1}^L p_i q_i = I.$$

Example with two products...

# Framework

---

$$\max_{q_1, q_2} U = \beta_0 q_1^{\beta_1} q_2^{\beta_2}$$

subject to

$$p_1 q_1 + p_2 q_2 = I.$$

Lagrangian of the problem:

$$L(q_1, q_2, \lambda) = \beta_0 q_1^{\beta_1} q_2^{\beta_2} + \lambda(I - p_1 q_1 - p_2 q_2).$$

Necessary optimality condition

$$\nabla L(q_1, q_2, \lambda) = 0$$

# Framework

---

Necessary optimality conditions

$$\begin{aligned}\beta_0 \beta_1 q_1^{\beta_1 - 1} q_2^{\beta_2} &- \lambda p_1 &= 0 \\ \beta_0 \beta_2 q_1^{\beta_1} q_2^{\beta_2 - 1} &- \lambda p_2 &= 0 \\ p_1 q_1 + p_2 q_2 &- I &= 0.\end{aligned}$$

We have

$$\begin{aligned}\beta_0 \beta_1 q_1^{\beta_1} q_2^{\beta_2} &- \lambda p_1 q_1 &= 0 \\ \beta_0 \beta_2 q_1^{\beta_1} q_2^{\beta_2} &- \lambda p_2 q_2 &= 0\end{aligned}$$

so that

$$\lambda I = \beta_0 q_1^{\beta_1} q_2^{\beta_2} (\beta_1 + \beta_2)$$

# Framework

---

Therefore

$$\beta_0 q_1^{\beta_1} q_2^{\beta_2} = \frac{\lambda I}{(\beta_1 + \beta_2)}$$

As  $\beta_0 \beta_2 q_1^{\beta_1} q_2^{\beta_2} = \lambda p_2 q_2$ , we obtain (assuming  $\lambda \neq 0$ )

$$q_2 = \frac{I \beta_2}{p_2 (\beta_1 + \beta_2)}$$

Similarly, we obtain

$$q_1 = \frac{I \beta_1}{p_1 (\beta_1 + \beta_2)}$$

# Framework

---

$$q_1 = \frac{I\beta_1}{p_1(\beta_1 + \beta_2)}$$

$$q_2 = \frac{I\beta_2}{p_2(\beta_1 + \beta_2)}$$

Demand functions



# Framework

---

Discrete choice set

- Similarities with **Knapsack problem**
- Calculus cannot be used anymore

$$U = U(q_1, \dots, q_L)$$

with

$$q_i = \begin{cases} 1 & \text{if product } i \text{ is chosen} \\ 0 & \text{otherwise} \end{cases}$$

and

$$\sum_i q_i = 1.$$

# Framework

---

- Do not work with demand functions anymore
- Work with utility functions
- $U$  is the “global” utility
- Define  $U_i$  the utility associated with product  $i$ .
- It is a function of the attributes of the product (price, quality, etc.)
- We say that product  $i$  is chosen if

$$U_i \geq U_j \quad \forall j.$$

# Framework

---

Example: two transportation modes

$$U_1 = -\beta t_1 - \gamma c_1$$

$$U_2 = -\beta t_2 - \gamma c_2$$

with  $\beta, \gamma > 0$

$$U_1 \geq U_2 \text{ iff } -\beta t_1 - \gamma c_1 \geq -\beta t_2 - \gamma c_2$$

