Decision aid methodologies in Transportation

Tutorial on Column Generation
8th May, 2012

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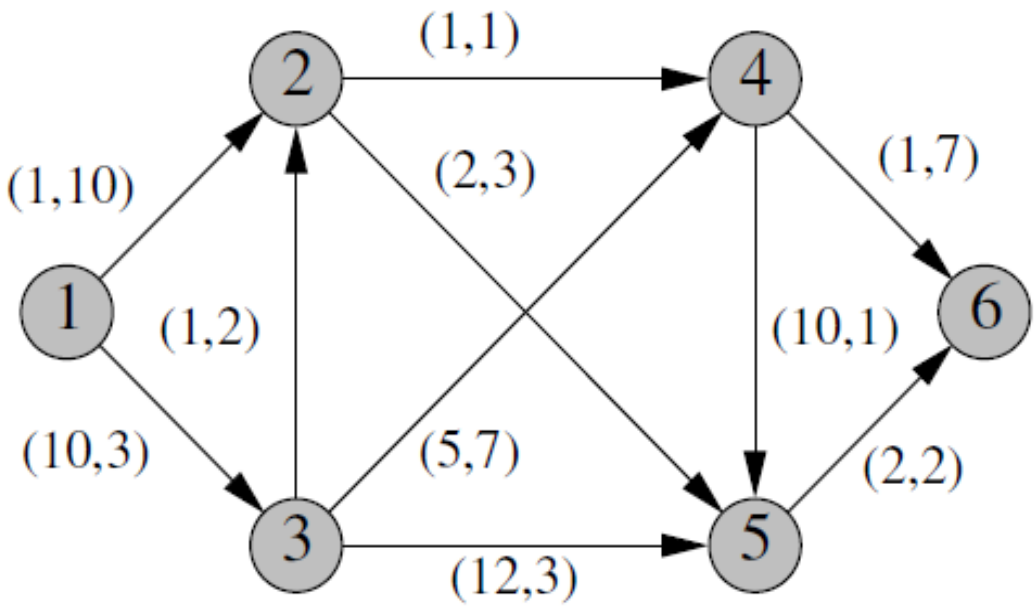
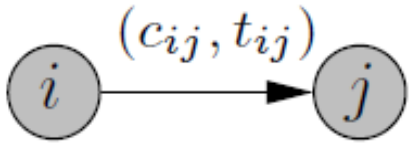
Time Constrained Shortest Path Problem





Find the shortest path from node 1 to node 6 such that the total travel time does not exceed 14 units (Ahuja et al., 1993, p.599**)**

Integer Programming Formulation

$$z^\star := \min \sum_{(i,j)\in A} c_{ij} x_{ij} \tag{1.1}$$

$$\text{subject to} \sum_{j:(1,j)\in A} x_{1j} = 1 \tag{1.2}$$

$$\sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} = 0 \qquad i = 2,3,4,5 \tag{1.3}$$

$$\sum_{i:(i,6)\in A} x_{i6} = 1 \tag{1.4}$$

$$\sum_{(i,j)\in A} t_{ij} x_{ij} \leq 14 \tag{1.5}$$

$$x_{ij} = 0 \text{ or } 1 \quad (i,j) \in A \tag{1.6}$$

Solution: The optimal integer solution is path 13246 of cost 13 with a traversal time of 13.

Source: Desrosiers et. al (2005)

Master problem

$$z^\star = \min \sum_{p \in P} \Big(\sum_{(i,j) \in A} c_{ij} x_{pij} \Big) \lambda_p \tag{1.10}$$

$$\text{subject to} \sum_{p \in P} \Big(\sum_{(i,j) \in A} t_{ij} x_{pij} \Big) \lambda_p \leq 14 \tag{1.11}$$

$$\sum_{p \in P} \lambda_p = 1 \tag{1.12}$$

$$\lambda_p \geq 0 \qquad p \in P \tag{1.13}$$

$$\sum_{p \in P} x_{pij} \lambda_p = x_{ij} \qquad (i,j) \in A \tag{1.14}$$

$$x_{ij} = 0 \text{ or } 1 \quad (i,j) \in A\,. \tag{1.15}$$

Source: Desrosiers et. al (2005)