that is

$$-\frac{\beta}{\gamma} t_1 - c_1 \geq -\frac{\beta}{\gamma} t_2 - c_2$$

or

$$c_1 - c_2 \leq -\frac{\beta}{\gamma} (t_1 - t_2)$$

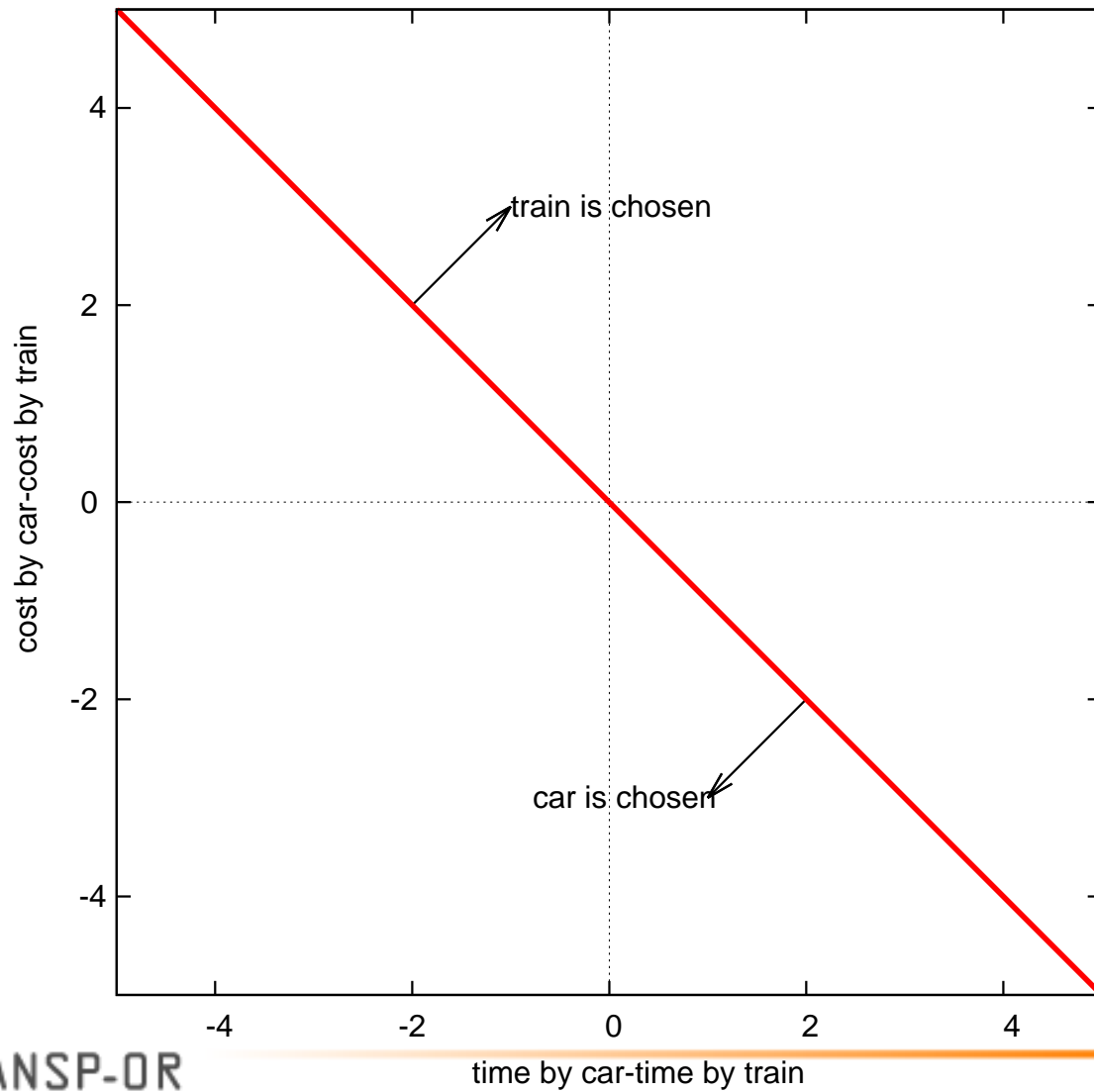
# Framework

---

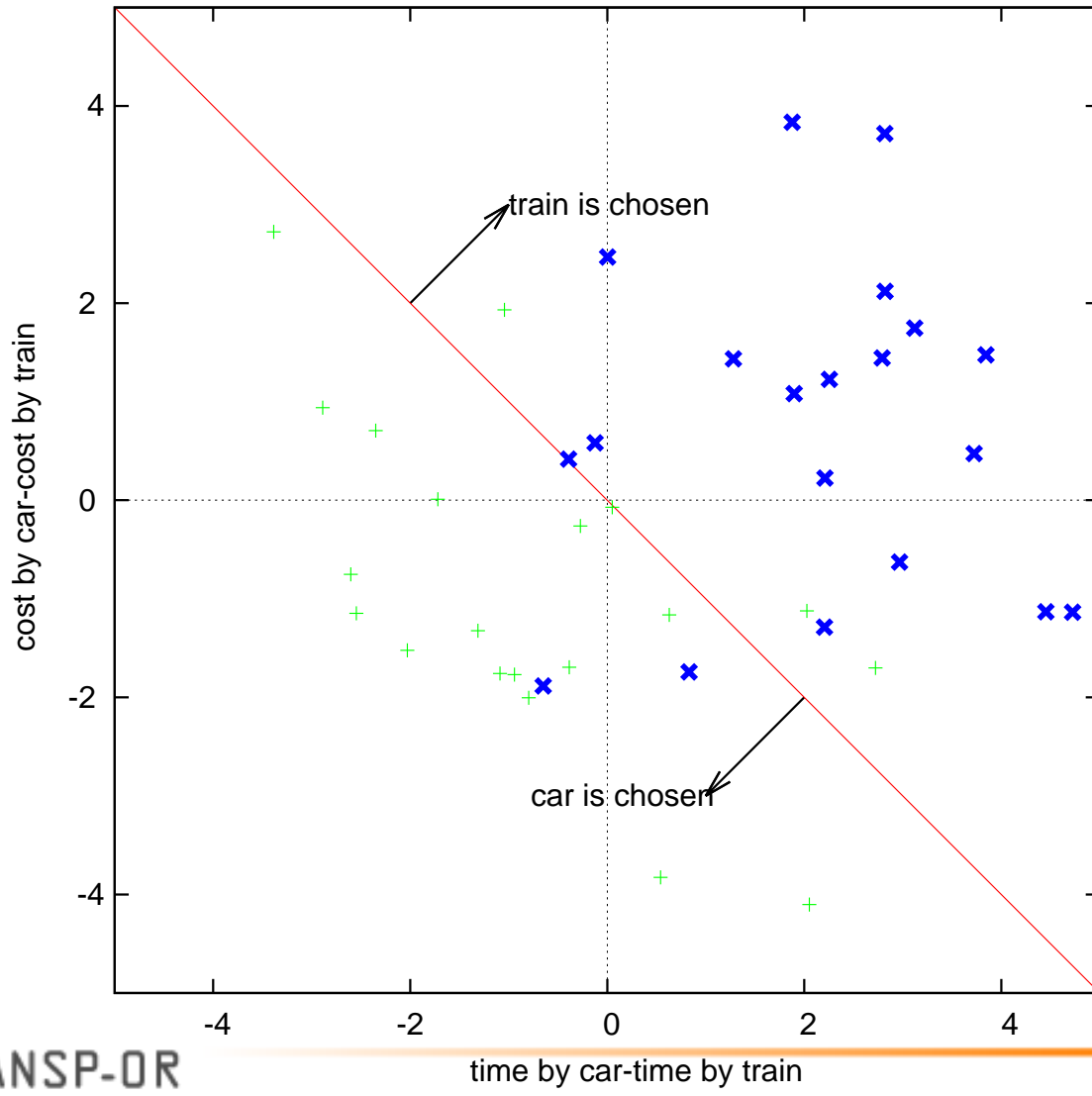
Obvious cases:

- $c_1 \geq c_2$  and  $t_1 \geq t_2$ : 2 dominates 1.
- $c_2 \geq c_1$  and  $t_2 \geq t_1$ : 1 dominates 2.
- Trade-offs in over quadrants

# Framework



# Framework



# Assumptions

---

## Decision rules

Neoclassical economic theory (ctd)

Decision-maker

- ✓ perfect discriminating capability
- ✓ full rationality
- ✓ permanent consistency

Analyst

- ✓ knowledge of all attributes
- ✓ perfect knowledge of  $\succsim$  (or  $U_n(\cdot)$ )
- ✓ no measurement error

# Assumptions

---

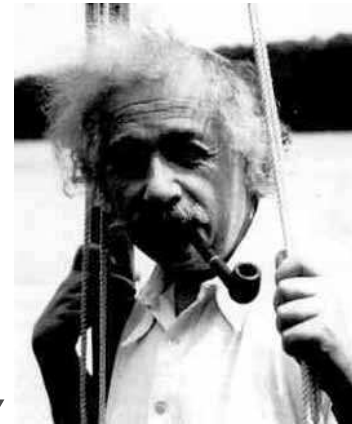
## Uncertainty

Source of uncertainty?

- ➔ Decision-maker: stochastic decision rules
- ➔ Analyst: lack of information



- ➔ Bohr: *“Nature is stochastic”*
- ➔ Einstein: *“God does not play dice”*