LP Relaxation of the master problem

$$\min \sum_{p \in P} (c_p \lambda_p)$$

$$\sum_{p \in P} t_p \lambda_p \leq 14$$

$$\sum_{p \in P} \lambda_p = 1$$

$$\lambda_p \geq 0 \qquad \forall p \in P$$

Subproblem

Find negative reduced cost columns to enter the basis

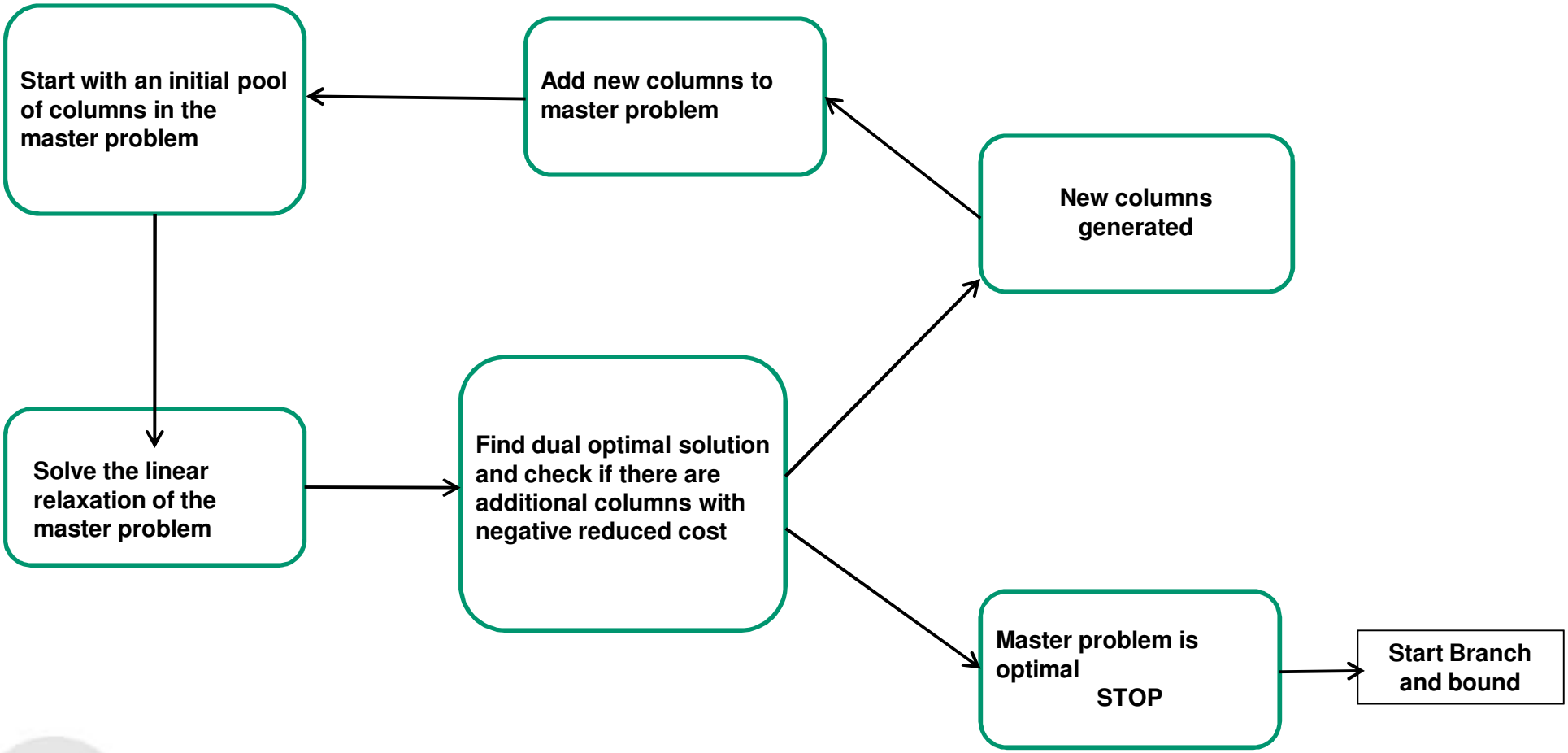
$$\bar{c}_p = \sum_{(i,j)\in A} c_{ij} x_{pij} - \pi_1 \Big(\sum_{(i,j)\in A} t_{ij} x_{pij} \Big) - \pi_0 \ < 0.$$

This is equivalent to solving the following subproblem:

$$\bar{c}^\star = \min_{(1.2)\text{–}(1.4),\ (1.6)} \sum_{(i,j)\in A} (c_{ij} - \pi_1 t_{ij}) x_{ij} - \pi_0.$$

Source: Desrosiers et. al (2005)

Column Generation Summary



Results

Iteration	Master Solution	$\bar{z}$	π_0	π_1	$\bar{c}^\star$	p	c_p	t_p
BB0.1	$y_0 = 1$	100.0	100.00	0.00	−97.0	1246	3	18
BB0.2	$y_0 = 0.22, \lambda_{1246} = 0.78$	24.6	100.00	−5.39	−32.9	1356	24	8
BB0.3	$\lambda_{1246} = 0.6, \lambda_{1356} = 0.4$	11.4	40.80	−2.10	−4.8	13256	15	10
BB0.4	$\lambda_{1246} = \lambda_{13256} = 0.5$	9.0	30.00	−1.50	−2.5	1256	5	15
BB0.5	$\lambda_{13256} = 0.2, \lambda_{1256} = 0.8$	7.0	35.00	−2.00	0			
Arc flows: $x_{12} = 0.8, x_{13} = x_{32} = 0.2, x_{25} = x_{56} = 1$								

Source: Desrosiers et. al (2005)

References

- Ahuja, R.K., Magnanti, T.L., and Orlin, J.B. (1993). *Network Flows: Theory, Algorithms and Applications.* Prentice Hall, Inc., Englewood Cliffs, New Jersey 07632.

- J. Desrosiers and M. E. Lübbecke, A primer in column generation, in Column Generation, G. Desaulniers, J. Desrosiers, and M. M. Solomon, eds., Springer-Verlag, New York, 2005, ch. 1, pp. 1–32.