# Assumptions

---

Lack of information: random utility models

**Manski 1973** The structure of Random Utility Models *Theory and Decision* 8:229–254

Sources of uncertainty:

- Unobserved attributes
- Unobserved taste variations
- Measurement errors
- Instrumental variables

For each individual  $n$ ,

$$U_{in} = V_{in} + \varepsilon_{in}$$

and

$$P(i|\mathcal{C}_n) = P[U_{in} = \max_{j \in \mathcal{C}_n} U_{jn}] = P(U_{in} \geq U_{jn} \forall j \in \mathcal{C}_n)$$

# Random utility models

---

$$U_{in} = V_{in} + \varepsilon_{in}$$

- Dependent variable is latent
- Only differences matter

$$\begin{aligned} P(i|\mathcal{C}_n) &= P(U_{in} \geq U_{jn} \forall j \in \mathcal{C}_n) \\ &= P(U_{in} + K \geq U_{jn} + K \forall j \in \mathcal{C}_n) \quad \forall K \in \mathbb{R} \end{aligned}$$

$$\begin{aligned} P(i|\mathcal{C}_n) &= P(U_{in} \geq U_{jn} \forall j \in \mathcal{C}_n) \\ &= P(\lambda U_{in} \geq \lambda U_{jn} \forall j \in \mathcal{C}_n) \quad \forall \lambda > 0 \end{aligned}$$